Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.2-trig^m-a-trig+b-trig-^n

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [294]. This is test number [136].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (294)	% 0. (0)
Mathematica	% 100. (294)	% 0. (0)
Maple	% 98.3 (289)	% 1.7 (5)
Maxima	% 78.23 (230)	% 21.77 (64)
Fricas	% 98.64 (290)	% 1.36 (4)
Sympy	% 20.75 (61)	% 79.25 (233)
Giac	% 95.92 (282)	% 4.08 (12)

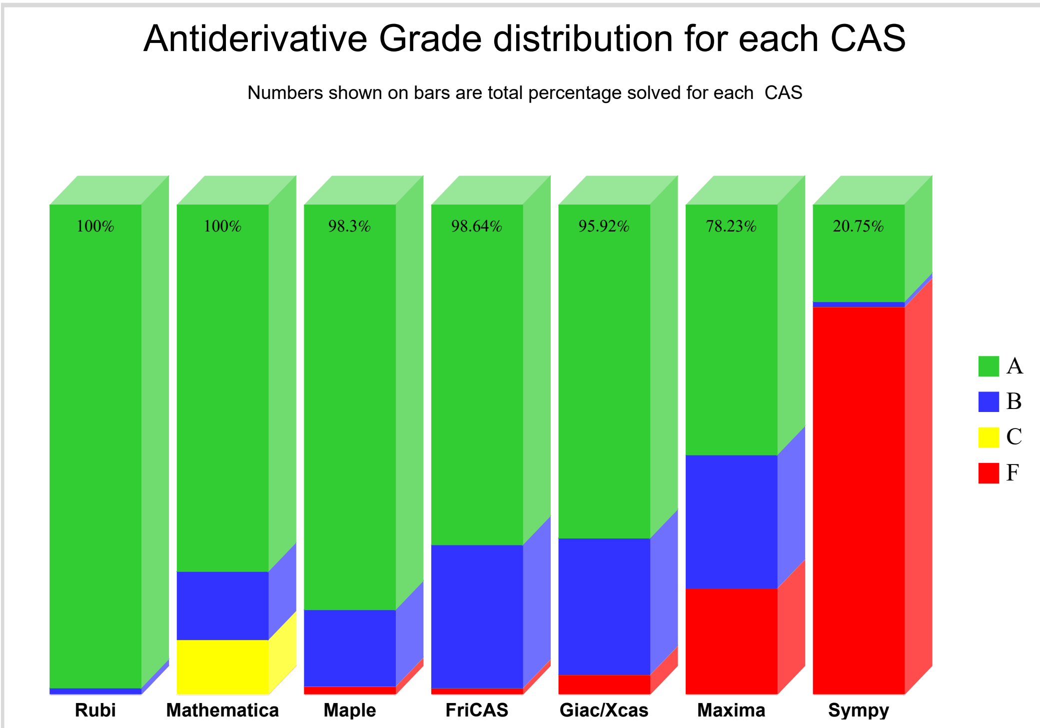
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

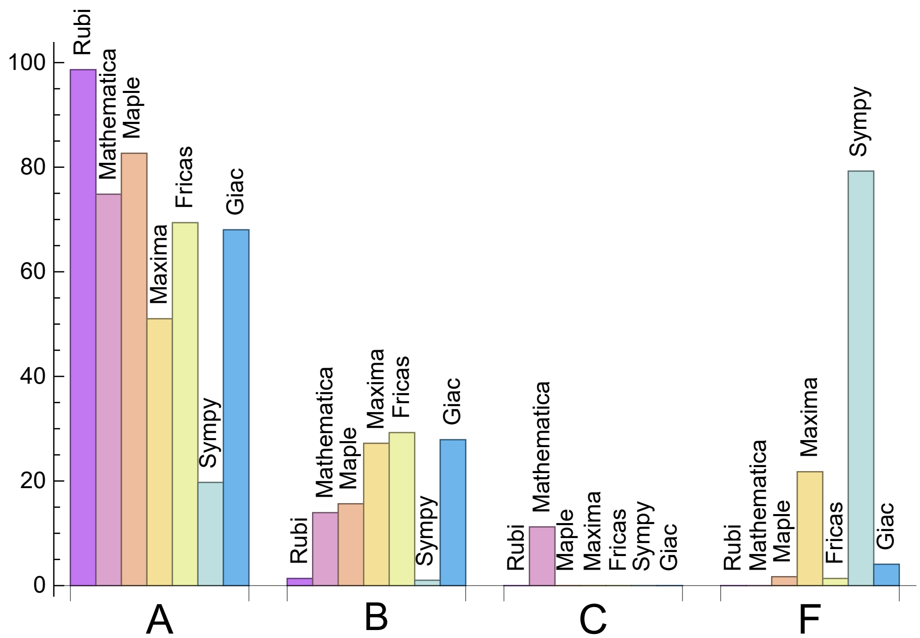
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	98.64	1.36	0.	0.
Mathematica	74.83	13.95	11.22	0.
Maple	82.65	15.65	0.	1.7
Maxima	51.02	27.21	0.	21.77
Fricas	69.39	29.25	0.	1.36
Sympy	19.73	1.02	0.	79.25
Giac	68.03	27.89	0.	4.08

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.19	112.64	1.04	86.5	1.
Mathematica	0.95	154.64	1.47	95.	1.02
Maple	0.12	158.35	1.43	109.	1.22
Maxima	1.29	200.23	2.63	138.	1.61
Fricas	0.56	404.23	4.16	278.5	3.2
Sympy	6.78	211.11	2.01	153.	1.88
Giac	2.35	296.59	2.94	160.	1.99

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica $\{29, 100\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

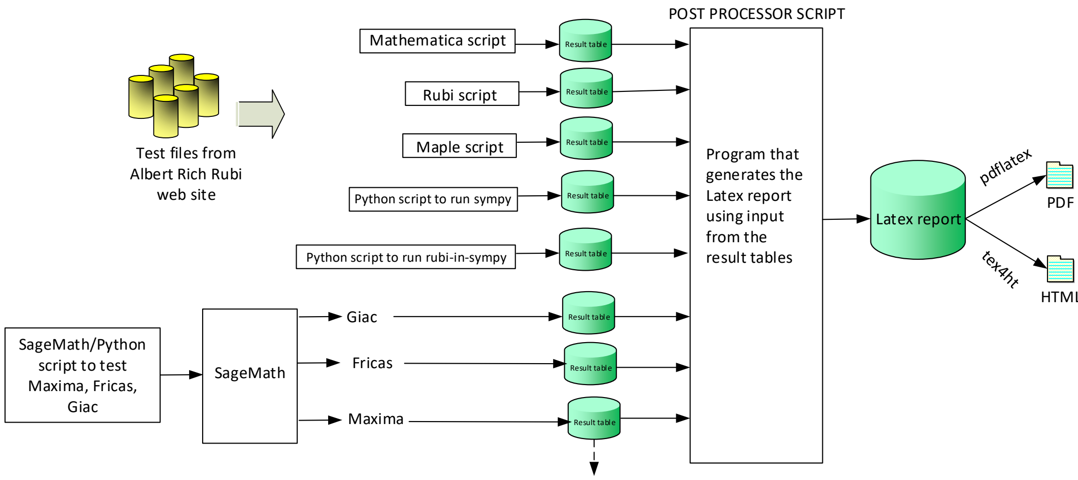
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { 15, 23, 131, 142 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 23, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 67, 69, 71, 73, 74, 75, 76, 78, 79, 80, 82, 87, 89, 90, 91, 92, 93, 95, 96, 97, 101, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 120, 122, 123, 125, 126, 127, 128, 130, 133, 136, 137, 138, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 189, 190, 194, 195, 196, 197, 198, 201, 202, 204, 206, 208, 210, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 269, 270, 271, 272, 273, 274, 275, 277, 279, 281, 283, 285, 287, 288, 289, 291, 293, 294 }

B grade: { 6, 21, 24, 63, 66, 68, 70, 72, 81, 84, 85, 86, 88, 98, 99, 100, 103, 104, 105, 119, 121, 134, 143, 169, 171, 173, 179, 185, 188, 191, 192, 193, 199, 200, 203, 205, 207, 209, 211, 241, 242 }

C grade: { 8, 10, 16, 22, 25, 29, 50, 65, 77, 83, 94, 102, 112, 124, 129, 131, 132, 135, 139, 141, 142, 215, 264, 267, 268, 276, 278, 280, 282, 284, 286, 290, 292 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20, 22, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 105, 106, 107, 108, 109, 111, 113, 114, 115, 116, 118, 120, 123, 124, 125, 126, 127, 128, 130, 131, 132, 134, 135, 136, 138, 140, 141, 143, 145, 147, 149, 150, 152, 153, 154, 155, 156, 158, 160, 162, 164, 165, 166, 167, 168, 169, 170, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 185, 188, 189, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 206, 207, 208, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 273, 275, 277, 278, 279, 281, 283, 284, 285, 286, 287, 289, 290, 291, 293, 294 }

B grade: { 8, 13, 21, 23, 24, 25, 67, 85, 103, 104, 110, 112, 117, 119, 121, 122, 129, 133, 137, 139, 142, 144, 146, 148, 151, 157, 159, 161, 163, 171, 173, 180, 184, 186, 190, 197, 203, 205, 209, 211, 249, 276, 280, 282, 288, 292 }

C grade: { }

F grade: { 29, 187, 271, 272, 274 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 16, 18, 20, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 124, 126, 128, 130, 143, 145, 147, 149, 155, 167, 168, 170, 172, 174, 178, 179, 180, 181, 193, 198, 200, 201, 202, 206, 208, 212, 214, 217, 218, 219, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 271, 284

B grade: { 8, 10, 12, 14, 22, 24, 26, 28, 67, 85, 104, 110, 112, 114, 116, 118, 120, 122, 132, 134, 136, 138, 140, 141, 156, 157, 158, 159, 160, 161, 162, 169, 171, 173, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 203, 204, 205, 207, 209, 210, 211, 213, 215, 216, 220, 221, 222, 223, 226, 228, 264, 265, 266, 267, 268, 269, 270, 276, 278, 280, 282, 286, 288, 290, 292 }

C grade: { }

F grade: { 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 111, 113, 115, 117, 119, 121, 123, 125, 127, 129, 131, 133, 135, 137, 139, 142, 144, 146, 148, 150, 151, 152, 153, 154, 163, 164, 165, 166, 175, 176, 177, 187, 257, 258, 259, 260, 261, 262, 263, 272, 273, 274, 275, 277, 279, 281, 283, 285, 287, 289, 291, 293, 294 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 12, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 126, 139, 150, 151, 152, 153, 154, 155, 156, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 188, 189, 190, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 216, 217, 218, 219, 221, 222, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 273, 276, 277, 278, 280, 281, 282, 283, 284, 286, 288, 290, 292 }

B grade: { 6, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 37, 67, 85, 104, 113, 115, 117, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 157, 159, 161, 171, 172, 173, 184, 185, 186, 191, 192, 199, 205, 211, 215, 220, 223, 225, 226, 264, 265, 266, 267, 268, 269, 270, 271, 275, 279, 285, 287, 289, 291, 293, 294 }

C grade: { }

F grade: { 29, 187, 272, 274 }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 6, 7, 18, 30, 31, 32, 33, 34, 35, 43, 44, 45, 46, 47, 48, 57, 58, 59, 60, 61, 62, 74, 75, 76, 77, 78, 79, 92, 93, 94, 95, 96, 97, 114, 150, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 175, 176, 177, 178, 179, 180, 202, 232 }

B grade: { 1, 188, 195 }

C grade: { }

F grade: { 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 42, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 87, 89, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 106, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 134, 136, 137, 138, 139, 140, 143, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 189, 191, 192, 193, 194, 196, 198, 199, 200, 201, 202, 204, 206, 207, 208, 210, 212, 213, 214, 216, 217, 218, 219, 221, 223, 225, 228, 232, 241, 242, 252, 253, 254, 255, 256, 258, 259, 260, 263, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 290, 291, 294 }

B grade: { 5, 6, 13, 16, 22, 23, 25, 37, 39, 41, 53, 55, 67, 68, 70, 72, 84, 85, 86, 88, 90, 97, 104, 105, 107, 109, 121, 123, 132, 133, 135, 141, 142, 144, 146, 156, 157, 173, 179, 188, 190, 195, 197, 203, 205, 209, 211, 215, 220, 222, 224, 226, 227, 230, 231, 233, 234, 235, 237, 238, 239, 240, 247, 248, 249, 250, 251, 257, 261, 262, 264, 265, 266, 267, 268, 269, 270, 283, 284, 289, 292, 293 }

C grade: { }

F grade: { 29, 187, 229, 236, 243, 244, 245, 246, 271, 272, 273, 274 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	34	28	34	113	75	45
normalized size	1	1.	0.94	0.78	0.94	3.14	2.08	1.25
time (sec)	N/A	0.049	0.006	0.023	1.204	0.491	0.759	1.141

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	27	77	27	34
normalized size	1	1.	1.08	0.83	1.12	3.21	1.12	1.42
time (sec)	N/A	0.041	0.004	0.021	1.128	0.483	0.357	1.108

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	28	66	37	26
normalized size	1	1.	1.	0.84	1.12	2.64	1.48	1.04
time (sec)	N/A	0.027	0.004	0.018	1.095	0.473	0.204	1.114

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	14	30	8	14
normalized size	1	1.	1.	1.1	1.4	3.	0.8	1.4
time (sec)	N/A	0.006	0.002	0.01	1.002	0.46	0.059	1.137

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	12	34	8	32
normalized size	1	1.	1.	1.11	1.33	3.78	0.89	3.56
time (sec)	N/A	0.019	0.005	0.037	1.167	0.489	1.557	1.095

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	25	19	32	116	24	45
normalized size	1	1.	2.08	1.58	2.67	9.67	2.	3.75
time (sec)	N/A	0.033	0.007	0.037	1.104	0.49	3.856	1.143

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	20	59	17	18
normalized size	1	1.	1.	0.93	1.33	3.93	1.13	1.2
time (sec)	N/A	0.043	0.008	0.041	1.189	0.459	13.241	1.115

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	94	173	282	223	0	200
normalized size	1	1.	1.03	1.9	3.1	2.45	0.	2.2
time (sec)	N/A	0.107	0.178	0.091	1.751	0.519	0.	1.167

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	62	84	0	350	0	127
normalized size	1	1.	0.91	1.24	0.	5.15	0.	1.87
time (sec)	N/A	0.078	0.155	0.081	0.	0.516	0.	1.252

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	47	54	119	112	0	74
normalized size	1	1.	1.34	1.54	3.4	3.2	0.	2.11
time (sec)	N/A	0.057	0.055	0.058	1.718	0.501	0.	1.186

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	0	242	0	82
normalized size	1	1.	1.06	0.97	0.	6.72	0.	2.28
time (sec)	N/A	0.018	0.025	0.072	0.	0.491	0.	1.208

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	20	21	65	123	0	30
normalized size	1	1.	0.87	0.91	2.83	5.35	0.	1.3
time (sec)	N/A	0.07	0.045	0.076	1.143	0.504	0.	1.186

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	67	107	0	370	0	146
normalized size	1	1.	1.22	1.95	0.	6.73	0.	2.65
time (sec)	N/A	0.068	0.123	0.105	0.	0.579	0.	1.261

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	48	64	161	281	0	105
normalized size	1	1.	0.87	1.16	2.93	5.11	0.	1.91
time (sec)	N/A	0.121	0.152	0.09	1.109	0.533	0.	1.2

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	107	283	107	141	0	570	0	251
normalized size	1	2.64	1.	1.32	0.	5.33	0.	2.35
time (sec)	N/A	1.169	0.425	0.107	0.	0.547	0.	1.204

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	121	99	158	305	0	188
normalized size	1	1.	1.89	1.55	2.47	4.77	0.	2.94
time (sec)	N/A	0.118	0.247	0.087	1.693	0.509	0.	1.112

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	62	97	0	398	0	139
normalized size	1	1.	1.03	1.62	0.	6.63	0.	2.32
time (sec)	N/A	0.046	0.154	0.092	0.	0.499	0.	1.195

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	19	95	75	18
normalized size	1	1.	1.	0.82	1.12	5.59	4.41	1.06
time (sec)	N/A	0.013	0.021	0.08	1.129	0.464	99.08	1.102

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	72	106	0	563	0	147
normalized size	1	1.	1.14	1.68	0.	8.94	0.	2.33
time (sec)	N/A	0.06	0.296	0.126	0.	0.596	0.	1.211

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	76	60	84	346	0	85
normalized size	1	1.	1.55	1.22	1.71	7.06	0.	1.73
time (sec)	N/A	0.076	0.195	0.118	1.259	0.528	0.	1.116

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	270	224	0	859	0	290
normalized size	1	1.	2.29	1.9	0.	7.28	0.	2.46
time (sec)	N/A	0.18	1.731	0.144	0.	0.709	0.	1.254

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	114	193	485	632	0	327
normalized size	1	1.	1.16	1.97	4.95	6.45	0.	3.34
time (sec)	N/A	0.198	0.772	0.108	1.672	0.536	0.	1.182

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	300	92	212	0	656	0	266
normalized size	1	3.26	1.	2.3	0.	7.13	0.	2.89
time (sec)	N/A	0.695	0.394	0.12	0.	0.529	0.	1.243

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	19	47	29	113	257	0	27
normalized size	1	1.27	3.13	1.93	7.53	17.13	0.	1.8
time (sec)	N/A	0.026	0.087	0.097	1.142	0.478	0.	1.178

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	101	157	0	539	0	224
normalized size	1	1.	1.38	2.15	0.	7.38	0.	3.07
time (sec)	N/A	0.035	0.154	0.117	0.	0.518	0.	1.2

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	96	73	232	524	0	104
normalized size	1	1.	1.63	1.24	3.93	8.88	0.	1.76
time (sec)	N/A	0.082	0.197	0.12	1.243	0.554	0.	1.183

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	193	333	0	1084	0	286
normalized size	1	1.	1.05	1.81	0.	5.89	0.	1.55
time (sec)	N/A	0.224	0.744	0.162	0.	0.721	0.	1.275

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	208	151	416	887	0	197
normalized size	1	1.	1.78	1.29	3.56	7.58	0.	1.68
time (sec)	N/A	0.137	0.761	0.143	1.269	0.592	0.	1.195

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	66	66	367	0	0	0	0	0
normalized size	1	1.	5.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	3.452	0.631	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	84	161	219	128
normalized size	1	1.	0.66	0.71	0.97	1.85	2.52	1.47
time (sec)	N/A	0.091	0.104	0.042	1.224	0.495	4.502	1.097

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	46	66	126	87	115
normalized size	1	1.	1.	0.77	1.1	2.1	1.45	1.92
time (sec)	N/A	0.07	0.016	0.04	1.258	0.478	2.233	1.127

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	65	127	150	88
normalized size	1	1.	0.95	0.8	1.	1.95	2.31	1.35
time (sec)	N/A	0.078	0.091	0.04	1.165	0.48	1.174	1.116

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	47	90	63	74
normalized size	1	1.	1.	0.82	1.07	2.05	1.43	1.68
time (sec)	N/A	0.065	0.012	0.037	1.185	0.474	0.513	1.13

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	50	86	73	47
normalized size	1	1.	1.07	0.95	1.16	2.	1.7	1.09
time (sec)	N/A	0.043	0.049	0.033	1.236	0.468	0.276	1.092

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	46	25	32	51	31	32
normalized size	1	1.	1.92	1.04	1.33	2.12	1.29	1.33
time (sec)	N/A	0.013	0.012	0.017	1.062	0.468	0.159	1.093

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	24	41	46	0	36
normalized size	1	1.	1.	1.41	2.41	2.71	0.	2.12
time (sec)	N/A	0.026	0.014	0.065	1.236	0.489	0.	1.123

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	34	54	144	0	73
normalized size	1	1.	1.	1.42	2.25	6.	0.	3.04
time (sec)	N/A	0.045	0.013	0.066	1.121	0.493	0.	1.128

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	41	81	0	34
normalized size	1	1.	1.	0.89	1.46	2.89	0.	1.21
time (sec)	N/A	0.058	0.013	0.072	1.057	0.456	0.	1.158

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	52	54	82	203	0	134
normalized size	1	1.	1.	1.04	1.58	3.9	0.	2.58
time (sec)	N/A	0.067	0.017	0.075	1.096	0.498	0.	1.191

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	55	116	0	65
normalized size	1	1.	0.93	0.86	1.25	2.64	0.	1.48
time (sec)	N/A	0.063	0.082	0.079	1.189	0.459	0.	1.138

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	68	74	116	242	0	190
normalized size	1	1.	0.92	1.	1.57	3.27	0.	2.57
time (sec)	N/A	0.081	0.197	0.078	1.167	0.513	0.	1.199

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	72	147	0	95
normalized size	1	1.	0.88	0.8	1.2	2.45	0.	1.58
time (sec)	N/A	0.071	0.148	0.078	1.143	0.472	0.	1.193

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	154	108	132	221	187	209
normalized size	1	1.	1.12	0.79	0.96	1.61	1.36	1.53
time (sec)	N/A	0.138	0.378	0.064	1.144	0.5	7.901	1.167

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	147	118	138	223	384	178
normalized size	1	1.	0.84	0.68	0.79	1.28	2.21	1.02
time (sec)	N/A	0.17	0.241	0.061	1.228	0.503	4.898	1.152

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	116	88	104	169	138	154
normalized size	1	1.	1.13	0.85	1.01	1.64	1.34	1.5
time (sec)	N/A	0.122	0.171	0.059	1.094	0.486	2.353	1.162

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	98	97	101	170	260	115
normalized size	1	1.	0.78	0.77	0.8	1.35	2.06	0.91
time (sec)	N/A	0.135	0.225	0.058	1.124	0.482	1.369	1.173

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	64	52	70	120	85	99
normalized size	1	1.	0.96	0.78	1.04	1.79	1.27	1.48
time (sec)	N/A	0.091	0.388	0.051	1.124	0.472	0.545	1.154

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	92	120	128	68
normalized size	1	1.	0.95	1.27	1.67	2.18	2.33	1.24
time (sec)	N/A	0.02	0.101	0.058	1.179	0.48	0.322	1.114

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	84	63	81	155	0	120
normalized size	1	1.	1.53	1.15	1.47	2.82	0.	2.18
time (sec)	N/A	0.071	0.142	0.085	1.187	0.506	0.	1.173

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	69	57	66	146	0	59
normalized size	1	1.	1.77	1.46	1.69	3.74	0.	1.51
time (sec)	N/A	0.056	0.124	0.095	1.694	0.494	0.	1.16

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	98	120	234	0	165
normalized size	1	1.	1.	1.46	1.79	3.49	0.	2.46
time (sec)	N/A	0.094	0.047	0.102	1.199	0.498	0.	1.19

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	46	48	61	131	0	55
normalized size	1	1.	1.53	1.6	2.03	4.37	0.	1.83
time (sec)	N/A	0.046	0.04	0.103	1.048	0.463	0.	1.167

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	120	143	174	289	0	336
normalized size	1	1.	1.	1.19	1.45	2.41	0.	2.8
time (sec)	N/A	0.142	0.076	0.105	1.093	0.525	0.	1.193

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	54	82	95	184	0	108
normalized size	1	1.	0.64	0.96	1.12	2.16	0.	1.27
time (sec)	N/A	0.077	0.185	0.111	1.145	0.474	0.	1.152

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	104	189	243	343	0	463
normalized size	1	1.	0.62	1.12	1.45	2.04	0.	2.76
time (sec)	N/A	0.17	0.555	0.11	1.094	0.526	0.	1.208

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	104	110	123	231	0	159
normalized size	1	1.	0.83	0.88	0.98	1.85	0.	1.27
time (sec)	N/A	0.104	0.655	0.114	1.121	0.488	0.	1.177

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	265	265	235	175	220	350	508	294
normalized size	1	1.	0.89	0.66	0.83	1.32	1.92	1.11
time (sec)	N/A	0.246	0.475	0.076	1.098	0.534	22.569	1.181

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	204	145	170	278	233	266
normalized size	1	1.	1.17	0.83	0.97	1.59	1.33	1.52
time (sec)	N/A	0.184	0.412	0.081	1.121	0.506	8.98	1.179

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	171	155	177	292	454	212
normalized size	1	1.	0.79	0.72	0.82	1.35	2.1	0.98
time (sec)	N/A	0.213	0.302	0.078	1.226	0.516	6.824	1.184

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	150	125	144	230	182	196
normalized size	1	1.	1.07	0.89	1.03	1.64	1.3	1.4
time (sec)	N/A	0.163	0.289	0.07	1.24	0.494	2.782	1.148

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	94	114	123	228	299	140
normalized size	1	1.	1.21	1.46	1.58	2.92	3.83	1.79
time (sec)	N/A	0.064	0.398	0.066	1.19	0.496	1.59	1.155

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	81	75	113	173	117	123
normalized size	1	1.	1.4	1.29	1.95	2.98	2.02	2.12
time (sec)	N/A	0.023	0.333	0.066	1.207	0.48	0.753	1.13

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	401	123	123	181	0	126
normalized size	1	1.	4.41	1.35	1.35	1.99	0.	1.38
time (sec)	N/A	0.119	0.734	0.109	1.246	0.516	0.	1.201

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	131	126	113	273	0	203
normalized size	1	1.	1.52	1.47	1.31	3.17	0.	2.36
time (sec)	N/A	0.112	1.064	0.119	1.226	0.512	0.	1.198

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	79	93	115	215	0	96
normalized size	1	1.	1.1	1.29	1.6	2.99	0.	1.33
time (sec)	N/A	0.093	0.268	0.122	1.755	0.506	0.	1.204

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	293	187	159	304	0	231
normalized size	1	1.	2.84	1.82	1.54	2.95	0.	2.24
time (sec)	N/A	0.123	1.587	0.124	1.24	0.51	0.	1.208

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	57	72	117	181	0	77
normalized size	1	1.	1.9	2.4	3.9	6.03	0.	2.57
time (sec)	N/A	0.047	0.169	0.121	1.185	0.467	0.	1.232

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	464	256	212	362	0	450
normalized size	1	1.	2.94	1.62	1.34	2.29	0.	2.85
time (sec)	N/A	0.181	1.318	0.131	1.254	0.52	0.	1.238

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	54	127	165	246	0	151
normalized size	1	1.	0.45	1.06	1.38	2.05	0.	1.26
time (sec)	N/A	0.098	0.373	0.13	1.187	0.51	0.	1.2

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	637	328	281	410	0	628
normalized size	1	1.	3.03	1.56	1.34	1.95	0.	2.99
time (sec)	N/A	0.22	2.024	0.133	1.228	0.555	0.	1.201

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	115	173	208	304	0	224
normalized size	1	1.	0.66	0.99	1.2	1.75	0.	1.29
time (sec)	N/A	0.14	0.598	0.128	1.289	0.513	0.	1.184

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	810	399	335	481	0	806
normalized size	1	1.	3.13	1.54	1.29	1.86	0.	3.11
time (sec)	N/A	0.268	4.109	0.139	1.216	0.581	0.	1.215

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	177	219	248	344	0	297
normalized size	1	1.	0.83	1.03	1.16	1.62	0.	1.39
time (sec)	N/A	0.179	1.989	0.129	1.291	0.553	0.	1.201

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	237	236	251	413	367	363
normalized size	1	1.	0.85	0.85	0.9	1.48	1.32	1.3
time (sec)	N/A	0.258	0.718	0.084	1.561	0.551	21.754	1.212

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	381	222	250	269	424	736	331
normalized size	1	1.	0.58	0.66	0.71	1.11	1.93	0.87
time (sec)	N/A	0.389	0.6	0.086	1.148	0.539	15.751	1.22

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	204	206	208	336	286	309
normalized size	1	1.	0.93	0.94	0.95	1.53	1.3	1.4
time (sec)	N/A	0.234	0.52	0.081	1.146	0.526	8.205	1.212

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	178	219	230	342	614	252
normalized size	1	1.	0.59	0.73	0.76	1.14	2.04	0.84
time (sec)	N/A	0.303	0.427	0.08	1.247	0.537	5.412	1.239

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	146	142	166	277	206	223
normalized size	1	1.	0.88	0.86	1.01	1.68	1.25	1.35
time (sec)	N/A	0.181	0.379	0.069	1.154	0.517	2.458	1.194

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	107	153	184	273	381	165
normalized size	1	1.	0.99	1.42	1.7	2.53	3.53	1.53
time (sec)	N/A	0.044	0.402	0.071	1.159	0.504	1.536	1.105

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	181	163	170	289	0	293
normalized size	1	1.	1.21	1.09	1.13	1.93	0.	1.95
time (sec)	N/A	0.153	0.994	0.12	1.178	0.524	0.	1.229

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	477	210	182	312	0	173
normalized size	1	1.	4.01	1.76	1.53	2.62	0.	1.45
time (sec)	N/A	0.183	6.263	0.133	1.785	0.524	0.	1.211

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	268	211	192	363	0	278
normalized size	1	1.	1.77	1.4	1.27	2.4	0.	1.84
time (sec)	N/A	0.164	2.294	0.136	1.239	0.523	0.	1.273

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	105	145	157	282	0	140
normalized size	1	1.	1.02	1.41	1.52	2.74	0.	1.36
time (sec)	N/A	0.156	0.392	0.14	1.837	0.515	0.	1.199

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	936	297	259	392	0	439
normalized size	1	1.	5.57	1.77	1.54	2.33	0.	2.61
time (sec)	N/A	0.195	6.238	0.147	1.209	0.513	0.	1.255

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	73	96	139	250	0	99
normalized size	1	1.	2.43	3.2	4.63	8.33	0.	3.3
time (sec)	N/A	0.048	0.309	0.146	1.232	0.485	0.	1.208

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	258	258	1342	394	339	458	0	724
normalized size	1	1.	5.2	1.53	1.31	1.78	0.	2.81
time (sec)	N/A	0.294	6.258	0.146	1.194	0.544	0.	1.287

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	54	171	204	333	0	194
normalized size	1	1.	0.38	1.2	1.43	2.33	0.	1.36
time (sec)	N/A	0.123	0.558	0.149	1.122	0.512	0.	1.176

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	1732	491	435	533	0	953
normalized size	1	1.	5.25	1.49	1.32	1.62	0.	2.89
time (sec)	N/A	0.343	6.386	0.159	1.171	0.585	0.	1.288

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	115	236	261	400	0	289
normalized size	1	1.	0.57	1.17	1.3	1.99	0.	1.44
time (sec)	N/A	0.171	0.848	0.147	1.244	0.54	0.	1.236

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	408	408	242	590	516	626	0	1188
normalized size	1	1.	0.59	1.45	1.26	1.53	0.	2.91
time (sec)	N/A	0.398	1.33	0.143	1.238	0.628	0.	1.313

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	254	254	175	300	315	471	0	383
normalized size	1	1.	0.69	1.18	1.24	1.85	0.	1.51
time (sec)	N/A	0.219	1.742	0.142	1.233	0.579	0.	1.215

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	307	335	392	595	979	462
normalized size	1	1.	0.6	0.65	0.76	1.16	1.9	0.9
time (sec)	N/A	0.485	1.266	0.303	1.231	0.606	48.446	1.375

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	337	337	278	291	302	504	440	423
normalized size	1	1.	0.82	0.86	0.9	1.5	1.31	1.26
time (sec)	N/A	0.3	1.026	0.213	1.203	0.562	32.26	1.434

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	426	426	259	305	308	509	821	375
normalized size	1	1.	0.61	0.72	0.72	1.19	1.93	0.88
time (sec)	N/A	0.413	0.877	0.197	1.253	0.566	27.413	1.412

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	236	261	262	429	357	350
normalized size	1	1.	0.86	0.95	0.95	1.56	1.3	1.27
time (sec)	N/A	0.281	0.768	0.201	1.09	0.538	10.632	1.395

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	188	236	252	417	663	285
normalized size	1	1.	1.49	1.87	2.	3.31	5.26	2.26
time (sec)	N/A	0.09	0.617	0.18	1.094	0.537	9.868	1.324

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	156	175	232	354	267	252
normalized size	1	1.	1.66	1.86	2.47	3.77	2.84	2.68
time (sec)	N/A	0.047	0.463	0.154	1.233	0.513	2.981	1.157

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	711	272	230	362	0	269
normalized size	1	1.	4.18	1.6	1.35	2.13	0.	1.58
time (sec)	N/A	0.222	6.441	0.236	1.097	0.55	0.	1.26

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	632	251	219	429	0	382
normalized size	1	1.	3.08	1.22	1.07	2.09	0.	1.86
time (sec)	N/A	0.216	6.307	0.251	1.22	0.537	0.	1.275

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	169	169	571	291	242	414	0	234
normalized size	1	1.	3.38	1.72	1.43	2.45	0.	1.38
time (sec)	N/A	0.231	6.378	0.253	1.785	0.536	0.	1.273

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	397	327	244	455	0	379
normalized size	1	1.	1.95	1.6	1.2	2.23	0.	1.86
time (sec)	N/A	0.208	5.96	0.259	1.083	0.537	0.	1.294

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	126	202	235	367	0	194
normalized size	1	1.	0.86	1.37	1.6	2.5	0.	1.32
time (sec)	N/A	0.23	0.754	0.264	1.743	0.534	0.	1.277

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	1219	440	311	478	0	554
normalized size	1	1.	5.44	1.96	1.39	2.13	0.	2.47
time (sec)	N/A	0.233	6.322	0.26	1.134	0.571	0.	1.345

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	89	120	224	329	0	120
normalized size	1	1.	2.97	4.	7.47	10.97	0.	4.
time (sec)	N/A	0.048	0.48	0.257	1.206	0.507	0.	1.313

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	1677	564	390	556	0	918
normalized size	1	1.	5.27	1.77	1.23	1.75	0.	2.89
time (sec)	N/A	0.337	6.33	0.266	1.188	0.573	0.	1.318

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	54	217	301	408	0	238
normalized size	1	1.	0.31	1.23	1.7	2.31	0.	1.34
time (sec)	N/A	0.152	0.443	0.267	1.286	0.546	0.	1.332

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	391	391	331	688	486	643	0	1199
normalized size	1	1.	0.85	1.76	1.24	1.64	0.	3.07
time (sec)	N/A	0.389	2.215	0.266	1.223	0.629	0.	1.338

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	115	299	371	491	0	354
normalized size	1	1.	0.48	1.24	1.53	2.03	0.	1.46
time (sec)	N/A	0.222	1.207	0.256	1.247	0.599	0.	1.322

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	472	472	374	814	567	720	0	1480
normalized size	1	1.	0.79	1.72	1.2	1.53	0.	3.14
time (sec)	N/A	0.467	1.857	0.259	1.232	0.669	0.	1.324

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	218	524	761	471	0	435
normalized size	1	1.	0.96	2.31	3.35	2.07	0.	1.92
time (sec)	N/A	0.214	0.41	0.123	1.815	0.563	0.	1.199

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	137	221	0	598	0	386
normalized size	1	1.	0.83	1.33	0.	3.6	0.	2.33
time (sec)	N/A	0.175	1.015	0.135	0.	0.558	0.	1.319

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	143	236	383	278	0	246
normalized size	1	1.	1.2	1.98	3.22	2.34	0.	2.07
time (sec)	N/A	0.129	0.221	0.122	1.736	0.525	0.	1.149

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	79	90	0	436	0	159
normalized size	1	1.	0.87	0.99	0.	4.79	0.	1.75
time (sec)	N/A	0.082	0.18	0.132	0.	0.52	0.	1.288

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	41	74	167	144	299	100
normalized size	1	1.	0.91	1.64	3.71	3.2	6.64	2.22
time (sec)	N/A	0.066	0.066	0.106	1.785	0.495	19.243	1.195

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	0	312	0	100
normalized size	1	1.	0.96	0.91	0.	6.64	0.	2.13
time (sec)	N/A	0.023	0.033	0.116	0.	0.484	0.	1.253

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	18	19	139	144	0	26
normalized size	1	1.	0.44	0.46	3.39	3.51	0.	0.63
time (sec)	N/A	0.082	0.017	0.138	1.091	0.513	0.	1.159

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	109	174	0	471	0	184
normalized size	1	1.	1.36	2.17	0.	5.89	0.	2.3
time (sec)	N/A	0.084	0.138	0.174	0.	0.591	0.	1.333

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	52	72	321	294	0	73
normalized size	1	1.	0.59	0.82	3.65	3.34	0.	0.83
time (sec)	N/A	0.141	0.141	0.16	1.105	0.525	0.	1.156

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	321	488	0	633	0	375
normalized size	1	1.	2.1	3.19	0.	4.14	0.	2.45
time (sec)	N/A	0.157	1.99	0.202	0.	0.838	0.	1.35

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	99	162	624	439	0	162
normalized size	1	1.	0.63	1.03	3.95	2.78	0.	1.03
time (sec)	N/A	0.222	1.209	0.158	1.094	0.568	0.	1.199

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	661	994	0	834	0	748
normalized size	1	1.	2.52	3.79	0.	3.18	0.	2.85
time (sec)	N/A	0.256	5.248	0.201	0.	1.416	0.	1.388

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	149	292	381	614	0	338
normalized size	1	1.	1.03	2.01	2.63	4.23	0.	2.33
time (sec)	N/A	0.293	0.896	0.161	1.761	0.565	0.	1.139

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	231	130	172	0	697	0	386
normalized size	1	1.67	0.94	1.25	0.	5.05	0.	2.8
time (sec)	N/A	1.048	0.775	0.185	0.	0.554	0.	1.283

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	192	130	177	389	0	215
normalized size	1	1.	2.34	1.59	2.16	4.74	0.	2.62
time (sec)	N/A	0.137	0.39	0.155	1.797	0.518	0.	1.14

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	79	118	0	502	0	186
normalized size	1	1.	0.95	1.42	0.	6.05	0.	2.24
time (sec)	N/A	0.067	0.213	0.167	0.	0.507	0.	1.265

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	28	132	0	27
normalized size	1	1.	1.	0.66	0.88	4.12	0.	0.84
time (sec)	N/A	0.017	0.032	0.142	1.067	0.497	0.	1.114

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	120	174	0	699	0	224
normalized size	1	1.	1.3	1.89	0.	7.6	0.	2.43
time (sec)	N/A	0.079	0.735	0.22	0.	0.599	0.	1.288

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	78	81	440	0	96
normalized size	1	1.	0.68	1.04	1.08	5.87	0.	1.28
time (sec)	N/A	0.096	0.26	0.204	1.192	0.536	0.	1.138

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	709	440	0	860	0	378
normalized size	1	1.	3.96	2.46	0.	4.8	0.	2.11
time (sec)	N/A	0.24	6.112	0.247	0.	0.697	0.	1.322

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	122	174	155	656	0	201
normalized size	1	1.	0.87	1.23	1.1	4.65	0.	1.43
time (sec)	N/A	0.15	2.815	0.216	1.136	0.582	0.	1.146

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	492	211	283	0	1073	0	539
normalized size	1	2.28	0.98	1.31	0.	4.97	0.	2.5
time (sec)	N/A	1.742	1.035	0.213	0.	0.609	0.	1.378

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	154	219	649	748	0	358
normalized size	1	1.	1.26	1.8	5.32	6.13	0.	2.93
time (sec)	N/A	0.212	1.16	0.184	1.655	0.547	0.	1.263

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	225	119	280	0	795	0	396
normalized size	1	1.89	1.	2.35	0.	6.68	0.	3.33
time (sec)	N/A	0.588	0.655	0.206	0.	0.587	0.	1.358

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	30	57	21	231	313	0	27
normalized size	1	1.36	2.59	0.95	10.5	14.23	0.	1.23
time (sec)	N/A	0.031	0.118	0.168	1.139	0.483	0.	1.231

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	0	679	0	298
normalized size	1	1.	1.28	1.85	0.	6.59	0.	2.89
time (sec)	N/A	0.049	0.262	0.184	0.	0.517	0.	1.263

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	57	84	425	647	0	84
normalized size	1	1.	0.66	0.98	4.94	7.52	0.	0.98
time (sec)	N/A	0.103	0.525	0.234	1.2	0.563	0.	1.27

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	396	611	0	1175	0	424
normalized size	1	1.	1.52	2.35	0.	4.52	0.	1.63
time (sec)	N/A	0.286	2.445	0.29	0.	0.719	0.	1.61

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	140	184	880	814	0	189
normalized size	1	1.	0.87	1.14	5.47	5.06	0.	1.17
time (sec)	N/A	0.168	3.105	0.267	1.33	0.598	0.	1.343

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	383	383	688	1125	0	1299	0	689
normalized size	1	1.	1.8	2.94	0.	3.39	0.	1.8
time (sec)	N/A	0.785	2.295	0.308	0.	0.856	0.	1.416

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	272	321	1422	1076	0	328
normalized size	1	1.	1.17	1.38	6.13	4.64	0.	1.41
time (sec)	N/A	0.244	1.331	0.269	1.45	0.69	0.	1.311

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	419	304	520	1256	0	500
normalized size	1	1.	2.54	1.84	3.15	7.61	0.	3.03
time (sec)	N/A	0.303	6.211	0.212	1.825	0.611	0.	1.176

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	B	C	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	362	165	494	0	1173	0	707
normalized size	1	2.31	1.05	3.15	0.	7.47	0.	4.5
time (sec)	N/A	1.173	1.055	0.234	0.	0.596	0.	1.277

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	124	21	72	552	0	27
normalized size	1	1.	4.13	0.7	2.4	18.4	0.	0.9
time (sec)	N/A	0.053	0.662	0.2	1.178	0.526	0.	1.116

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	128	383	0	945	0	575
normalized size	1	1.	0.91	2.72	0.	6.7	0.	4.08
time (sec)	N/A	0.111	0.699	0.233	0.	0.567	0.	1.302

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	115	470	0	68
normalized size	1	1.	0.87	0.65	1.17	4.8	0.	0.69
time (sec)	N/A	0.04	0.29	0.202	1.188	0.502	0.	1.153

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	290	1367	0	1655	0	711
normalized size	1	1.	1.26	5.92	0.	7.16	0.	3.08
time (sec)	N/A	0.17	3.257	0.316	0.	1.035	0.	1.34

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	133	188	194	1191	0	186
normalized size	1	1.	0.96	1.36	1.41	8.63	0.	1.35
time (sec)	N/A	0.16	2.205	0.285	1.27	0.703	0.	1.199

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	400	400	538	1255	0	1843	0	740
normalized size	1	1.	1.34	3.14	0.	4.61	0.	1.85
time (sec)	N/A	0.796	3.206	0.344	0.	0.917	0.	1.357

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	295	330	293	1251	0	336
normalized size	1	1.	1.27	1.42	1.26	5.39	0.	1.45
time (sec)	N/A	0.249	2.013	0.306	1.195	0.703	0.	1.136

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	82	137	0	242	221	157
normalized size	1	1.	0.83	1.38	0.	2.44	2.23	1.59
time (sec)	N/A	0.151	0.136	0.119	0.	0.473	0.861	1.135

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	111	141	0	200	197	161
normalized size	1	1.	1.59	2.01	0.	2.86	2.81	2.3
time (sec)	N/A	0.128	0.068	0.112	0.	0.467	0.761	1.109

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	60	98	0	159	153	134
normalized size	1	1.	0.8	1.31	0.	2.12	2.04	1.79
time (sec)	N/A	0.128	0.101	0.115	0.	0.467	0.439	1.125

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	73	75	0	122	128	90
normalized size	1	1.	1.4	1.44	0.	2.35	2.46	1.73
time (sec)	N/A	0.12	0.071	0.11	0.	0.466	0.442	1.125

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	38	59	0	86	61	81
normalized size	1	1.	0.83	1.28	0.	1.87	1.33	1.76
time (sec)	N/A	0.029	0.068	0.098	0.	0.46	0.544	1.144

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	39	35	31	28
normalized size	1	1.	1.	0.79	1.34	1.21	1.07	0.97
time (sec)	N/A	0.016	0.034	0.09	1.187	0.457	0.152	1.093

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	136	65	0	80
normalized size	1	1.	1.	0.96	5.91	2.83	0.	3.48
time (sec)	N/A	0.07	0.065	0.121	1.168	0.493	0.	1.145

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	35	85	112	217	0	81
normalized size	1	1.	1.13	2.74	3.61	7.	0.	2.61
time (sec)	N/A	0.094	0.194	0.145	1.176	0.474	0.	1.152

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	35	26	146	88	0	36
normalized size	1	1.	1.03	0.76	4.29	2.59	0.	1.06
time (sec)	N/A	0.107	0.184	0.135	1.187	0.438	0.	1.165

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	54	258	251	505	0	136
normalized size	1	1.	0.9	4.3	4.18	8.42	0.	2.27
time (sec)	N/A	0.117	0.24	0.163	1.174	0.482	0.	1.175

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	53	47	285	208	0	63
normalized size	1	1.	1.02	0.9	5.48	4.	0.	1.21
time (sec)	N/A	0.116	0.272	0.148	1.213	0.452	0.	1.179

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	66	430	390	810	0	189
normalized size	1	1.	0.79	5.12	4.64	9.64	0.	2.25
time (sec)	N/A	0.133	0.477	0.169	1.277	0.497	0.	1.178

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	67	68	423	333	0	90
normalized size	1	1.	0.96	0.97	6.04	4.76	0.	1.29
time (sec)	N/A	0.122	0.35	0.153	1.283	0.461	0.	1.156

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	149	174	0	244	233	196
normalized size	1	1.	1.75	2.05	0.	2.87	2.74	2.31
time (sec)	N/A	0.186	0.104	0.138	0.	0.469	1.815	1.114

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	82	117	0	198	190	139
normalized size	1	1.	0.81	1.16	0.	1.96	1.88	1.38
time (sec)	N/A	0.101	0.127	0.134	0.	0.47	0.582	1.145

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	111	108	0	161	165	126
normalized size	1	1.	1.63	1.59	0.	2.37	2.43	1.85
time (sec)	N/A	0.176	0.085	0.127	0.	0.473	0.882	1.164

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	60	79	0	126	119	97
normalized size	1	1.	0.67	0.89	0.	1.42	1.34	1.09
time (sec)	N/A	0.082	0.105	0.127	0.	0.467	0.407	1.151

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	73	57	61	86	94	63
normalized size	1	1.	1.4	1.1	1.17	1.65	1.81	1.21
time (sec)	N/A	0.114	0.057	0.112	1.048	0.464	0.417	1.113

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	30	49	46	41
normalized size	1	1.	1.	0.74	0.97	1.58	1.48	1.32
time (sec)	N/A	0.016	0.043	0.099	1.014	0.46	0.238	1.143

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	184	63	158	161	0	80
normalized size	1	1.	4.	1.37	3.43	3.5	0.	1.74
time (sec)	N/A	0.115	0.231	0.154	1.683	0.483	0.	1.176

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	71	35	41	192	0	138
normalized size	1	1.	1.29	0.64	0.75	3.49	0.	2.51
time (sec)	N/A	0.071	0.414	0.161	1.115	0.486	0.	1.205

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	146	170	225	375	0	131
normalized size	1	1.	2.61	3.04	4.02	6.7	0.	2.34
time (sec)	N/A	0.146	0.395	0.184	1.051	0.481	0.	1.199

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	68	36	47	139	0	47
normalized size	1	1.	2.	1.06	1.38	4.09	0.	1.38
time (sec)	N/A	0.064	0.251	0.172	1.003	0.448	0.	1.103

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	215	342	398	667	0	207
normalized size	1	1.	2.56	4.07	4.74	7.94	0.	2.46
time (sec)	N/A	0.192	0.948	0.197	1.1	0.487	0.	1.206

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	77	47	63	267	0	63
normalized size	1	1.	1.1	0.67	0.9	3.81	0.	0.9
time (sec)	N/A	0.078	0.407	0.175	0.999	0.457	0.	1.132

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	106	137	0	247	226	161
normalized size	1	1.	0.85	1.1	0.	1.98	1.81	1.29
time (sec)	N/A	0.111	0.19	0.202	0.	0.472	0.925	1.157

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	149	141	0	205	199	161
normalized size	1	1.	1.41	1.33	0.	1.93	1.88	1.52
time (sec)	N/A	0.234	0.089	0.149	0.	0.471	1.169	1.195

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	84	98	0	166	156	108
normalized size	1	1.	0.64	0.75	0.	1.27	1.19	0.82
time (sec)	N/A	0.141	0.117	0.142	0.	0.464	0.639	1.159

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	111	90	93	128	133	99
normalized size	1	1.	1.23	1.	1.03	1.42	1.48	1.1
time (sec)	N/A	0.221	0.079	0.137	1.136	0.461	0.643	1.193

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	77	23	69	86	97	77
normalized size	1	1.	2.41	0.72	2.16	2.69	3.03	2.41
time (sec)	N/A	0.032	0.059	0.115	1.154	0.463	0.419	1.121

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	57	39	49	46	49
normalized size	1	1.	1.	1.84	1.26	1.58	1.48	1.58
time (sec)	N/A	0.016	0.04	0.128	1.018	0.458	0.199	1.106

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	91	40	134	157	0	138
normalized size	1	1.	1.49	0.66	2.2	2.57	0.	2.26
time (sec)	N/A	0.064	0.262	0.185	1.617	0.48	0.	1.145

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	109	108	444	302	0	151
normalized size	1	1.	1.76	1.74	7.16	4.87	0.	2.44
time (sec)	N/A	0.162	0.315	0.196	1.633	0.493	0.	1.204

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	110	52	404	317	0	176
normalized size	1	1.	1.47	0.69	5.39	4.23	0.	2.35
time (sec)	N/A	0.081	0.599	0.196	1.648	0.484	0.	1.189

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	64	258	290	521	0	154
normalized size	1	1.	0.84	3.39	3.82	6.86	0.	2.03
time (sec)	N/A	0.182	0.471	0.215	1.085	0.487	0.	1.174

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	90	47	324	177	0	63
normalized size	1	1.	2.65	1.38	9.53	5.21	0.	1.85
time (sec)	N/A	0.063	0.465	0.197	1.089	0.449	0.	1.22

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	115	430	460	836	0	224
normalized size	1	1.	1.11	4.13	4.42	8.04	0.	2.15
time (sec)	N/A	0.23	0.444	0.224	1.106	0.501	0.	1.212

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	90	0	0	0	0	0
normalized size	1	1.	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	2.073	0.586	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	42	23	17	30
normalized size	1	1.	3.2	1.2	8.4	4.6	3.4	6.
time (sec)	N/A	0.023	0.018	0.068	0.997	0.479	0.173	1.133

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	19	11	73	36	0	14
normalized size	1	1.	1.9	1.1	7.3	3.6	0.	1.4
time (sec)	N/A	0.068	0.021	0.089	1.577	0.471	0.	1.095

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	4	15	41	16	0	19
normalized size	1	1.	1.	3.75	10.25	4.	0.	4.75
time (sec)	N/A	0.06	0.018	0.091	1.644	0.468	0.	1.125

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	13	38	88	0	16
normalized size	1	1.	2.27	1.18	3.45	8.	0.	1.45
time (sec)	N/A	0.053	0.034	0.065	1.662	0.464	0.	1.125

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	20	10	31	82	0	14
normalized size	1	1.	2.22	1.11	3.44	9.11	0.	1.56
time (sec)	N/A	0.078	0.023	0.099	1.628	0.491	0.	1.134

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	20	62	0	14
normalized size	1	1.	2.3	1.1	2.	6.2	0.	1.4
time (sec)	N/A	0.024	0.017	0.061	1.07	0.461	0.	1.121

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	20	8	34	47	0	16
normalized size	1	1.	1.82	0.73	3.09	4.27	0.	1.45
time (sec)	N/A	0.055	0.022	0.089	1.116	0.486	0.	1.12

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	18	8	39	26	17	27
normalized size	1	1.	2.	0.89	4.33	2.89	1.89	3.
time (sec)	N/A	0.027	0.02	0.067	1.097	0.483	0.233	1.115

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	13	73	38	0	19
normalized size	1	1.	1.64	0.93	5.21	2.71	0.	1.36
time (sec)	N/A	0.082	0.022	0.083	1.664	0.482	0.	1.109

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	15	41	16	0	19
normalized size	1	1.	1.	2.5	6.83	2.67	0.	3.17
time (sec)	N/A	0.06	0.021	0.099	1.483	0.47	0.	1.121

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	29	15	38	89	0	19
normalized size	1	1.	1.93	1.	2.53	5.93	0.	1.27
time (sec)	N/A	0.06	0.036	0.067	1.496	0.468	0.	1.142

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	18	8	31	81	0	11
normalized size	1	1.	2.57	1.14	4.43	11.57	0.	1.57
time (sec)	N/A	0.089	0.022	0.105	1.505	0.49	0.	1.156

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	20	61	0	14
normalized size	1	1.	2.27	1.	1.82	5.55	0.	1.27
time (sec)	N/A	0.029	0.016	0.059	0.982	0.457	0.	1.159

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	22	8	34	49	0	19
normalized size	1	1.	1.69	0.62	2.62	3.77	0.	1.46
time (sec)	N/A	0.062	0.022	0.09	1.015	0.488	0.	1.126

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	15	24	30	51	27	22
normalized size	1	1.	0.65	1.04	1.3	2.22	1.17	0.96
time (sec)	N/A	0.094	0.014	0.027	1.044	0.458	6.244	1.125

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	14	19	51	16	0	24
normalized size	1	1.	2.33	3.17	8.5	2.67	0.	4.
time (sec)	N/A	0.07	0.007	0.083	1.666	0.465	0.	1.096

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	20	11	46	45	0	14
normalized size	1	1.	2.	1.1	4.6	4.5	0.	1.4
time (sec)	N/A	0.067	0.007	0.082	1.6	0.485	0.	1.122

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	36	21	53	66	0	30
normalized size	1	1.	5.14	3.	7.57	9.43	0.	4.29
time (sec)	N/A	0.085	0.027	0.086	1.641	0.49	0.	1.139

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	10	9	31	51	0	11
normalized size	1	1.	0.83	0.75	2.58	4.25	0.	0.92
time (sec)	N/A	0.054	0.018	0.057	1.666	0.487	0.	1.132

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	25	6	39	53	0	16
normalized size	1	1.	2.27	0.55	3.55	4.82	0.	1.45
time (sec)	N/A	0.06	0.009	0.078	1.108	0.485	0.	1.141

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	12	28	0	5
normalized size	1	1.	0.67	0.56	1.33	3.11	0.	0.56
time (sec)	N/A	0.026	0.019	0.05	1.076	0.458	0.	1.171

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	4	4	14	19	51	16	0	24
normalized size	1	1.	3.5	4.75	12.75	4.	0.	6.
time (sec)	N/A	0.07	0.007	0.079	1.532	0.47	0.	1.121

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	20	9	62	45	0	14
normalized size	1	1.	2.	0.9	6.2	4.5	0.	1.4
time (sec)	N/A	0.068	0.01	0.078	1.662	0.481	0.	1.092

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	46	19	53	65	0	27
normalized size	1	1.	9.2	3.8	10.6	13.	0.	5.4
time (sec)	N/A	0.094	0.016	0.095	1.672	0.488	0.	1.134

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	31	45	0	16
normalized size	1	1.	1.	0.81	1.94	2.81	0.	1.
time (sec)	N/A	0.06	0.015	0.063	1.642	0.465	0.	1.093

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	25	6	55	54	0	19
normalized size	1	1.	1.92	0.46	4.23	4.15	0.	1.46
time (sec)	N/A	0.061	0.01	0.081	1.093	0.48	0.	1.104

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	14	30	0	11
normalized size	1	1.	0.67	0.75	1.17	2.5	0.	0.92
time (sec)	N/A	0.03	0.007	0.054	1.118	0.455	0.	1.131

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	23	23	61	21	238	119	0	92
normalized size	1	1.	2.65	0.91	10.35	5.17	0.	4.
time (sec)	N/A	0.039	0.185	0.077	1.663	0.485	0.	1.247

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	30	30	340	140	0	111
normalized size	1	1.	0.59	0.59	6.67	2.75	0.	2.18
time (sec)	N/A	0.172	0.072	0.087	1.692	0.494	0.	1.199

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	20	17	22	43	0	22
normalized size	1	1.	1.11	0.94	1.22	2.39	0.	1.22
time (sec)	N/A	0.031	0.107	0.042	1.674	0.493	0.	1.152

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	42	47	111	0	50
normalized size	1	1.	0.83	1.45	1.62	3.83	0.	1.72
time (sec)	N/A	0.038	0.035	0.102	1.61	0.519	0.	1.197

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	13	15	32	0	15
normalized size	1	1.	1.	1.18	1.36	2.91	0.	1.36
time (sec)	N/A	0.025	0.036	0.045	1.665	0.486	0.	1.14

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	30	19	53	77	0	107
normalized size	1	1.	1.88	1.19	3.31	4.81	0.	6.69
time (sec)	N/A	0.032	0.048	0.05	1.577	0.497	0.	1.231

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	22	20	331	128	0	97
normalized size	1	1.	0.46	0.42	6.9	2.67	0.	2.02
time (sec)	N/A	0.108	0.025	0.07	1.835	0.485	0.	1.18

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	38	27	0	38
normalized size	1	1.	1.	1.3	3.8	2.7	0.	3.8
time (sec)	N/A	0.025	0.009	0.076	1.048	0.449	0.	1.101

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	23	24	86	72	0	24
normalized size	1	1.	1.64	1.71	6.14	5.14	0.	1.71
time (sec)	N/A	0.147	0.008	0.079	1.623	0.47	0.	1.12

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	32	31	0	35
normalized size	1	1.	1.	1.08	2.67	2.58	0.	2.92
time (sec)	N/A	0.031	0.007	0.041	1.11	0.487	0.	1.154

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	34	60	62	163	0	65
normalized size	1	1.	1.	1.76	1.82	4.79	0.	1.91
time (sec)	N/A	0.039	0.016	0.103	1.119	0.498	0.	1.195

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	35	76	0	38
normalized size	1	1.	1.	1.09	3.18	6.91	0.	3.45
time (sec)	N/A	0.026	0.002	0.075	1.073	0.489	0.	1.185

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	23	32	0	62
normalized size	1	1.	1.	0.93	1.53	2.13	0.	4.13
time (sec)	N/A	0.032	0.012	0.046	1.098	0.48	0.	1.144

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	59	42	0	14
normalized size	1	1.	1.	1.1	5.9	4.2	0.	1.4
time (sec)	N/A	0.086	0.004	0.067	1.058	0.448	0.	1.144

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	38	69	0	0
normalized size	1	1.	1.	0.88	1.15	2.09	0.	0.
time (sec)	N/A	0.062	0.012	0.032	1.1	0.487	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	38	29	38	68	0	134
normalized size	1	1.	1.15	0.88	1.15	2.06	0.	4.06
time (sec)	N/A	0.054	0.115	0.03	1.066	0.486	0.	1.211

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	29	40	26	34	62	0	138
normalized size	1	1.32	1.82	1.18	1.55	2.82	0.	6.27
time (sec)	N/A	0.03	0.011	0.023	1.107	0.477	0.	1.191

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	37	31	34	59	37	36
normalized size	1	1.	1.42	1.19	1.31	2.27	1.42	1.38
time (sec)	N/A	0.013	0.018	0.014	1.065	0.501	0.56	1.126

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	25	43	80	0	144
normalized size	1	1.	1.	1.	1.72	3.2	0.	5.76
time (sec)	N/A	0.028	0.013	0.028	1.116	0.501	0.	1.167

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	36	63	0	96
normalized size	1	1.	1.	0.89	1.29	2.25	0.	3.43
time (sec)	N/A	0.05	0.02	0.03	1.025	0.468	0.	1.158

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	43	66	0	131
normalized size	1	1.	1.	0.85	1.3	2.	0.	3.97
time (sec)	N/A	0.057	0.023	0.033	1.054	0.472	0.	1.167

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	77	100	92	211	0	0
normalized size	1	1.	0.73	0.94	0.87	1.99	0.	0.
time (sec)	N/A	0.349	0.396	0.063	1.119	0.5	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	82	86	89	178	0	6967
normalized size	1	1.	0.95	1.	1.03	2.07	0.	81.01
time (sec)	N/A	0.19	0.223	0.059	1.017	0.499	0.	7.746

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	117	83	103	207	0	7713
normalized size	1	1.	1.34	0.95	1.18	2.38	0.	88.66
time (sec)	N/A	0.321	0.152	0.052	1.113	0.529	0.	5.721

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	116	99	113	279	0	3715
normalized size	1	1.	1.51	1.29	1.47	3.62	0.	48.25
time (sec)	N/A	0.116	0.605	0.045	1.571	0.525	0.	2.314

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	75	123	138	305	0	231
normalized size	1	1.	0.83	1.37	1.53	3.39	0.	2.57
time (sec)	N/A	0.426	0.193	0.063	1.717	0.545	0.	2.759

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	201	109	111	294	0	213
normalized size	1	1.	2.03	1.1	1.12	2.97	0.	2.15
time (sec)	N/A	0.467	1.146	0.073	1.699	0.523	0.	2.905

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	336	169	174	325	0	305
normalized size	1	1.	2.69	1.35	1.39	2.6	0.	2.44
time (sec)	N/A	0.44	0.636	0.08	1.13	0.525	0.	2.9

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	114	109	128	240	0	0
normalized size	1	1.	1.48	1.42	1.66	3.12	0.	0.
time (sec)	N/A	0.189	0.249	0.078	1.161	0.508	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	106	128	127	247	0	0
normalized size	1	1.	0.88	1.07	1.06	2.06	0.	0.
time (sec)	N/A	0.194	0.186	0.076	1.165	0.53	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	98	147	117	311	0	0
normalized size	1	1.	0.88	1.31	1.04	2.78	0.	0.
time (sec)	N/A	0.177	0.272	0.06	1.12	0.538	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	102	164	153	300	0	0
normalized size	1	1.	0.88	1.41	1.32	2.59	0.	0.
time (sec)	N/A	0.104	0.31	0.056	1.046	0.536	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	213	147	297	0	691
normalized size	1	1.	0.87	1.85	1.28	2.58	0.	6.01
time (sec)	N/A	0.242	0.544	0.078	1.117	0.55	0.	100.792

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	97	204	130	258	0	567
normalized size	1	1.	0.87	1.84	1.17	2.32	0.	5.11
time (sec)	N/A	0.254	1.978	0.09	1.086	0.542	0.	105.274

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	99	252	173	270	0	579
normalized size	1	1.	0.83	2.12	1.45	2.27	0.	4.87
time (sec)	N/A	0.22	0.316	0.096	1.124	0.536	0.	104.493

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	100	111	254	288	0	409
normalized size	1	1.	0.88	0.98	2.25	2.55	0.	3.62
time (sec)	N/A	0.341	0.376	0.124	1.642	0.633	0.	1.259

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	80	93	174	239	0	257
normalized size	1	1.	0.87	1.01	1.89	2.6	0.	2.79
time (sec)	N/A	0.279	0.225	0.12	1.659	0.589	0.	1.247

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	70	80	138	190	0	180
normalized size	1	1.	0.88	1.	1.72	2.38	0.	2.25
time (sec)	N/A	0.237	0.1	0.101	1.563	0.564	0.	1.216

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	75	96	173	0	135
normalized size	1	1.	0.85	1.01	1.3	2.34	0.	1.82
time (sec)	N/A	0.08	0.085	0.099	1.136	0.534	0.	1.213

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	64	75	99	174	0	136
normalized size	1	1.	0.85	1.	1.32	2.32	0.	1.81
time (sec)	N/A	0.186	0.062	0.123	1.06	0.539	0.	1.205

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	103	95	169	236	0	180
normalized size	1	1.	1.1	1.01	1.8	2.51	0.	1.91
time (sec)	N/A	0.27	0.151	0.137	1.16	0.77	0.	1.268

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	92	110	213	360	0	257
normalized size	1	1.	0.85	1.02	1.97	3.33	0.	2.38
time (sec)	N/A	0.284	0.292	0.132	1.075	0.912	0.	1.264

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	243	243	164	291	0	1839	0	639
normalized size	1	1.	0.67	1.2	0.	7.57	0.	2.63
time (sec)	N/A	0.611	3.06	0.186	0.	0.701	0.	1.29

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	151	255	0	1507	0	447
normalized size	1	1.	0.67	1.12	0.	6.64	0.	1.97
time (sec)	N/A	0.491	2.166	0.171	0.	0.635	0.	1.228

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	131	155	0	1134	0	381
normalized size	1	1.	0.6	0.71	0.	5.18	0.	1.74
time (sec)	N/A	0.385	1.294	0.145	0.	0.596	0.	1.212

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	1164	0	390
normalized size	1	1.	0.63	0.8	0.	5.73	0.	1.92
time (sec)	N/A	0.399	1.246	0.148	0.	0.591	0.	1.13

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	127	162	0	1164	0	389
normalized size	1	1.	0.93	1.19	0.	8.56	0.	2.86
time (sec)	N/A	0.318	1.137	0.17	0.	0.599	0.	1.214

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	121	155	0	1131	0	383
normalized size	1	1.	0.92	1.18	0.	8.63	0.	2.92
time (sec)	N/A	0.334	0.763	0.188	0.	0.584	0.	1.205

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	196	276	0	1906	0	478
normalized size	1	1.	0.85	1.19	0.	8.25	0.	2.07
time (sec)	N/A	0.437	2.002	0.186	0.	2.544	0.	1.294

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	713	333	923	2583	0	1145
normalized size	1	1.	2.88	1.34	3.72	10.42	0.	4.62
time (sec)	N/A	0.902	6.347	0.18	1.897	1.802	0.	1.489

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	204	295	795	2279	0	913
normalized size	1	1.	0.88	1.27	3.43	9.82	0.	3.94
time (sec)	N/A	0.762	6.179	0.174	1.207	1.194	0.	1.378

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	184	220	801	2115	0	932
normalized size	1	1.	0.87	1.04	3.8	10.02	0.	4.42
time (sec)	N/A	0.668	5.478	0.142	1.309	1.108	0.	1.366

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	696	322	811	2344	0	1080
normalized size	1	1.	3.04	1.41	3.54	10.24	0.	4.72
time (sec)	N/A	0.48	6.311	0.151	1.213	1.226	0.	1.23

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	231	231	703	324	813	2338	0	1081
normalized size	1	1.	3.04	1.4	3.52	10.12	0.	4.68
time (sec)	N/A	0.621	6.313	0.177	1.194	1.174	0.	1.386

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	217	251	805	2021	0	930
normalized size	1	1.	1.02	1.18	3.8	9.53	0.	4.39
time (sec)	N/A	0.364	6.273	0.194	1.267	1.112	0.	1.396

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	217	256	798	2142	0	911
normalized size	1	1.	0.95	1.12	3.5	9.39	0.	4.
time (sec)	N/A	0.408	6.262	0.183	1.238	1.161	0.	1.393

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	A	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	246	0	243	876	0	0
normalized size	1	1.	1.59	0.	1.57	5.65	0.	0.
time (sec)	N/A	0.385	1.512	3.616	1.129	0.612	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	401	0	0	0	0	0
normalized size	1	1.	1.52	0.	0.	0.	0.	0.
time (sec)	N/A	0.764	12.386	1.85	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	35	40	0	81	0	0
normalized size	1	1.	0.9	1.03	0.	2.08	0.	0.
time (sec)	N/A	0.073	0.079	0.056	0.	0.495	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	106	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.432	0.39	0.547	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	81	0	348	0	127
normalized size	1	1.	0.94	1.25	0.	5.35	0.	1.95
time (sec)	N/A	0.065	0.13	0.079	0.	0.523	0.	1.261

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	153	174	285	221	0	205
normalized size	1	1.	1.66	1.89	3.1	2.4	0.	2.23
time (sec)	N/A	0.139	0.324	0.069	1.642	0.514	0.	1.139

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	113	166	0	494	0	257
normalized size	1	1.	0.93	1.36	0.	4.05	0.	2.11
time (sec)	N/A	0.166	1.007	0.092	0.	0.539	0.	1.208

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	82	175	286	223	0	211
normalized size	1	1.	0.88	1.88	3.08	2.4	0.	2.27
time (sec)	N/A	0.133	0.303	0.069	1.645	0.514	0.	1.143

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	115	168	0	498	0	259
normalized size	1	1.	1.03	1.5	0.	4.45	0.	2.31
time (sec)	N/A	0.196	0.655	0.091	0.	0.535	0.	1.229

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	178	363	582	396	0	371
normalized size	1	1.	1.01	2.06	3.31	2.25	0.	2.11
time (sec)	N/A	0.278	0.507	0.075	1.789	0.555	0.	1.121

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	112	170	0	494	0	271
normalized size	1	1.	0.91	1.38	0.	4.02	0.	2.2
time (sec)	N/A	0.158	1.069	0.091	0.	0.537	0.	1.218

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	287	363	572	397	0	369
normalized size	1	1.	1.64	2.07	3.27	2.27	0.	2.11
time (sec)	N/A	0.278	0.758	0.078	1.654	0.558	0.	1.173

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	223	305	0	706	0	487
normalized size	1	1.	1.16	1.58	0.	3.66	0.	2.52
time (sec)	N/A	0.359	1.552	0.111	0.	0.584	0.	1.236

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	87	144	120	159	323	0	194
normalized size	1	1.24	2.06	1.71	2.27	4.61	0.	2.77
time (sec)	N/A	0.166	0.239	0.089	1.711	0.506	0.	1.091

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	152	111	142	0	597	0	282
normalized size	1	1.38	1.01	1.29	0.	5.43	0.	2.56
time (sec)	N/A	0.239	0.589	0.114	0.	0.544	0.	1.206

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	198	226	243	350	531	0	301
normalized size	1	1.53	1.75	1.88	2.71	4.12	0.	2.33
time (sec)	N/A	0.506	1.438	0.101	1.704	0.56	0.	1.12

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	151	110	144	0	586	0	275
normalized size	1	1.39	1.01	1.32	0.	5.38	0.	2.52
time (sec)	N/A	0.259	0.682	0.121	0.	0.543	0.	1.24

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	186	145	260	347	540	0	296
normalized size	1	1.42	1.11	1.98	2.65	4.12	0.	2.26
time (sec)	N/A	0.545	1.585	0.105	1.697	0.566	0.	1.091

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	238	200	269	0	818	0	462
normalized size	1	1.38	1.16	1.56	0.	4.76	0.	2.69
time (sec)	N/A	0.677	1.217	0.133	0.	0.6	0.	1.214

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	196	221	241	346	562	0	289
normalized size	1	1.53	1.73	1.88	2.7	4.39	0.	2.26
time (sec)	N/A	0.408	1.285	0.098	1.7	0.56	0.	1.132

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	238	198	261	0	838	0	452
normalized size	1	1.35	1.12	1.48	0.	4.76	0.	2.57
time (sec)	N/A	0.697	1.223	0.131	0.	0.604	0.	1.216

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	289	409	515	616	822	0	587
normalized size	1	1.38	1.95	2.45	2.93	3.91	0.	2.8
time (sec)	N/A	1.251	2.758	0.108	1.653	0.65	0.	1.116

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	76	63	0	347	0	122
normalized size	1	1.	1.62	1.34	0.	7.38	0.	2.6
time (sec)	N/A	0.079	0.117	0.101	0.	0.591	0.	1.245

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	60	49	0	363	0	101
normalized size	1	1.	1.25	1.02	0.	7.56	0.	2.1
time (sec)	N/A	0.078	0.083	0.106	0.	0.574	0.	1.193

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [286] had the largest ratio of [0.7222]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.	14	0.357
2	A	6	4	1.	14	0.286
3	A	6	5	1.	12	0.417
4	A	3	2	1.	9	0.222
5	A	3	2	1.	12	0.167
6	A	5	4	1.	14	0.286
7	A	6	5	1.	14	0.357
8	A	5	5	1.	16	0.312
9	A	4	4	1.	16	0.25
10	A	2	2	1.	14	0.143
11	A	2	2	1.	11	0.182
12	A	3	3	1.	14	0.214
13	A	4	4	1.	16	0.25
14	A	6	6	1.	16	0.375
15	B	19	11	2.64	16	0.688
16	A	4	4	1.	16	0.25
17	A	3	3	1.	14	0.214
18	A	1	1	1.	11	0.091
19	A	4	4	1.	14	0.286
20	A	3	2	1.	16	0.125
21	A	11	7	1.	16	0.438
22	A	5	5	1.	16	0.312
23	B	13	7	3.26	16	0.438
24	A	2	2	1.27	14	0.143
25	A	3	3	1.	11	0.273
26	A	3	2	1.	14	0.143
27	A	12	7	1.	16	0.438
28	A	3	2	1.	16	0.125
29	A	1	1	1.	33	0.03
30	A	8	5	1.	26	0.192
31	A	6	4	1.	26	0.154

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	7	5	1.	26	0.192
33	A	6	4	1.	26	0.154
34	A	6	5	1.	24	0.208
35	A	3	2	1.	17	0.118
36	A	3	2	1.	24	0.083
37	A	5	4	1.	26	0.154
38	A	6	5	1.	26	0.192
39	A	6	5	1.	26	0.192
40	A	6	4	1.	26	0.154
41	A	7	5	1.	26	0.192
42	A	6	4	1.	26	0.154
43	A	9	6	1.	28	0.214
44	A	12	6	1.	28	0.214
45	A	9	6	1.	28	0.214
46	A	10	6	1.	28	0.214
47	A	8	5	1.	26	0.192
48	A	2	2	1.	19	0.105
49	A	7	6	1.	26	0.231
50	A	3	3	1.	28	0.107
51	A	7	5	1.	28	0.179
52	A	2	2	1.	28	0.071
53	A	9	6	1.	28	0.214
54	A	3	2	1.	28	0.071
55	A	11	6	1.	28	0.214
56	A	3	2	1.	28	0.071
57	A	17	7	1.	28	0.25
58	A	12	7	1.	28	0.25
59	A	15	8	1.	28	0.286
60	A	12	6	1.	28	0.214
61	A	4	4	1.	26	0.154
62	A	2	1	1.	19	0.053
63	A	7	6	1.	26	0.231

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
64	A	10	8	1.	28	0.286
65	A	4	4	1.	28	0.143
66	A	9	5	1.	28	0.179
67	A	2	2	1.	28	0.071
68	A	12	7	1.	28	0.25
69	A	3	2	1.	28	0.071
70	A	14	7	1.	28	0.25
71	A	3	2	1.	28	0.071
72	A	16	7	1.	28	0.25
73	A	3	2	1.	28	0.071
74	A	15	7	1.	28	0.25
75	A	22	7	1.	28	0.25
76	A	15	7	1.	28	0.25
77	A	19	8	1.	28	0.286
78	A	14	6	1.	26	0.231
79	A	3	2	1.	19	0.105
80	A	14	8	1.	26	0.308
81	A	7	6	1.	28	0.214
82	A	14	9	1.	28	0.321
83	A	5	5	1.	28	0.179
84	A	12	5	1.	28	0.179
85	A	2	2	1.	28	0.071
86	A	16	7	1.	28	0.25
87	A	3	2	1.	28	0.071
88	A	19	7	1.	28	0.25
89	A	3	2	1.	28	0.071
90	A	22	7	1.	28	0.25
91	A	3	2	1.	28	0.071
92	A	29	10	1.	28	0.357
93	A	18	7	1.	28	0.25
94	A	25	8	1.	28	0.286
95	A	18	7	1.	28	0.25

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.	26	0.154
97	A	3	2	1.	19	0.105
98	A	8	6	1.	26	0.231
99	A	17	10	1.	28	0.357
100	A	7	6	1.	28	0.214
101	A	17	10	1.	28	0.357
102	A	6	5	1.	28	0.179
103	A	15	6	1.	28	0.214
104	A	2	2	1.	28	0.071
105	A	19	8	1.	28	0.286
106	A	3	2	1.	28	0.071
107	A	22	8	1.	28	0.286
108	A	3	2	1.	28	0.071
109	A	25	8	1.	28	0.286
110	A	9	5	1.	28	0.179
111	A	7	5	1.	28	0.179
112	A	5	5	1.	28	0.179
113	A	4	4	1.	28	0.143
114	A	2	2	1.	26	0.077
115	A	2	2	1.	19	0.105
116	A	3	3	1.	26	0.115
117	A	4	4	1.	28	0.143
118	A	6	6	1.	28	0.214
119	A	7	5	1.	28	0.179
120	A	9	6	1.	28	0.214
121	A	11	5	1.	28	0.179
122	A	7	6	1.	28	0.214
123	A	11	6	1.67	28	0.214
124	A	4	4	1.	28	0.143
125	A	3	3	1.	26	0.115
126	A	1	1	1.	19	0.053
127	A	4	4	1.	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
128	A	3	2	1.	28	0.071
129	A	11	7	1.	28	0.25
130	A	3	2	1.	28	0.071
131	B	15	7	2.28	28	0.25
132	A	5	5	1.	28	0.179
133	A	6	4	1.89	28	0.143
134	A	2	2	1.36	26	0.077
135	A	3	3	1.	19	0.158
136	A	3	2	1.	26	0.077
137	A	12	7	1.	28	0.25
138	A	3	2	1.	28	0.071
139	A	31	8	1.	28	0.286
140	A	3	2	1.	28	0.071
141	A	6	5	1.	28	0.179
142	B	7	4	2.31	28	0.143
143	A	2	2	1.	28	0.071
144	A	5	5	1.	26	0.192
145	A	2	2	1.	19	0.105
146	A	8	5	1.	26	0.192
147	A	3	2	1.	28	0.071
148	A	32	8	1.	28	0.286
149	A	3	2	1.	28	0.071
150	A	9	6	1.	31	0.194
151	A	7	5	1.	31	0.161
152	A	8	6	1.	31	0.194
153	A	7	5	1.	31	0.161
154	A	2	2	1.	29	0.069
155	A	1	1	1.	22	0.045
156	A	4	3	1.	29	0.103
157	A	6	5	1.	31	0.161
158	A	7	6	1.	31	0.194
159	A	7	6	1.	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	7	5	1.	31	0.161
161	A	8	6	1.	31	0.194
162	A	7	5	1.	31	0.161
163	A	10	7	1.	31	0.226
164	A	5	4	1.	31	0.129
165	A	10	7	1.	31	0.226
166	A	3	2	1.	31	0.065
167	A	9	6	1.	29	0.207
168	A	1	1	1.	22	0.045
169	A	8	7	1.	29	0.241
170	A	4	3	1.	31	0.097
171	A	8	6	1.	31	0.194
172	A	3	3	1.	31	0.097
173	A	10	7	1.	31	0.226
174	A	4	3	1.	31	0.097
175	A	5	4	1.	31	0.129
176	A	13	8	1.	31	0.258
177	A	4	2	1.	31	0.065
178	A	13	7	1.	31	0.226
179	A	2	2	1.	29	0.069
180	A	1	1	1.	22	0.045
181	A	4	3	1.	29	0.103
182	A	11	9	1.	31	0.29
183	A	4	3	1.	31	0.097
184	A	10	6	1.	31	0.194
185	A	3	3	1.	31	0.097
186	A	13	8	1.	31	0.258
187	A	1	1	1.	33	0.03
188	A	3	3	1.	7	0.429
189	A	4	3	1.	10	0.3
190	A	3	3	1.	10	0.3
191	A	3	3	1.	10	0.3

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
192	A	4	4	1.	10	0.4
193	A	2	2	1.	10	0.2
194	A	5	5	1.	10	0.5
195	A	3	3	1.	9	0.333
196	A	4	3	1.	12	0.25
197	A	3	3	1.	12	0.25
198	A	3	3	1.	12	0.25
199	A	4	4	1.	12	0.333
200	A	2	2	1.	12	0.167
201	A	5	5	1.	12	0.417
202	A	4	4	1.	20	0.2
203	A	3	3	1.	10	0.3
204	A	4	3	1.	10	0.3
205	A	4	4	1.	10	0.4
206	A	3	3	1.	10	0.3
207	A	5	5	1.	10	0.5
208	A	2	2	1.	10	0.2
209	A	3	3	1.	12	0.25
210	A	4	3	1.	12	0.25
211	A	4	4	1.	12	0.333
212	A	3	3	1.	12	0.25
213	A	5	5	1.	12	0.417
214	A	2	2	1.	12	0.167
215	A	3	3	1.	15	0.2
216	A	4	2	1.	22	0.091
217	A	2	2	1.	22	0.091
218	A	4	3	1.	22	0.136
219	A	2	2	1.	22	0.091
220	A	3	2	1.	22	0.091
221	A	2	1	1.	22	0.045
222	A	3	3	1.	17	0.176
223	A	3	2	1.	24	0.083

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
224	A	2	2	1.	24	0.083
225	A	3	2	1.	24	0.083
226	A	2	2	1.	24	0.083
227	A	2	1	1.	24	0.042
228	A	2	1	1.	24	0.042
229	A	6	4	1.	26	0.154
230	A	6	5	1.	26	0.192
231	A	5	5	1.32	24	0.208
232	A	3	2	1.	17	0.118
233	A	5	5	1.	24	0.208
234	A	6	5	1.	26	0.192
235	A	6	4	1.	26	0.154
236	A	6	5	1.	28	0.179
237	A	5	5	1.	28	0.179
238	A	7	7	1.	26	0.269
239	A	10	8	1.	19	0.421
240	A	7	7	1.	26	0.269
241	A	7	7	1.	28	0.25
242	A	9	9	1.	28	0.321
243	A	4	3	1.	28	0.107
244	A	5	4	1.	28	0.143
245	A	5	4	1.	26	0.154
246	A	4	3	1.	19	0.158
247	A	5	4	1.	26	0.154
248	A	5	4	1.	28	0.143
249	A	5	4	1.	28	0.143
250	A	5	4	1.	28	0.143
251	A	5	4	1.	28	0.143
252	A	5	4	1.	26	0.154
253	A	4	3	1.	19	0.158
254	A	7	5	1.	26	0.192
255	A	5	4	1.	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
256	A	5	4	1.	28	0.143
257	A	12	8	1.	28	0.286
258	A	11	7	1.	28	0.25
259	A	11	7	1.	26	0.269
260	A	11	7	1.	19	0.368
261	A	6	6	1.	26	0.231
262	A	6	6	1.	28	0.214
263	A	12	8	1.	28	0.286
264	A	6	5	1.	28	0.179
265	A	6	5	1.	28	0.179
266	A	6	5	1.	26	0.192
267	A	5	4	1.	19	0.21
268	A	6	5	1.	26	0.192
269	A	6	5	1.	28	0.179
270	A	5	4	1.	28	0.143
271	A	4	3	1.	28	0.107
272	A	8	7	1.	28	0.25
273	A	6	4	1.	26	0.154
274	A	7	4	1.	28	0.143
275	A	5	5	1.	16	0.312
276	A	7	7	1.	18	0.389
277	A	9	8	1.	18	0.444
278	A	7	7	1.	18	0.389
279	A	10	8	1.	20	0.4
280	A	13	8	1.	20	0.4
281	A	9	8	1.	18	0.444
282	A	13	9	1.	20	0.45
283	A	17	9	1.	20	0.45
284	A	6	5	1.24	16	0.312
285	A	13	8	1.38	18	0.444
286	A	17	13	1.53	18	0.722
287	A	13	8	1.39	18	0.444

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
288	A	21	10	1.42	20	0.5
289	A	33	12	1.38	20	0.6
290	A	17	13	1.53	18	0.722
291	A	33	12	1.35	20	0.6
292	A	48	12	1.38	20	0.6
293	A	5	4	1.	14	0.286
294	A	5	4	1.	14	0.286

Chapter 3

Listing of integrals

3.1 $\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=36

$$\frac{1}{4}a\sin^4(x)+\frac{3bx}{8}-\frac{1}{4}b\sin^3(x)\cos(x)-\frac{3}{8}b\sin(x)\cos(x)$$

```
[Out] (3*b*x)/8 - (3*b*Cos[x]*Sin[x])/8 - (b*Cos[x]*Sin[x]^3)/4 + (a*Sin[x]^4)/4
```

Rubi [A] time = 0.0486186, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3089, 2564, 30, 2635, 8}

$$\frac{1}{4}a\sin^4(x)+\frac{3bx}{8}-\frac{1}{4}b\sin^3(x)\cos(x)-\frac{3}{8}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (3*b*x)/8 - (3*b*Cos[x]*Sin[x])/8 - (b*Cos[x]*Sin[x]^3)/4 + (a*Sin[x]^4)/4
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
```

```
gerQ[m] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$
\begin{aligned}
\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin^3(x)+b\sin^4(x)\right)\,dx\\
&= a\int \cos(x)\sin^3(x)\,dx+b\int \sin^4(x)\,dx\\
&= -\frac{1}{4}b\cos(x)\sin^3(x)+a\,\mathrm{Subst}\left(\int x^3\,dx,x,\sin(x)\right)+\frac{1}{4}(3b)\int \sin^2(x)\,dx\\
&= -\frac{3}{8}b\cos(x)\sin(x)-\frac{1}{4}b\cos(x)\sin^3(x)+\frac{1}{4}a\sin^4(x)+\frac{1}{8}(3b)\int 1\,dx\\
&= \frac{3bx}{8}-\frac{3}{8}b\cos(x)\sin(x)-\frac{1}{4}b\cos(x)\sin^3(x)+\frac{1}{4}a\sin^4(x)
\end{aligned}
$$

Mathematica [A] time = 0.0063702, size = 34, normalized size = 0.94

$$\frac{1}{4}a\sin^4(x)+\frac{3bx}{8}-\frac{1}{4}b\sin(2x)+\frac{1}{32}b\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (3*b*x)/8 + (a*Sin[x]^4)/4 - (b*Sin[2*x])/4 + (b*Sin[4*x])/32
```

Maple [A] time = 0.023, size = 28, normalized size = 0.8

$$b\left(-\frac{\cos(x)}{4}\left((\sin(x))^3+\frac{3\sin(x)}{2}\right)+\frac{3x}{8}\right)+\frac{a(\sin(x))^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] b*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+1/4*a*sin(x)^4
```

Maxima [A] time = 1.20377, size = 34, normalized size = 0.94

$$\frac{1}{4}a\sin(x)^4+\frac{1}{32}b(12x+\sin(4x)-8\sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*sin(x)^4 + 1/32*b*(12*x + sin(4*x) - 8*sin(2*x))
```

Fricas [A] time = 0.490686, size = 113, normalized size = 3.14

$$\frac{1}{4}a\cos(x)^4-\frac{1}{2}a\cos(x)^2+\frac{3}{8}bx+\frac{1}{8}\left(2b\cos(x)^3-5b\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*a*cos(x)^4 - 1/2*a*cos(x)^2 + 3/8*b*x + 1/8*(2*b*cos(x)^3 - 5*b*cos(x))
*sin(x)
```

Sympy [B] time = 0.758967, size = 75, normalized size = 2.08

$$\frac{a\sin^4(x)}{4}+\frac{3bx\sin^4(x)}{8}+\frac{3bx\sin^2(x)\cos^2(x)}{4}+\frac{3bx\cos^4(x)}{8}-\frac{5b\sin^3(x)\cos(x)}{8}-\frac{3b\sin(x)\cos^3(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*sin(x)**4/4 + 3*b*x*sin(x)**4/8 + 3*b*x*sin(x)**2*cos(x)**2/4 + 3*b*x*cos
(x)**4/8 - 5*b*sin(x)**3*cos(x)/8 - 3*b*sin(x)*cos(x)**3/8
```

Giac [A] time = 1.14065, size = 45, normalized size = 1.25

$$\frac{3}{8}\,bx+\frac{1}{32}\,a\cos(4\,x)-\frac{1}{8}\,a\cos(2\,x)+\frac{1}{32}\,b\sin(4\,x)-\frac{1}{4}\,b\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/32*a*cos(4*x) - 1/8*a*cos(2*x) + 1/32*b*sin(4*x) - 1/4*b*sin(2*
x)
```

3.2 $\int \sin^2(x)(a\cos(x) + b\sin(x))\,dx$

Optimal. Leaf size=24

$$\frac{1}{3}a\sin^3(x) + \frac{1}{3}b\cos^3(x) - b\cos(x)$$

```
[Out] -(b*Cos[x]) + (b*Cos[x]^3)/3 + (a*Sin[x]^3)/3
```

Rubi [A] time = 0.041309, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3089, 2564, 30, 2633}

$$\frac{1}{3}a\sin^3(x) + \frac{1}{3}b\cos^3(x) - b\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cos[x]) + (b*Cos[x]^3)/3 + (a*Sin[x]^3)/3
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin^2(x)+b\sin^3(x)\right)\,dx\\
&= a\int \cos(x)\sin^2(x)\,dx+b\int \sin^3(x)\,dx\\
&= a\,\text{Subst}\left(\int x^2\,dx,x,\sin(x)\right)-b\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(x)\right)\\
&= -b\cos(x)+\frac{1}{3}b\cos^3(x)+\frac{1}{3}a\sin^3(x)
\end{aligned}$$

Mathematica [A] time = 0.0038364, size = 26, normalized size = 1.08

$$\frac{1}{3}a\sin^3(x)-\frac{3}{4}b\cos(x)+\frac{1}{12}b\cos(3x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-3*b*Cos[x])/4 + (b*Cos[3*x])/12 + (a*Sin[x]^3)/3
```

Maple [A] time = 0.021, size = 20, normalized size = 0.8

$$-\frac{b\left(2+(\sin(x))^2\right)\cos(x)}{3}+\frac{a(\sin(x))^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/3*b*(2+sin(x)^2)*cos(x)+1/3*a*sin(x)^3
```

Maxima [A] time = 1.12823, size = 27, normalized size = 1.12

$$\frac{1}{3} a \sin(x)^3 + \frac{1}{3} \left(\cos(x)^3 - 3 \cos(x)\right) b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*sin(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*b
```

Fricas [A] time = 0.483446, size = 77, normalized size = 3.21

$$\frac{1}{3} b \cos(x)^3 - b \cos(x) - \frac{1}{3} \left(a \cos(x)^2 - a\right) \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*b*cos(x)^3 - b*cos(x) - 1/3*(a*cos(x)^2 - a)*sin(x)
```

Sympy [A] time = 0.35743, size = 27, normalized size = 1.12

$$\frac{a \sin^3(x)}{3} - b \sin^2(x) \cos(x) - \frac{2b \cos^3(x)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*sin(x)**3/3 - b*sin(x)**2*cos(x) - 2*b*cos(x)**3/3
```

Giac [A] time = 1.10847, size = 34, normalized size = 1.42

$$\frac{1}{12} b \cos(3 x) - \frac{3}{4} b \cos(x) - \frac{1}{12} a \sin(3 x) + \frac{1}{4} a \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")

[Out] 1/12*b*cos(3*x) - 3/4*b*cos(x) - 1/12*a*sin(3*x) + 1/4*a*sin(x)
```

3.3 $\int \sin(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}a\sin^2(x)+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

```
[Out] (b*x)/2 - (b*Cos[x]*Sin[x])/2 + (a*Sin[x]^2)/2
```

Rubi [A] time = 0.026968, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3089, 2564, 30, 2635, 8}

$$\frac{1}{2}a\sin^2(x)+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*x)/2 - (b*Cos[x]*Sin[x])/2 + (a*Sin[x]^2)/2
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sin(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin(x)+b\sin^2(x)\right)\,dx\\ &= a\int \cos(x)\sin(x)\,dx + b\int \sin^2(x)\,dx\\ &= -\frac{1}{2}b\cos(x)\sin(x) + a\,\mathrm{Subst}\left(\int x\,dx, x, \sin(x)\right) + \frac{1}{2}b\int 1\,dx\\ &= \frac{bx}{2} - \frac{1}{2}b\cos(x)\sin(x) + \frac{1}{2}a\sin^2(x)\end{aligned}$$

Mathematica [A] time = 0.0040089, size = 25, normalized size = 1.

$$-\frac{1}{2}a\cos^2(x) + \frac{bx}{2} - \frac{1}{4}b\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*x)/2 - (a*Cos[x]^2)/2 - (b*Sin[2*x])/4
```

Maple [A] time = 0.018, size = 21, normalized size = 0.8

$$b\left(-\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) - \frac{(\cos(x))^2 a}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a*cos(x)+b*sin(x)),x)
```

```
[Out] b*(-1/2*cos(x)*sin(x)+1/2*x)-1/2*cos(x)^2*a
```

Maxima [A] time = 1.09505, size = 28, normalized size = 1.12

$$-\frac{1}{2}\,a\cos\left(x\right)^{2}+\frac{1}{4}\,b(2\,x-\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*a*cos(x)^2 + 1/4*b*(2*x - sin(2*x))
```

Fricas [A] time = 0.472956, size = 66, normalized size = 2.64

$$-\frac{1}{2}\,a\cos\left(x\right)^{2}-\frac{1}{2}\,b\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*a*cos(x)^2 - 1/2*b*cos(x)*sin(x) + 1/2*b*x
```

Sympy [A] time = 0.204378, size = 37, normalized size = 1.48

$$-\frac{a\cos^{2}\left(x\right)}{2}+\frac{bx\sin^{2}\left(x\right)}{2}+\frac{bx\cos^{2}\left(x\right)}{2}-\frac{b\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a*cos(x)**2/2 + b*x*sin(x)**2/2 + b*x*cos(x)**2/2 - b*sin(x)*cos(x)/2
```

Giac [A] time = 1.11448, size = 26, normalized size = 1.04

$$\frac{1}{2}\,bx - \frac{1}{4}\,a\cos\left(2\,x\right) - \frac{1}{4}\,b\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 1/2*b*x - 1/4*a*cos(2*x) - 1/4*b*sin(2*x)
```

3.4 $\int (a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=10

$$a\sin(x)-b\cos(x)$$

```
[Out] -(b*Cos[x]) + a*Sin[x]
```

Rubi [A] time = 0.00609, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2637, 2638}

$$a\sin(x)-b\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[x] + b*Sin[x],x]
```

```
[Out] -(b*Cos[x]) + a*Sin[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(x)+b\sin(x))\,dx &= a\int \cos(x)\,dx+b\int \sin(x)\,dx\\ &= -b\cos(x)+a\sin(x)\end{aligned}$$

Mathematica [A] time = 0.0018502, size = 10, normalized size = 1.

$$a\sin(x)-b\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[x] + b*Sin[x],x]
```

```
[Out] -(b*Cos[x]) + a*Sin[x]
```

Maple [A] time = 0.01, size = 11, normalized size = 1.1

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(x)+b*sin(x),x)
```

```
[Out] -b*cos(x)+a*sin(x)
```

Maxima [A] time = 1.00208, size = 14, normalized size = 1.4

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="maxima")
```

```
[Out] -b*cos(x) + a*sin(x)
```

Fricas [A] time = 0.46018, size = 30, normalized size = 3.

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="fricas")
```

```
[Out] -b*cos(x) + a*sin(x)
```

Sympy [A] time = 0.059447, size = 8, normalized size = 0.8

$$a\sin(x) - b\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x)
```

```
[Out] a*sin(x) - b*cos(x)
```

Giac [A] time = 1.13656, size = 14, normalized size = 1.4

$$-b\cos(x) + a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="giac")
```

```
[Out] -b*cos(x) + a*sin(x)
```

3.5 $\int \csc(x)(a\cos(x) + b\sin(x))\,dx$

Optimal. Leaf size=9

$$a\log(\sin(x)) + bx$$

```
[Out] b*x + a*Log[Sin[x]]
```

Rubi [A] time = 0.0190863, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3085, 3475}

$$a\log(\sin(x)) + bx$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] b*x + a*Log[Sin[x]]
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc(x)(a\cos(x) + b\sin(x))\,dx &= \int (b + a\cot(x))\,dx \\
&= bx + a\int \cot(x)\,dx \\
&= bx + a\log(\sin(x))
\end{aligned}$$

Mathematica [A] time = 0.0051956, size = 9, normalized size = 1.

$$a\log(\sin(x)) + bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] b*x + a*Log[Sin[x]]
```

Maple [A] time = 0.037, size = 10, normalized size = 1.1

$$bx + a\ln(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*(a*cos(x)+b*sin(x)),x)
```

```
[Out] b*x+a*ln(sin(x))
```

Maxima [A] time = 1.16693, size = 12, normalized size = 1.33

$$bx + a\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] b*x + a*log(sin(x))
```

Fricas [A] time = 0.489148, size = 34, normalized size = 3.78

$$bx + a\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] b*x + a*log(1/2*sin(x))
```

Sympy [A] time = 1.55686, size = 8, normalized size = 0.89

$$a\log\left(\sin\left(x\right)\right) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*log(sin(x)) + b*x
```

Giac [B] time = 1.0948, size = 32, normalized size = 3.56

$$bx - a\log\left(\tan\left(\frac{1}{2}\,x\right)^{2} + 1\right) + a\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] b*x - a*log(tan(1/2*x)^2 + 1) + a*log(abs(tan(1/2*x)))
```

3.6 $\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=12

$$-a\csc(x)-b\tanh^{-1}(\cos(x))$$

```
[Out] -(b*ArcTanh[Cos[x]]) - a*Csc[x]
```

Rubi [A] time = 0.0333175, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3089, 3770, 2606, 8}

$$-a\csc(x)-b\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*ArcTanh[Cos[x]]) - a*Csc[x]
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx &= \int (b\csc(x)+a\cot(x)\csc(x))\,dx\\ &= a\int \cot(x)\csc(x)\,dx + b\int \csc(x)\,dx\\ &= -b\tanh^{-1}(\cos(x)) - a\,\mathrm{Subst}\left(\int 1\,dx, x, \csc(x)\right)\\ &= -b\tanh^{-1}(\cos(x)) - a\csc(x)\end{aligned}$$

Mathematica [B] time = 0.0067673, size = 25, normalized size = 2.08

$$-a\csc(x)+b\log\left(\sin\left(\frac{x}{2}\right)\right)-b\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(a*Csc[x]) - b*Log[Cos[x/2]] + b*Log[Sin[x/2]]
```

Maple [A] time = 0.037, size = 19, normalized size = 1.6

$$-\frac{a}{\sin(x)}+b\ln(-\cot(x)+\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/sin(x)+b*ln(-cot(x)+csc(x))
```

Maxima [A] time = 1.10404, size = 32, normalized size = 2.67

$$-\frac{1}{2}\,b(\log(\cos(x)+1)-\log(\cos(x)-1))-\frac{a}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(cos(x) + 1) - log(cos(x) - 1)) - a/sin(x)
```

Fricas [B] time = 0.490314, size = 116, normalized size = 9.67

$$-\frac{b \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) \sin(x) - b \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) \sin(x) + 2\,a}{2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + 2*
a)/sin(x)
```

Sympy [A] time = 3.85581, size = 24, normalized size = 2.

$$-\frac{a}{\sin(x)} + \frac{b \log(\cos(x) - 1)}{2} - \frac{b \log(\cos(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/sin(x) + b*log(cos(x) - 1)/2 - b*log(cos(x) + 1)/2
```

Giac [B] time = 1.14332, size = 45, normalized size = 3.75

$$b \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right) - \frac{1}{2}\,a \tan\left(\frac{1}{2}\,x\right) - \frac{2\,b \tan\left(\frac{1}{2}\,x\right) + a}{2 \tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] b*log(abs(tan(1/2*x))) - 1/2*a*tan(1/2*x) - 1/2*(2*b*tan(1/2*x) + a)/tan(1/
2*x)
```

3.7 $\int \csc^3(x)(a\cos(x) + b\sin(x))\,dx$

Optimal. Leaf size=15

$$-\frac{1}{2}a\csc^2(x) - b\cot(x)$$

```
[Out] -(b*Cot[x]) - (a*Csc[x]^2)/2
```

Rubi [A] time = 0.043457, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3089, 3767, 8, 2606, 30}

$$-\frac{1}{2}a\csc^2(x) - b\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cot[x]) - (a*Csc[x]^2)/2
```

Rule 3089

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(b\csc^2(x)+a\cot(x)\csc^2(x)\right)\,dx\\
&= a\int \cot(x)\csc^2(x)\,dx + b\int \csc^2(x)\,dx\\
&= -(a\,\text{Subst}(\int x\,dx, x, \csc(x))) - b\,\text{Subst}(\int 1\,dx, x, \cot(x))\\
&= -b\cot(x) - \frac{1}{2}a\csc^2(x)
\end{aligned}
$$

Mathematica [A] time = 0.0079179, size = 15, normalized size = 1.

$$-\frac{1}{2}a\csc^2(x) - b\cot(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cot[x]) - (a*Csc[x]^2)/2
```

Maple [A] time = 0.041, size = 14, normalized size = 0.9

$$-\frac{a}{2\,(\sin(x))^2} - b\cot(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/2*a/sin(x)^2-b*cot(x)
```

Maxima [A] time = 1.18897, size = 20, normalized size = 1.33

$$-\frac{b}{\tan(x)} - \frac{a}{2\,\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -b/tan(x) - 1/2*a/sin(x)^2
```

Fricas [A] time = 0.458895, size = 59, normalized size = 3.93

$$\frac{2\,b\cos(x)\sin(x) + a}{2\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*cos(x)*sin(x) + a)/(cos(x)^2 - 1)
```

Sympy [A] time = 13.2413, size = 17, normalized size = 1.13

$$-\frac{a}{2\sin^2(x)} - \frac{b\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/(2*sin(x)**2) - b*cos(x)/sin(x)
```

Giac [A] time = 1.11529, size = 18, normalized size = 1.2

$$-\frac{2\,b\tan(x) + a}{2\,\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*tan(x) + a)/tan(x)^2
```

3.8 $\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=91

$$\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{a\sin^2(x)}{2(a^2+b^2)}-\frac{b\sin(x)\cos(x)}{2(a^2+b^2)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

```
[Out] (a^2*b*x)/(a^2 + b^2)^2 + (b*x)/(2*(a^2 + b^2)) - (a^3*Log[a*Cos[x] + b*Sin
[x]])/(a^2 + b^2)^2 - (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) - (a*Sin[x]^2)/(2*(
a^2 + b^2))
```

Rubi [A] time = 0.106942, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3099, 3097, 3133, 2635, 8}

$$\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{a\sin^2(x)}{2(a^2+b^2)}-\frac{b\sin(x)\cos(x)}{2(a^2+b^2)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 + b^2)^2 + (b*x)/(2*(a^2 + b^2)) - (a^3*Log[a*Cos[x] + b*Sin
[x]])/(a^2 + b^2)^2 - (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) - (a*Sin[x]^2)/(2*(
a^2 + b^2))
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
```

```
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \sin^2(x)\,dx}{a^2+b^2}\\
&= \frac{a^2bx}{\left(a^2+b^2\right)^2}-\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}-\frac{a^3\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{b\int 1\,dx}{2\left(a^2+b^2\right)}\\
&= \frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}-\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}-\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}
\end{aligned}$$

Mathematica [C] time = 0.178337, size = 94, normalized size = 1.03

$$\frac{a\left(a^2+b^2\right)\cos(2x)+6a^2bx-a^2b\sin(2x)-2a^3\log\left((a\cos(x)+b\sin(x))^2\right)-4ia^3x+4ia^3\tan^{-1}(\tan(x))+2b^3x-b^3\sin(}{4\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] ((-4*I)*a^3*x + 6*a^2*b*x + 2*b^3*x + (4*I)*a^3*ArcTan[Tan[x]] + a*(a^2 + b
^2)*Cos[2*x] - 2*a^3*Log[(a*Cos[x] + b*Sin[x])^2] - a^2*b*Sin[2*x] - b^3*Si
n[2*x])/(4*(a^2 + b^2)^2)
```

Maple [B] time = 0.091, size = 173, normalized size = 1.9

$$-\frac{a^3\ln(a+b\tan(x))}{(a^2+b^2)^2}-\frac{\tan(x)\,a^2b}{2\,(a^2+b^2)^2\,((\tan(x))^2+1)}-\frac{\tan(x)\,b^3}{2\,(a^2+b^2)^2\,((\tan(x))^2+1)}+\frac{a^3}{2\,(a^2+b^2)^2\,((\tan(x))^2+1)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a^3/(a^2+b^2)^2*ln(a+b*tan(x))-1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*a^2*b-1
/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*b^3+1/2/(a^2+b^2)^2/(tan(x)^2+1)*a^3+1/2
/(a^2+b^2)^2/(tan(x)^2+1)*a*b^2+1/2/(a^2+b^2)^2*a^3*ln(tan(x)^2+1)+3/2/(a^2
+b^2)^2*arctan(tan(x))*a^2*b+1/2/(a^2+b^2)^2*arctan(tan(x))*b^3
```

Maxima [B] time = 1.7512, size = 282, normalized size = 3.1

$$-\frac{a^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}+\frac{a^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(3\,a^2b+b^3)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4}-\frac{\frac{b\sin(x)}{\cos(x)+1}+\frac{2\,a\sin(x)^2}{(\cos(x)+1)^2}}{a^2+b^2+\frac{2\,(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 + 2
*a^2*b^2 + b^4) + a^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 + b
^4) + (3*a^2*b + b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4) -
 (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x)
+ 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*si
n(x)^4/(cos(x) + 1)^4)
```

Fricas [A] time = 0.518957, size = 223, normalized size = 2.45

$$-\frac{a^3\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\left(a^3+ab^2\right)\cos\left(x\right)^2+\left(a^2b+b^3\right)\cos\left(x\right)\sin\left(x\right)-\left(3\,a^2b+b^3\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(a^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 + a*
b^2)*cos(x)^2 + (a^2*b + b^3)*cos(x)*sin(x) - (3*a^2*b + b^3)*x)/(a^4 + 2*a
^2*b^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16685, size = 200, normalized size = 2.2

$$-\frac{a^3b\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{a^3\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{\left(3\,a^2b+b^3\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{a^3\tan\left(x\right)^2+a^2b\tan\left(x\right)+b^3\tan\left(x\right)-ab^2}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^3*log(tan(x
)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(3*a^2*b + b^3)*x/(a^4 + 2*a^2*b^2 +
 b^4) - 1/2*(a^3*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a*b^2)/((a^4 + 2*a^
2*b^2 + b^4)*(tan(x)^2 + 1))
```

3.9 $\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=68

$$-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}-\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -((a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) -
 (b*Cos[x])/(a^2 + b^2) - (a*Sin[x])/(a^2 + b^2)
```

Rubi [A] time = 0.0778495, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3099, 3074, 206, 2638}

$$-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}-\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) -
 (b*Cos[x])/(a^2 + b^2) - (a*Sin[x])/(a^2 + b^2)
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{a\sin(x)}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \sin(x)\,dx}{a^2+b^2}\\
&= -\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2+b^2}\\
&= -\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}
\end{aligned}$$

Mathematica [A] time = 0.155249, size = 62, normalized size = 0.91

$$\frac{2a^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\sin(x)+b\cos(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*C
os[x] + a*Sin[x])/(a^2 + b^2)
```

Maple [A] time = 0.081, size = 84, normalized size = 1.2

$$2\,\frac{-a\tan(x/2)-b}{\left(a^2+b^2\right)\left((\tan(x/2))^2+1\right)}+8\,\frac{a^2}{\left(4\,a^2+4\,b^2\right)\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tan(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 2/(a^2+b^2)*(-a*tan(1/2*x)-b)/(tan(1/2*x)^2+1)+8*a^2/(4*a^2+4*b^2)/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.515675, size = 350, normalized size = 5.15

$$\frac{\sqrt{a^2+b^2}a^2\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-2\left(a^2b+b^3\right)\cos(x)-2\left(a^3+ab^2\right)\sin(x)}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 -
 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x
) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^2*b + b^3)*cos(x) - 2*(a^3 + a*b^2)
*sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.25161, size = 127, normalized size = 1.87

$$-\frac{a^2 \log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a\tan\left(\frac{1}{2}\,x\right)+b\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*tan(1/2*x) + b)/((a^2 +
 b^2)*(tan(1/2*x)^2 + 1))
```

3.10 $\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=35

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

```
[Out] (b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

Rubi [A] time = 0.0566863, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3097, 3133}

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{bx}{a^2+b^2} - \frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$
$$= \frac{bx}{a^2+b^2} - \frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

Mathematica [C] time = 0.0550763, size = 47, normalized size = 1.34

$$\frac{2x(b-ia)-a\log\left((a\cos(x)+b\sin(x))^2\right)+2ia\tan^{-1}(\tan(x))}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*((-I)*a + b)*x + (2*I)*a*ArcTan[Tan[x]] - a*Log[(a*Cos[x] + b*Sin[x])^2]
)/(2*(a^2 + b^2))
```

Maple [A] time = 0.058, size = 54, normalized size = 1.5

$$-\frac{a\ln(a+b\tan(x))}{a^2+b^2}+\frac{a\ln\left((\tan(x))^2+1\right)}{2a^2+2b^2}+\frac{b\arctan(\tan(x))}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/(a^2+b^2)*ln(a+b*tan(x))+1/2/(a^2+b^2)*a*ln(tan(x)^2+1)+1/(a^2+b^2)*b*ar
ctan(tan(x))
```

Maxima [B] time = 1.71766, size = 119, normalized size = 3.4

$$\frac{2b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}-\frac{a\log\left(-a-\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2}+\frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - a*log(-a - 2*b*sin(x)/(cos(x)
 + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + a*log(sin(x)^2/(cos(x) + 1
)^2 + 1)/(a^2 + b^2)
```

Fricas [A] time = 0.501445, size = 112, normalized size = 3.2

$$\frac{2\,bx - a\log\left(2\,ab\cos\left(x\right)\sin\left(x\right) + \left(a^2 - b^2\right)\cos\left(x\right)^2 + b^2\right)}{2\left(a^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x - a*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/(a^2
+ b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.18595, size = 74, normalized size = 2.11

$$-\frac{ab\log\left(\left|b\tan\left(x\right) + a\right|\right)}{a^2b + b^3} + \frac{bx}{a^2 + b^2} + \frac{a\log\left(\tan\left(x\right)^2 + 1\right)}{2\left(a^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(b*tan(x) + a))/(a^2*b + b^3) + b*x/(a^2 + b^2) + 1/2*a*log(tan
(x)^2 + 1)/(a^2 + b^2)
```

3.11 $\int \frac{1}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0181187, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a\cos(x)+b\sin(x)}\,dx = -\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)$$
$$= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Mathematica [A] time = 0.0251666, size = 38, normalized size = 1.06

$$\frac{2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Maple [A] time = 0.072, size = 35, normalized size = 1.

$$2\,\frac{1}{\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tan\left(x/2\right)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.491015, size = 242, normalized size = 6.72

$$\frac{\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt
(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x
)^2 + b^2))/sqrt(a^2 + b^2)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.2085, size = 82, normalized size = 2.28

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b
 + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

3.12 $\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}$$

```
[Out] Log[Sin[x]]/a - Log[a*Cos[x] + b*Sin[x]]/a
```

Rubi [A] time = 0.0698094, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3101, 3475, 3133}

$$\frac{\log(\sin(x))}{a}-\frac{\log(a\cos(x)+b\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] Log[Sin[x]]/a - Log[a*Cos[x] + b*Sin[x]]/a
```

Rule 3101

```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/a, Int[Cot[c + d*x], x], x] - Dis
t[1/a, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
```

```
), 0]
```

Rubi steps

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{\int \cot(x)\,dx}{a} - \frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a}$$
$$= \frac{\log(\sin(x))}{a} - \frac{\log(a\cos(x)+b\sin(x))}{a}$$

Mathematica [A] time = 0.0453793, size = 20, normalized size = 0.87

$$\frac{\log(\sin(x)) - \log(a\cos(x)+b\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]])/a
```

Maple [A] time = 0.076, size = 21, normalized size = 0.9

$$-\frac{\ln(a+b\tan(x))}{a} + \frac{\ln(\tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/a*ln(a+b*tan(x))+1/a*ln(tan(x))
```

Maxima [B] time = 1.14257, size = 65, normalized size = 2.83

$$-\frac{\log\left(-a - \frac{2\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a} + \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(
x)/(cos(x) + 1))/a
```

Fricas [A] time = 0.503741, size = 123, normalized size = 5.35

$$-\frac{\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\log\left(-\frac{1}{4}\,\cos\left(x\right)^2+\frac{1}{4}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(-1/4*cos(
x)^2 + 1/4))/a
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(x\right)}{a\cos\left(x\right)+b\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x)), x)
```

Giac [A] time = 1.18594, size = 30, normalized size = 1.3

$$-\frac{\log\left(|b\tan\left(x\right)+a|\right)}{a}+\frac{\log\left(|\tan\left(x\right)|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(b*tan(x) + a))/a + log(abs(tan(x)))/a
```

3.13 $\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=55

$$-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}$$

```
[Out] (b*ArcTanh[Cos[x]])/a^2 - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sq
rt[a^2 + b^2]])/a^2 - Csc[x]/a
```

Rubi [A] time = 0.0675421, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3103, 3770, 3074, 206}

$$-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*ArcTanh[Cos[x]])/a^2 - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sq
rt[a^2 + b^2]])/a^2 - Csc[x]/a
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{\csc(x)}{a}-\frac{b\int\csc(x)\,dx}{a^2}+\frac{\left(a^2+b^2\right)\int\frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\ &= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}-\frac{\left(a^2+b^2\right)\operatorname{Subst}\left(\int\frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2}\\ &= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.122909, size = 67, normalized size = 1.22

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)-a\csc(x)+b\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]] - a*Csc[x] +
b*(Log[Cos[x/2]] - Log[Sin[x/2]]))/a^2
```

Maple [B] time = 0.105, size = 107, normalized size = 2.

$$-\frac{1}{2a}\tan\left(\frac{x}{2}\right)+2\frac{1}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2a\tan(x/2)-2b}{\sqrt{a^2+b^2}}\right)+2\frac{b^2}{\sqrt{a^2+b^2}a^2}\mathrm{Artanh}\left(1/2\,\frac{2a\tan(x/2)-2b}{\sqrt{a^2+b^2}}\right)-\frac{1}{2a}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/2/a*tan(1/2*x)+2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b
^2)^(1/2))+2/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)
^(1/2))*b^2-1/2/a/tan(1/2*x)-b/a^2*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.579315, size = 370, normalized size = 6.73

$$\frac{b \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) \sin(x) - b \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) \sin(x) + \sqrt{a^2 + b^2} \log\left(-\frac{2\,ab\cos(x)\sin(x) + \left(a^2 - b^2\right)\cos(x)^2 - 2\,a^2 - b^2 + 2\,\sqrt{a^2 + b^2}(b}{2\,ab\cos(x)\sin(x) + \left(a^2 - b^2\right)\cos(x)^2 + b^2}\right.}{2\,a^2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + sqr
t(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2
 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b
^2)*cos(x)^2 + b^2))*sin(x) - 2*a)/(a^2*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{a\cos(x) + b\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x)), x)
```

Giac [B] time = 1.26104, size = 146, normalized size = 2.65

$$-\frac{b\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}x\right)}{2a}-\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2a\tan\left(\frac{1}{2}x\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}x\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{a^2}+\frac{2b\tan\left(\frac{1}{2}x\right)-a}{2a^2\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(tan(1/2*x)))/a^2 - 1/2*tan(1/2*x)/a - sqrt(a^2 + b^2)*log(abs(2*
a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a
^2 + b^2)))/a^2 + 1/2*(2*b*tan(1/2*x) - a)/(a^2*tan(1/2*x))
```

3.14 $\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=55

$$\frac{(a^2+b^2)\log(\sin(x))}{a^3}-\frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}+\frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}$$

```
[Out] (b*Cot[x])/a^2 - Csc[x]^2/(2*a) + ((a^2 + b^2)*Log[Sin[x]])/a^3 - ((a^2 + b
^2)*Log[a*Cos[x] + b*Sin[x]])/a^3
```

Rubi [A] time = 0.120739, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3103, 3767, 8, 3101, 3475, 3133}

$$\frac{(a^2+b^2)\log(\sin(x))}{a^3}-\frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}+\frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*Cot[x])/a^2 - Csc[x]^2/(2*a) + ((a^2 + b^2)*Log[Sin[x]])/a^3 - ((a^2 + b
^2)*Log[a*Cos[x] + b*Sin[x]])/a^3
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3101

```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/a, Int[Cot[c + d*x], x], x] - Dis
t[1/a, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{\csc^2(x)}{2a}-\frac{b\int \csc^2(x)\,dx}{a^2}+\frac{\left(a^2+b^2\right)\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\
&= -\frac{\csc^2(x)}{2a}+\frac{b\,\text{Subst}\left(\int 1\,dx,x,\cot(x)\right)}{a^2}+\frac{\left(a^2+b^2\right)\int \cot(x)\,dx}{a^3}-\frac{\left(a^2+b^2\right)\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^3}\\
&= \frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}+\frac{\left(a^2+b^2\right)\log(\sin(x))}{a^3}-\frac{\left(a^2+b^2\right)\log(a\cos(x)+b\sin(x))}{a^3}
\end{aligned}
$$

Mathematica [A] time = 0.152216, size = 48, normalized size = 0.87

$$
\frac{2\left(a^2+b^2\right)(\log(\sin(x))-\log(a\cos(x)+b\sin(x)))-a^2\csc^2(x)+2ab\cot(x)}{2a^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a*b*Cot[x] - a^2*Csc[x]^2 + 2*(a^2 + b^2)*(Log[Sin[x]] - Log[a*Cos[x] +
b*Sin[x]]))/(2*a^3)
```

Maple [A] time = 0.09, size = 64, normalized size = 1.2

$$-\frac{\ln(a+b\tan(x))}{a}-\frac{\ln(a+b\tan(x))\,b^2}{a^3}-\frac{1}{2\,a\,(\tan(x))^2}+\frac{\ln(\tan(x))}{a}+\frac{\ln(\tan(x))\,b^2}{a^3}+\frac{b}{a^2\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -1/a*ln(a+b*tan(x))-1/a^3*ln(a+b*tan(x))*b^2-1/2/a/tan(x)^2+1/a*ln(tan(x))+
1/a^3*ln(tan(x))*b^2+b/a^2/tan(x)
```

Maxima [B] time = 1.10915, size = 161, normalized size = 2.93

$$-\frac{\frac{4\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8\,a^2}-\frac{\left(a^2+b^2\right)\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}+\frac{\left(a^2+b^2\right)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}-\frac{\left(a-\frac{4\,b\sin(x)}{\cos(x)+1}\right)(\cos(x)+1)^2}{8\,a^2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^2 - (a^2 + b^2
)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^3 + (a^2
+ b^2)*log(sin(x)/(cos(x) + 1))/a^3 - 1/8*(a - 4*b*sin(x)/(cos(x) + 1))*(co
s(x) + 1)^2/(a^2*sin(x)^2)
```

Fricas [B] time = 0.532769, size = 281, normalized size = 5.11

$$-\frac{2\,ab\cos(x)\sin(x)-a^2+\left(\left(a^2+b^2\right)\cos(x)^2-a^2-b^2\right)\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)-\left(\left(a^2+b^2\right)\mathrm{co}\right.}{2\left(a^3\cos(x)^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(x)*sin(x) - a^2 + ((a^2 + b^2)*cos(x)^2 - a^2 - b^2)*log(2*
a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - ((a^2 + b^2)*cos(x)^2 - a
^2 - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*cos(x)^2 - a^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{a\cos(x) + b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x)), x)
```

Giac [A] time = 1.19962, size = 105, normalized size = 1.91

$$\frac{\left(a^2+b^2\right)\log\left(|\tan(x)|\right)}{a^3} - \frac{\left(a^2b+b^3\right)\log\left(|b\tan(x)+a|\right)}{a^3b} - \frac{3\,a^2\tan(x)^2 + 3\,b^2\tan(x)^2 - 2\,ab\tan(x) + a^2}{2\,a^3\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(tan(x)))/a^3 - (a^2*b + b^3)*log(abs(b*tan(x) + a))/(a^
3*b) - 1/2*(3*a^2*tan(x)^2 + 3*b^2*tan(x)^2 - 2*a*b*tan(x) + a^2)/(a^3*tan(
x)^2)
```

3.15 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=107

$$\frac{-b\left(a^2+b^2\right)\sin(2x)+a\left(a^2+b^2\right)\cos(2x)+3a\left(a^2-b^2\right)}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{6a^2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] (6*a^2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (3
*a*(a^2 - b^2) + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/(2*(a^2 +
 b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rubi [B] time = 1.16922, antiderivative size = 283, normalized size of antiderivative = 2.64, number of steps used = 19, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {4401, 2637, 2638, 6742, 639, 203, 638, 618, 206, 3100, 3074}

$$\frac{3a^3\sin(x)}{b^3\left(a^2+b^2\right)}+\frac{3a^2\cos(x)}{b^2\left(a^2+b^2\right)}+\frac{2a^2\left(a+b\tan\left(\frac{x}{2}\right)\right)}{\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}-\frac{2a^3\cos^2\left(\frac{x}{2}\right)\left(\left(a^2-b^2\right)\tan\left(\frac{x}{2}\right)+2ab\right)}{b^3\left(a^2+b^2\right)^2}+\frac{2a^2\,(\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(3/2
)) - (2*a^2*b*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
+ (2*a^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b*(a^2 +
 b^2)^(5/2)) - Cos[x]/b^2 + (3*a^2*Cos[x])/(b^2*(a^2 + b^2)) - (2*a*Sin[x])
/b^3 + (3*a^3*Sin[x])/(b^3*(a^2 + b^2)) - (2*a^3*Cos[x/2]^2*(2*a*b + (a^2 -
 b^2)*Tan[x/2]))/(b^3*(a^2 + b^2)^2) + (2*a^2*(a + b*Tan[x/2]))/((a^2 + b^2
)^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \int \left(-\frac{2a\cos(x)}{b^3}+\frac{\sin(x)}{b^2}-\frac{a^3\cos^3(x)}{b^3(a\cos(x)+b\sin(x))^2}+\frac{3a^2\cos^2(x)}{b^3(a\cos(x)+b\sin(x))}\right)dx$$

$$= -\frac{(2a)\int\cos(x)\,dx}{b^3}+\frac{(3a^2)\int\frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{b^3}-\frac{a^3\int\frac{\cos^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{b^3}+\frac{\int\sin(x)\,dx}{b^2}$$

$$= -\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(x)}{b^3}-\frac{(2a^3)\,\text{Subst}\left(\int\frac{(1-x^2)^3}{(1+x^2)^2(a+2bx-ax^2)^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{b^3}+$$

$$= -\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(x)}{b^3}+\frac{3a^3\sin(x)}{b^3(a^2+b^2)}-\frac{(2a^3)\,\text{Subst}\left(\int\left(\frac{2(a^2-b^2-2abx)}{(a^2+b^2)^2(1+x^2)^2}+\right.\right.}{}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}-\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(x)}{b^3}+\frac{3a^3\sin(x)}{b^3(a^2+b^2)}-\frac{(4a^3)}{}$$

$$= \frac{a^3(a^2-b^2)x}{b^3(a^2+b^2)^2}-\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}-\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(x)}{b^3}+\frac{3a^3}{b^3(a}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}+\frac{2a^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{5/2}}-\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x}{b^2(a^2+b}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}-\frac{2a^2b\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{2a^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan}{\sqrt{a^2+}}\right.}{b(a^2+b^2)^{5/2}}$$

Mathematica [A] time = 0.425441, size = 107, normalized size = 1.

$$\frac{-b(a^2+b^2)\sin(2x)+a(a^2+b^2)\cos(2x)+3a(a^2-b^2)}{2(a^2+b^2)^2(a\cos(x)+b\sin(x))}+\frac{6a^2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (6*a^2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (3
*a*(a^2 - b^2) + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/(2*(a^2 +
```

```
b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.107, size = 141, normalized size = 1.3

$$4\,\frac{-ab\tan(x/2)+1/2\,a^2-1/2\,b^2}{\left(a^4+2\,a^2b^2+b^4\right)\left((\tan(x/2))^2+1\right)}-4\,\frac{a^2}{a^4+2\,a^2b^2+b^4}\left(\frac{1/2\,b\tan(x/2)+a/2}{(\tan(x/2))^2\,a-2\,b\tan(x/2)-a}-3/2\,\frac{b}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 4/(a^4+2*a^2*b^2+b^4)*(-a*b*tan(1/2*x)+1/2*a^2-1/2*b^2)/(tan(1/2*x)^2+1)-4*
a^2/(a^4+2*a^2*b^2+b^4)*((1/2*b*tan(1/2*x)+1/2*a)/(tan(1/2*x)^2*a-2*b*tan(1
/2*x)-a)-3/2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(
1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.546829, size = 570, normalized size = 5.33

$$\frac{2\,a^5-2\,a^3b^2-4\,ab^4+2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^2-2\left(a^4b+2\,a^2b^3+b^5\right)\cos(x)\sin(x)+3\left(a^3b\cos(x)+a^2b^2\sin(x)\right)}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)\cos(x)+\left(a^6b+3\,a^4b^3+3\,a^2b^5\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5 - 2*a^3*b^2 - 4*a*b^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 - 2
*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)*sin(x) + 3*(a^3*b*cos(x) + a^2*b^2*sin(x)
)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2
- b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^
2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) +
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20356, size = 251, normalized size = 2.35

$$-\frac{3\,a^2 b \log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,x\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,x\right)^2+a^2b\tan\left(\frac{1}{2}\,x\right)-2\,b^3\tan\left(\frac{1}{2}\,x\right)+2\,a^3-ab\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -3*a^2*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*
x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) -
2*(3*a^2*b*tan(1/2*x)^3 - 3*a*b^2*tan(1/2*x)^2 + a^2*b*tan(1/2*x) - 2*b^3*t
an(1/2*x) + 2*a^3 - a*b^2)/((a*tan(1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*tan(1/
2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

3.16 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=64

$$-\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)\left(a\cot(x)+b\right)}-\frac{2ab\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] -(((a^2 - b^2)*x)/(a^2 + b^2)^2) + a/((a^2 + b^2)*(b + a*Cot[x])) - (2*a*b*
Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2
```

Rubi [A] time = 0.118366, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3085, 3483, 3531, 3530}

$$-\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)\left(a\cot(x)+b\right)}-\frac{2ab\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(((a^2 - b^2)*x)/(a^2 + b^2)^2) + a/((a^2 + b^2)*(b + a*Cot[x])) - (2*a*b*
Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \int \frac{1}{(b+a\cot(x))^2}\,dx\\
&= \frac{a}{\left(a^2+b^2\right)(b+a\cot(x))}+\frac{\int \frac{b-a\cot(x)}{b+a\cot(x)}\,dx}{a^2+b^2}\\
&= -\frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(b+a\cot(x))}-\frac{(2ab)\int \frac{-a+b\cot(x)}{b+a\cot(x)}\,dx}{\left(a^2+b^2\right)^2}\\
&= -\frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2}+\frac{a}{\left(a^2+b^2\right)(b+a\cot(x))}-\frac{2ab\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}
\end{aligned}
$$

Mathematica [C] time = 0.24657, size = 121, normalized size = 1.89

$$\frac{\sin(x)\left(-a^2bx+a^3+ab^2(1-2ix)-ab^2\log\left((a\cos(x)+b\sin(x))^2\right)+b^3x\right)-a\cos(x)\left(ab\log\left((a\cos(x)+b\sin(x))^2\right)+x\right.}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-(a*Cos[x]*((a + I*b)^2*x + a*b*Log[(a*Cos[x] + b*Sin[x])^2])) + (a^3 + a*
b^2*(1 - (2*I)*x) - a^2*b*x + b^3*x - a*b^2*Log[(a*Cos[x] + b*Sin[x])^2])*S
in[x] + (2*I)*a*b*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]))/((a^2 + b^2)^2*(a*C
os[x] + b*Sin[x]))
```

Maple [A] time = 0.087, size = 99, normalized size = 1.6

$$-\frac{a^2}{\left(a^2+b^2\right)b\left(a+b\tan\left(x\right)\right)}-2\,\frac{ab\ln\left(a+b\tan\left(x\right)\right)}{\left(a^2+b^2\right)^2}-\frac{\arctan\left(\tan\left(x\right)\right)a^2}{\left(a^2+b^2\right)^2}+\frac{\arctan\left(\tan\left(x\right)\right)b^2}{\left(a^2+b^2\right)^2}+\frac{ab\ln\left(\left(\tan\left(x\right)\right)^2+1\right)}{\left(a^2+b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -a^2/(a^2+b^2)/b/(a+b*tan(x))-2*a*b/(a^2+b^2)^2*ln(a+b*tan(x))-1/(a^2+b^2)^
2*arctan(tan(x))*a^2+1/(a^2+b^2)^2*arctan(tan(x))*b^2+1/(a^2+b^2)^2*a*b*ln(
tan(x)^2+1)
```

Maxima [A] time = 1.69264, size = 158, normalized size = 2.47

$$-\frac{2\,ab\log\left(b\tan\left(x\right)+a\right)}{a^4+2\,a^2b^2+b^4}+\frac{ab\log\left(\tan\left(x\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{a^2}{a^3b+ab^3+\left(a^2b^2+b^4\right)\tan\left(x\right)}-\frac{\left(a^2-b^2\right)x}{a^4+2\,a^2b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(b*tan(x) + a)/(a^4 + 2*a^2*b^2 + b^4) + a*b*log(tan(x)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - a^2/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(x)) - (a^2
 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4)
```

Fricas [B] time = 0.508699, size = 305, normalized size = 4.77

$$-\frac{\left(a^2b+\left(a^3-ab^2\right)x\right)\cos\left(x\right)+\left(a^2b\cos\left(x\right)+ab^2\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\left(a^3-\left(a^2b-\right.\right.}{\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(x\right)+\left(a^4b+2\,a^2b^3+b^5\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + (a^3 - a*b^2)*x)*cos(x) + (a^2*b*cos(x) + a*b^2*sin(x))*log(2*a*
b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 - (a^2*b - b^3)*x)*sin
(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.11171, size = 188, normalized size = 2.94

$$-\frac{2\,ab^2\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{ab\log\left(\tan\left(x\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(a^2-b^2\right)x}{a^4+2\,a^2b^2+b^4}+\frac{2\,ab^3\tan\left(x\right)-a^4+a^2b^2}{\left(a^4b+2\,a^2b^3+b^5\right)\left(b\tan\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + a*b*log(tan(x)^
2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4) + (2
*a*b^3*tan(x) - a^4 + a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(x) + a))
```

3.17 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=60

$$\frac{a}{\left(a^2+b^2\right)\left(a\cos(x)+b\sin(x)\right)}-\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -((b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) + a
/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.0464099, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3154, 3074, 206}

$$\frac{a}{\left(a^2+b^2\right)\left(a\cos(x)+b\sin(x)\right)}-\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) + a
/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{a}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}+\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\ &= \frac{a}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}-\frac{b\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2+b^2}\\ &= -\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}+\frac{a}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}\end{aligned}$$

Mathematica [A] time = 0.154449, size = 62, normalized size = 1.03

$$\frac{a}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}+\frac{2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + a/((a^
2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.092, size = 97, normalized size = 1.6

$$4\,{\frac {2\,b\tan \left(x/2 \right) +2\,a}{ \left(-4\,{a}^{2}-4\,{b}^{2} \right) \left(\left(\tan \left(x/2 \right) \right) ^{2}a-2\,b\tan \left(x/2 \right) -a \right) }}-8\,{\frac {b}{ \left(-4\,{a}^{2}-4\,{b}^{2} \right) \sqrt {{a}^{2}+{b}^{2}}}{\rm Artanh} \left(1/2\,{\frac {2\,a\tan \left(x/2 \right) -2\,b}{\sqrt {{a}^{2}+{b}^{2}}}} \right) }$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 4*(2*b*tan(1/2*x)+2*a)/(-4*a^2-4*b^2)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-8*b
/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^
(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.498819, size = 398, normalized size = 6.63

$$\frac{2a^3 + 2ab^2 + \left(ab\cos\left(x\right) + b^2\sin\left(x\right)\right)\sqrt{a^2 + b^2}\log\left(-\frac{2ab\cos(x)\sin(x) + \left(a^2 - b^2\right)\cos(x)^2 - 2a^2 - b^2 + 2\sqrt{a^2 + b^2}(b\cos(x) - a\sin(x))}{2ab\cos(x)\sin(x) + \left(a^2 - b^2\right)\cos(x)^2 + b^2}\right)}{2\left(\left(a^5 + 2a^3b^2 + ab^4\right)\cos\left(x\right) + \left(a^4b + 2a^2b^3 + b^5\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*
b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b
*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/(
(a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19466, size = 139, normalized size = 2.32

$$-\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b\tan\left(\frac{1}{2}\,x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2
*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*tan(1/2*x) + a)/((a*tan(1
/2*x)^2 - 2*b*tan(1/2*x) - a)*(a^2 + b^2))
```

3.18 $\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

```
[Out] Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.0127537, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {3075}

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx=\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Mathematica [A] time = 0.0205677, size = 17, normalized size = 1.

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-2),x]
```

```
[Out] Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.08, size = 14, normalized size = 0.8

$$-\frac{1}{b\left(a+b\tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -1/b/(a+b*tan(x))
```

Maxima [A] time = 1.12893, size = 19, normalized size = 1.12

$$-\frac{1}{b^{2}\tan\left(x\right)+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(b^2*tan(x) + a*b)
```

Fricas [B] time = 0.46397, size = 95, normalized size = 5.59

$$-\frac{b\cos\left(x\right)-a\sin\left(x\right)}{\left(a^{3}+ab^{2}\right)\cos\left(x\right)+\left(a^{2}b+b^{3}\right)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(x) - a*sin(x))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))
```

Sympy [A] time = 99.0802, size = 75, normalized size = 4.41

$$\begin{cases} -\frac{\tan^2\left(\frac{x}{2}\right)}{ab\tan^2\left(\frac{x}{2}\right)-ab-2b^2\tan\left(\frac{x}{2}\right)} + \frac{1}{ab\tan^2\left(\frac{x}{2}\right)-ab-2b^2\tan\left(\frac{x}{2}\right)} & \text{for } b \neq 0 \\ -\frac{2\tan\left(\frac{x}{2}\right)}{a^2\left(\tan^2\left(\frac{x}{2}\right)-1\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Piecewise((-tan(x/2)**2/(a*b*tan(x/2)**2 - a*b - 2*b**2*tan(x/2)) + 1/(a*b*
tan(x/2)**2 - a*b - 2*b**2*tan(x/2)), Ne(b, 0)), (-2*tan(x/2)/(a**2*(tan(x/
2)**2 - 1)), True))
```

Giac [A] time = 1.1023, size = 18, normalized size = 1.06

$$-\frac{1}{(b\tan(x) + a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(x) + a)*b)
```

3.19 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=63

$$\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])
/(a^2*Sqrt[a^2 + b^2]) + 1/(a*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.0601325, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3093, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])
/(a^2*Sqrt[a^2 + b^2]) + 1/(a*(a*Cos[x] + b*Sin[x]))
```

Rule 3093

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{1}{a(a\cos(x)+b\sin(x))}+\frac{\int \csc(x)\,dx}{a^2}-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\end{aligned}$$

Mathematica [A] time = 0.295683, size = 72, normalized size = 1.14

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{a\csc(x)}{a\cot(x)+b}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((-2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (a*Csc
[x])/(b + a*Cot[x]) - Log[Cos[x/2]] + Log[Sin[x/2]])/a^2
```

Maple [A] time = 0.126, size = 106, normalized size = 1.7

$$-2\,\frac{b\tan(x/2)}{a^2\left((\tan(x/2))^2a-2\,b\tan(x/2)-a\right)}-2\,\frac{1}{a\left((\tan(x/2))^2a-2\,b\tan(x/2)-a\right)}-2\,\frac{b}{\sqrt{a^2+b^2}a^2}\text{Artanh}\left(1/2\,\frac{2\,a\tan(x/2)}{\sqrt{a^2+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -2/a^2/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)*tan(1/2*x)*b-2/a/(tan(1/2*x)^2*a-2
*b*tan(1/2*x)-a)-2/a^2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(
a^2+b^2)^(1/2))+1/a^2*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.595954, size = 563, normalized size = 8.94

$$\frac{2\,a^3 + 2\,ab^2 + \left(ab\cos\left(x\right) + b^2\sin\left(x\right)\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(x)\sin(x) + \left(a^2-b^2\right)\cos(x)^2 - 2\,a^2 - b^2 - 2\,\sqrt{a^2+b^2}(b\cos(x) - a\sin(x))}{2\,ab\cos(x)\sin(x) + \left(a^2-b^2\right)\cos(x)^2 + b^2}\right) - \left(\left(a^3 + \cdots\right.\right.}{2\left(\left(a^5 + a^3b^2\right)\cos\left(x\right) + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log((2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*
cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - (
(a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(1/2*cos(x) + 1/2) + ((a^3
+ a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(-1/2*cos(x) + 1/2))/((a^5 + a^3
*b^2)*cos(x) + (a^4*b + a^2*b^3)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{(a\cos(x) + b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x))**2, x)
```

Giac [A] time = 1.21133, size = 147, normalized size = 2.33

$$\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^2} + \frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^2} - \frac{2\left(b\tan\left(\frac{1}{2}\,x\right) + a\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2 - 2\,b\tan\left(\frac{1}{2}\,x\right) - a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*
b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + log(abs(tan(1/2*x)))/a^2 -
2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)*a^2)
```

3.20 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=49

$$-\frac{\frac{b}{a^2}+\frac{1}{b}}{a+b\tan(x)}-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\cot(x)}{a^2}$$

```
[Out] -(Cot[x]/a^2) - (2*b*Log[Tan[x]])/a^3 + (2*b*Log[a + b*Tan[x]])/a^3 - (b^(-
1) + b/a^2)/(a + b*Tan[x])
```

Rubi [A] time = 0.0761826, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3087, 894}

$$-\frac{\frac{b}{a^2}+\frac{1}{b}}{a+b\tan(x)}-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\cot(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(Cot[x]/a^2) - (2*b*Log[Tan[x]])/a^3 + (2*b*Log[a + b*Tan[x]])/a^3 - (b^(-
1) + b/a^2)/(a + b*Tan[x])
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x^2(a+bx)^2}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2b}{a^3x}+\frac{a^2+b^2}{a^2(a+bx)^2}+\frac{2b^2}{a^3(a+bx)}\right)dx, x, \tan(x)\right)\\ &= -\frac{\cot(x)}{a^2}-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\frac{1}{b}+\frac{b}{a^2}}{a+b\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.194669, size = 76, normalized size = 1.55

$$\frac{a^2\left(-\cot^2(x)\right)+a^2+2b^2\log(a\cos(x)+b\sin(x))-ab\cot(x)(-2\log(a\cos(x)+b\sin(x))+2\log(\sin(x))+1)-2b^2\log(\text{si}}{a^3(a\cot(x)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a^2 + b^2 - a^2*Cot[x]^2 - 2*b^2*Log[Sin[x]] - a*b*Cot[x]*(1 + 2*Log[Sin[x
]] - 2*Log[a*Cos[x] + b*Sin[x]]) + 2*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^3*(b
+ a*Cot[x]))
```

Maple [A] time = 0.118, size = 60, normalized size = 1.2

$$-\frac{1}{b\left(a+b\tan(x)\right)}-\frac{b}{a^2\left(a+b\tan(x)\right)}+2\,\frac{b\ln\left(a+b\tan(x)\right)}{a^3}-\frac{1}{a^2\tan(x)}-2\,\frac{b\ln\left(\tan(x)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -1/b/(a+b*tan(x))-1/a^2*b/(a+b*tan(x))+2*b*ln(a+b*tan(x))/a^3-1/a^2/tan(x)-
2*b*ln(tan(x))/a^3
```

Maxima [A] time = 1.25928, size = 84, normalized size = 1.71

$$-\frac{ab+\left(a^2+2\,b^2\right)\tan(x)}{a^2b^2\tan(x)^2+a^3b\tan(x)}+\frac{2\,b\log\left(b\tan(x)+a\right)}{a^3}-\frac{2\,b\log\left(\tan(x)\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a*b + (a^2 + 2*b^2)*tan(x))/(a^2*b^2*tan(x)^2 + a^3*b*tan(x)) + 2*b*log(b
*tan(x) + a)/a^3 - 2*b*log(tan(x))/a^3
```

Fricas [B] time = 0.528474, size = 346, normalized size = 7.06

$$\frac{2\,a^2\cos(x)^2 + 2\,ab\cos(x)\sin(x) - a^2 + \left(b^2\cos(x)^2 - ab\cos(x)\sin(x) - b^2\right)\log\left(2\,ab\cos(x)\sin(x) + \left(a^2 - b^2\right)\cos(x}{a^3b\cos(x)^2 - a^4\cos(x)\sin(x) - a^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*cos(x)^2 + 2*a*b*cos(x)*sin(x) - a^2 + (b^2*cos(x)^2 - a*b*cos(x)*si
n(x) - b^2)*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (b^2*co
s(x)^2 - a*b*cos(x)*sin(x) - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*b*cos(x)^2
 - a^4*cos(x)*sin(x) - a^3*b)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{(a\cos(x) + b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x))**2, x)
```

Giac [A] time = 1.11624, size = 85, normalized size = 1.73

$$\frac{2\,b\log(|b\tan(x) + a|)}{a^3} - \frac{2\,b\log(|\tan(x)|)}{a^3} - \frac{a^2\tan(x) + 2\,b^2\tan(x) + ab}{\left(b\tan(x)^2 + a\tan(x)\right)a^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*tan(x) + a))/a^3 - 2*b*log(abs(tan(x)))/a^3 - (a^2*tan(x) + 2
*b^2*tan(x) + a*b)/((b*tan(x)^2 + a*tan(x))*a^2*b)
```

3.21 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=118

$$\frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))}-\frac{2b^2\tanh^{-1}(\cos(x))}{a^4}-\frac{\left(a^2+b^2\right)\tanh^{-1}(\cos(x))}{a^4}+\frac{3b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{2b\,\mathrm{cs}}{a}$$

```
[Out] -ArcTanh[Cos[x]]/(2*a^2) - (2*b^2*ArcTanh[Cos[x]])/a^4 - ((a^2 + b^2)*ArcTa
nh[Cos[x]])/a^4 + (3*b*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a
^2 + b^2]])/a^4 + (2*b*Csc[x])/a^3 - (Cot[x]*Csc[x])/(2*a^2) + (a^2 + b^2)/
(a^3*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.180463, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3105, 3093, 3770, 3074, 206, 3768, 3103}

$$\frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))}-\frac{2b^2\tanh^{-1}(\cos(x))}{a^4}-\frac{\left(a^2+b^2\right)\tanh^{-1}(\cos(x))}{a^4}+\frac{3b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{2b\,\mathrm{cs}}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```


```
[Out] -ArcTanh[Cos[x]]/(2*a^2) - (2*b^2*ArcTanh[Cos[x]])/a^4 - ((a^2 + b^2)*ArcTa
nh[Cos[x]])/a^4 + (3*b*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a
^2 + b^2]])/a^4 + (2*b*Csc[x])/a^3 - (Cot[x]*Csc[x])/(2*a^2) + (a^2 + b^2)/
(a^3*(a*Cos[x] + b*Sin[x]))
```


Rule 3105

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/a^2, Int[Sin[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/a^2, Int
[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*b)/a^2, Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3093

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\int \csc^3(x)\,dx}{a^2} - \frac{(2b)\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2} + \frac{\left(a^2+b^2\right)\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2}$$

$$= \frac{2b\csc(x)}{a^3} - \frac{\cot(x)\csc(x)}{2a^2} + \frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))} + \frac{\int \csc(x)\,dx}{2a^2} + \frac{\left(2b^2\right)\int \csc(x)\,dx}{a^4} +$$

$$= -\frac{\tanh^{-1}(\cos(x))}{2a^2} - \frac{2b^2\tanh^{-1}(\cos(x))}{a^4} - \frac{\left(a^2+b^2\right)\tanh^{-1}(\cos(x))}{a^4} + \frac{2b\csc(x)}{a^3} - \frac{\cot(x}{2}$$

$$= -\frac{\tanh^{-1}(\cos(x))}{2a^2} - \frac{2b^2\tanh^{-1}(\cos(x))}{a^4} - \frac{\left(a^2+b^2\right)\tanh^{-1}(\cos(x))}{a^4} + \frac{3b\sqrt{a^2+b^2}\tanh^{}}{a}$$

Mathematica [B] time = 1.73063, size = 270, normalized size = 2.29

$$-48b\sqrt{a^2+b^2}(a\cot(x)+b)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right) + a^2b\sec^2\left(\frac{x}{2}\right) + 12a^2b\log\left(\sin\left(\frac{x}{2}\right)\right) - 12a^2b\log\left(\cos\left(\frac{x}{2}\right)\right) + 8a^2b\tan\Big($$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-48*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(b + a*Co
t[x]) + 8*a^3*Csc[x] + 8*a*b^2*Csc[x] - 12*a^2*b*Log[Cos[x/2]] - 24*b^3*Log
[Cos[x/2]] - 12*a^3*Cot[x]*Log[Cos[x/2]] - 24*a*b^2*Cot[x]*Log[Cos[x/2]] +
12*a^2*b*Log[Sin[x/2]] + 24*b^3*Log[Sin[x/2]] + 12*a^3*Cot[x]*Log[Sin[x/2]]
 + 24*a*b^2*Cot[x]*Log[Sin[x/2]] + a^2*b*Sec[x/2]^2 + a^3*Cot[x]*Sec[x/2]^2
 - a*Csc[x/2]^2*(-4*a*b*Cos[x] + a^2*Cot[x] + b*(a - 4*b*Sin[x])) + 8*a*b^2
*Tan[x/2] + 8*a^2*b*Cot[x]*Tan[x/2])/(8*a^4*(b + a*Cot[x]))
```

Maple [B] time = 0.144, size = 224, normalized size = 1.9

$$\frac{1}{8a^2}\left(\tan\left(\frac{x}{2}\right)\right)^2 + \frac{b}{a^3}\tan\left(\frac{x}{2}\right) - 2\frac{b\tan(x/2)}{a^2\left((\tan(x/2))^2a - 2b\tan(x/2) - a\right)} - 2\frac{\tan(x/2)b^3}{a^4\left((\tan(x/2))^2a - 2b\tan(x/2) - a\right)} - 2\frac{}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 1/8/a^2*tan(1/2*x)^2+1/a^3*tan(1/2*x)*b-2/a^2/(tan(1/2*x)^2*a-2*b*tan(1/2*x
)-a)*tan(1/2*x)*b-2/a^4/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)*tan(1/2*x)*b^3-2/
a/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-2/a^3/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)
*b^2-6/a^4*b*(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/
2))-1/8/a^2/tan(1/2*x)^2+3/2/a^2*ln(tan(1/2*x))+3/a^4*ln(tan(1/2*x))*b^2+1/
a^3*b/tan(1/2*x)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.708885, size = 859, normalized size = 7.28

$$-\frac{6\,a^{2}b\cos\left(x\right)\sin\left(x\right)+4\,a^{3}+12\,ab^{2}-6\,\left(a^{3}+2\,ab^{2}\right)\cos\left(x\right)^{2}-6\,\left(ab\cos\left(x\right)^{3}-ab\cos\left(x\right)+\left(b^{2}\cos\left(x\right)^{2}-b^{2}\right)\sin\left(x\right)\right)\sqrt{a^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*b*cos(x)*sin(x) + 4*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*cos(x)^2
 - 6*(a*b*cos(x)^3 - a*b*cos(x) + (b^2*cos(x)^2 - b^2)*sin(x))*sqrt(a^2 + b
^2)*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(
a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 + b^2)) + 3*((a^3 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a^2*b
+ 2*b^3 - (a^2*b + 2*b^3)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - 3*((a^3
 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a^2*b + 2*b^3 - (a^2*b + 2
*b^3)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^5*cos(x)^3 - a^5*cos(x)
+ (a^4*b*cos(x)^2 - a^4*b)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{(a\cos(x) + b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x))**2, x)
```

Giac [A] time = 1.25371, size = 290, normalized size = 2.46

$$\frac{3\left(a^2+2\,b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{2\,a^4}+\frac{3\left(a^2b+b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a^4}+\frac{a^2\tan\left(\frac{1}{2}\,x\right)^2+8\,ab\tan\left(\frac{1}{2}\,x\right)}{8\,a^4}-\frac{2\left(a^2b\tan\right.}{\left(a\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*(a^2 + 2*b^2)*log(abs(tan(1/2*x)))/a^4 + 3*(a^2*b + b^3)*log(abs(2*a*ta
n(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 +
 b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/8*(a^2*tan(1/2*x)^2 + 8*a*b*tan(1/2*x))/a
^4 - 2*(a^2*b*tan(1/2*x) + b^3*tan(1/2*x) + a^3 + a*b^2)/((a*tan(1/2*x)^2 -
 2*b*tan(1/2*x) - a)*a^4) - 1/8*(18*a^2*tan(1/2*x)^2 + 36*b^2*tan(1/2*x)^2
- 8*a*b*tan(1/2*x) + a^2)/(a^4*tan(1/2*x)^2)
```

3.22 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=98

$$-\frac{bx\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3}+\frac{2ab}{\left(a^2+b^2\right)^2(a\cot(x)+b)}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}+\frac{a\left(a^2-3b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}$$

```
[Out] -((b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3) + a/(2*(a^2 + b^2)*(b + a*Cot[x])^2) +
 (2*a*b)/((a^2 + b^2)^2*(b + a*Cot[x])) + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b
*Sin[x]])/(a^2 + b^2)^3
```

Rubi [A] time = 0.198367, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3085, 3483, 3529, 3531, 3530}

$$-\frac{bx\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3}+\frac{2ab}{\left(a^2+b^2\right)^2(a\cot(x)+b)}+\frac{a}{2\left(a^2+b^2\right)(a\cot(x)+b)^2}+\frac{a\left(a^2-3b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3) + a/(2*(a^2 + b^2)*(b + a*Cot[x])^2) +
 (2*a*b)/((a^2 + b^2)^2*(b + a*Cot[x])) + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b
*Sin[x]])/(a^2 + b^2)^3
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \frac{1}{(b+a\cot(x))^3}\,dx$$

$$= \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{\int \frac{b-a\cot(x)}{(b+a\cot(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{\int \frac{-a^2+b^2-2ab\cot(x)}{b+a\cot(x)}\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{b\left(3a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{\left(a\left(a^2-3b^2\right)\right) \cdots}{\left(a^2 \cdots\right.}$$

$$= -\frac{b\left(3a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{a\left(a^2-3b^2\right)\log \cdots}{\left(\cdots\right.}$$

Mathematica [C] time = 0.771634, size = 114, normalized size = 1.16

$$\frac{bx\left(b^2-3a^2\right)}{\left(a^2+b^2\right)^3}+\frac{3ab\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{a\left(a^2-3b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}+\frac{a^3}{2(a-ib)^2(a+ib)^2(a\cos(x)+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (b*(-3*a^2 + b^2)*x)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b*Sin[
x]])/(a^2 + b^2)^3 + a^3/(2*(a - I*b)^2*(a + I*b)^2*(a*Cos[x] + b*Sin[x])^2
) + (3*a*b*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.108, size = 193, normalized size = 2.

$$\frac{a^3\ln(a+b\tan(x))}{\left(a^2+b^2\right)^3}-3\frac{a\ln(a+b\tan(x))b^2}{\left(a^2+b^2\right)^3}-\frac{a^4}{b^2\left(a^2+b^2\right)^2(a+b\tan(x))}-3\frac{a^2}{\left(a^2+b^2\right)^2(a+b\tan(x))}+\frac{}{2b^2\left(a^2+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] a^3/(a^2+b^2)^3*ln(a+b*tan(x))-3*a/(a^2+b^2)^3*ln(a+b*tan(x))*b^2-a^4/(a^2+
b^2)^2/b^2/(a+b*tan(x))-3*a^2/(a^2+b^2)^2/(a+b*tan(x))+1/2*a^3/b^2/(a^2+b^2
)/(a+b*tan(x))^2-1/2/(a^2+b^2)^3*ln(tan(x)^2+1)*a^3+3/2/(a^2+b^2)^3*ln(tan(
x)^2+1)*a*b^2-3/(a^2+b^2)^3*arctan(tan(x))*a^2*b+1/(a^2+b^2)^3*arctan(tan(x
))*b^3
```

Maxima [B] time = 1.67194, size = 485, normalized size = 4.95

$$-\frac{2\left(3a^2b-b^3\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(a^3-3ab^2\right)\log\left(-a-\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(a^3-3ab^2\right)\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{}{a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(3*a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) + (a^3 - 3*a*b^2)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(co
s(x) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3 - 3*a*b^2)*log(sin(
x)^2/(cos(x) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(2*a^2*b*s
in(x)/(cos(x) + 1) - 2*a^2*b*sin(x)^3/(cos(x) + 1)^3 + (a^3 + 5*a*b^2)*sin(
x)^2/(cos(x) + 1)^2)/(a^6 + 2*a^4*b^2 + a^2*b^4 + 4*(a^5*b + 2*a^3*b^3 + a*
b^5)*sin(x)/(cos(x) + 1) - 2*(a^6 - 3*a^2*b^4 - 2*b^6)*sin(x)^2/(cos(x) + 1
)^2 - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + 2*a^4*
b^2 + a^2*b^4)*sin(x)^4/(cos(x) + 1)^4)
```

Fricas [B] time = 0.535953, size = 632, normalized size = 6.45

$$\frac{a^5 + 7\,a^3b^2 - 2\left(6\,a^3b^2 + \left(3\,a^4b - 4\,a^2b^3 + b^5\right)x\right)\cos\left(x\right)^2 + 2\left(3\,a^4b - 3\,a^2b^3 - 2\left(3\,a^3b^2 - ab^4\right)x\right)\cos\left(x\right)\sin\left(x\right) - 2\left(3\,a^2\right.}{2\left(a^6b^2 + 3\,a^4b^4 + 3\,a^2b^6 + b^8 + \left(a^8 + 2\,a^6b^2 - 2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^5 + 7*a^3*b^2 - 2*(6*a^3*b^2 + (3*a^4*b - 4*a^2*b^3 + b^5)*x)*cos(x)
^2 + 2*(3*a^4*b - 3*a^2*b^3 - 2*(3*a^3*b^2 - a*b^4)*x)*cos(x)*sin(x) - 2*(3
*a^2*b^3 - b^5)*x + (a^3*b^2 - 3*a*b^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*cos(x)
^2 + 2*(a^4*b - 3*a^2*b^3)*cos(x)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 -
b^2)*cos(x)^2 + b^2))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8 + (a^8 + 2*a^6
*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7
)*cos(x)*sin(x))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.18225, size = 327, normalized size = 3.34

$$-\frac{\left(3\,a^2b-b^3\right)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(a^3-3\,ab^2\right)\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{\left(a^3b-3\,ab^3\right)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{3\,a^3b^4\tan\left(x\right)^2-9\,ab^6}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(a^3 - 3*a*b^2
)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^3*b - 3*a*b^3)
*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2*(3*a^3*
b^4*tan(x)^2 - 9*a*b^6*tan(x)^2 + 2*a^6*b*tan(x) + 14*a^4*b^3*tan(x) - 12*a
^2*b^5*tan(x) + a^7 + 9*a^5*b^2 - 4*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*
b^6 + b^8)*(b*tan(x) + a)^2)
```

3.23 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=92

$$\frac{a\left(\left(a^2+4b^2\right)\sin(x)+3ab\cos(x)\right)}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))^2}-\frac{\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] -(((a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5
/2)) + (a*(3*a*b*Cos[x] + (a^2 + 4*b^2)*Sin[x]))/(2*(a^2 + b^2)^2*(a*Cos[x]
 + b*Sin[x])^2)
```

Rubi [B] time = 0.695363, antiderivative size = 300, normalized size of antiderivative = 3.26, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4401, 1660, 12, 618, 206, 3155, 3074}

$$-\frac{ab\left(5a^2+2b^2\right)\tan\left(\frac{x}{2}\right)+3a^2b^2+4a^4+2b^4}{ab\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}+\frac{2\left(\left(a^2+2b^2\right)\tan\left(\frac{x}{2}\right)+ab\right)}{a\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)^2}+\frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (2*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b^2*(a^2 + b^2)^(3/
2)) - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(b^2*Sqrt[a^2 + b^2])
- (a^2*(2*a^2 - b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b^2*(a^2 +
 b^2)^(5/2)) + (2*a)/(b*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])) + (2*(a*b + (a^2
 + 2*b^2)*Tan[x/2]))/(a*(a^2 + b^2)*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2)^2) -
(4*a^4 + 3*a^2*b^2 + 2*b^4 + a*b*(5*a^2 + 2*b^2)*Tan[x/2])/(a*b*(a^2 + b^2)
^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \left(\frac{a^2\cos^2(x)}{b^2(a\cos(x)+b\sin(x))^3} - \frac{2a\cos(x)}{b^2(a\cos(x)+b\sin(x))^2} + \frac{1}{b^2(a\cos(x)+b\sin(x))}\right) dx$$

$$= \frac{\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{b^2} - \frac{(2a)\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{b^2} + \frac{a^2\int \frac{\cos^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx}{b^2}$$

$$= \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))} - \frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)}{b^2} + \frac{\left(2a^2\right)\text{S}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))} + \frac{2\left(ab+\left(a^2+2b^2\right)\right.}{a\left(a^2+b^2\right)\left(a+2b\tan\left(\frac{x}{2}\right.\right.}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{a^2\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{5/2}}$$

Mathematica [A] time = 0.394132, size = 92, normalized size = 1.

$$\frac{a\left(\left(a^2+4b^2\right)\sin(x)+3ab\cos(x)\right)}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))^2} - \frac{\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] -(((a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5
/2)) + (a*(3*a*b*Cos[x] + (a^2 + 4*b^2)*Sin[x]))/(2*(a^2 + b^2)^2*(a*Cos[x]
```

```
+ b*Sin[x])^2)
```

Maple [B] time = 0.12, size = 212, normalized size = 2.3

$$-8\frac{1}{\left((\tan(x/2))^2 a - 2b\tan(x/2) - a\right)^2}\left(-1/8\,\frac{a\left(a^2-2b^2\right)(\tan(x/2))^3}{a^4+2a^2b^2+b^4} + 3/8\,\frac{b\left(a^2-2b^2\right)(\tan(x/2))^2}{a^4+2a^2b^2+b^4} - 1/8\,\frac{\left(a^2+10b^2\right)a}{a^4+2a^2b}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] -8*(-1/8*a*(a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)*tan(1/2*x)^3+3/8*b*(a^2-2*b^2)/(
a^4+2*a^2*b^2+b^4)*tan(1/2*x)^2-1/8*(a^2+10*b^2)*a/(a^4+2*a^2*b^2+b^4)*tan(
1/2*x)-3/8*a^2*b/(a^4+2*a^2*b^2+b^4))/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)^2-(
a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-
2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.52851, size = 656, normalized size = 7.13

$$-\frac{\left(a^2b^2-2b^4+\left(a^4-3a^2b^2+2b^4\right)\cos(x)^2+2\left(a^3b-2ab^3\right)\cos(x)\sin(x)\right)\sqrt{a^2+b^2}\log\left(-\frac{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2}{2ab\cos(x)\sin(x)+(}\right.}{4\left(a^6b^2+3a^4b^4+3a^2b^6+b^8+\left(a^8+2a^6b^2-2a^2b^6-b^8\right)\cos(x)^2+2\left(a^7b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a^2*b^2 - 2*b^4 + (a^4 - 3*a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b - 2*
a*b^3)*cos(x)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^
2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(a^4*b + a^2*b^3)*cos(x)
- 2*(a^5 + 5*a^3*b^2 + 4*a*b^4)*sin(x))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
b^8 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5*b^3 +
 3*a^3*b^5 + a*b^7)*cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.24283, size = 266, normalized size = 2.89

$$\frac{\left(a^{2}-2\,b^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}+\frac{a^{3}\tan\left(\frac{1}{2}\,x\right)^{3}-2\,ab^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-3\,a^{2}b\tan\left(\frac{1}{2}\,x\right)^{2}+6\,b^{3}\tan\left(\frac{1}{2}\,x\right)^{2}+a^{3}\tan}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(a\tan\left(\frac{1}{2}\,x\right)^{2}-2\,b\tan\left(\frac{1}{2}\,x\right)-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 - 2*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)) + (a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - 3*a^2*b*tan(1/2*x)^2 +
6*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) + 10*a*b^2*tan(1/2*x) + 3*a^2*b)/((a^4
+ 2*a^2*b^2 + b^4)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2)
```

3.24 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3} \, dx$

Optimal. Leaf size=15

$$\frac{1}{2a(a\cot(x)+b)^2}$$

```
[Out] 1/(2*a*(b + a*Cot[x])^2)
```

Rubi [A] time = 0.0260682, antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3087, 37}

$$\frac{\tan^2(x)}{2a(a+b\tan(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] Tan[x]^2/(2*a*(a + b*Tan[x])^2)
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Subst}\left(\int \frac{x}{(a+bx)^3}\,dx, x, \tan(x)\right)$$
$$= \frac{\tan^2(x)}{2a(a+b\tan(x))^2}$$

Mathematica [B] time = 0.0874972, size = 47, normalized size = 3.13

$$\frac{a(a+b\sin(2x))+2b^2\sin^2(x)}{2a\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (2*b^2*Sin[x]^2 + a*(a + b*Sin[2*x]))/(2*a*(a^2 + b^2)*(a*Cos[x] + b*Sin[x]
)^2)
```

Maple [B] time = 0.097, size = 29, normalized size = 1.9

$$-\frac{1}{b^2\,(a+b\tan(x))}+\frac{a}{2\,b^2\,(a+b\tan(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] -1/b^2/(a+b*tan(x))+1/2*a/b^2/(a+b*tan(x))^2
```

Maxima [B] time = 1.14235, size = 113, normalized size = 7.53

$$\frac{2\,\sin(x)^2}{\left(a^3+\frac{4\,a^2b\sin(x)}{\cos(x)+1}-\frac{4\,a^2b\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\left(a^3-2\,ab^2\right)\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos(x)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2*sin(x)^2/((a^3 + 4*a^2*b*sin(x)/(cos(x) + 1) - 4*a^2*b*sin(x)^3/(cos(x) +
 1)^3 + a^3*sin(x)^4/(cos(x) + 1)^4 - 2*(a^3 - 2*a*b^2)*sin(x)^2/(cos(x) +
1)^2)*(cos(x) + 1)^2)
```

Fricas [B] time = 0.478099, size = 257, normalized size = 17.13

$$-\frac{4\,ab^2\cos\left(x\right)^2 - a^3 - 3\,ab^2 - 2\left(a^2b - b^3\right)\cos\left(x\right)\sin\left(x\right)}{2\left(a^4b^2 + 2\,a^2b^4 + b^6 + \left(a^6 + a^4b^2 - a^2b^4 - b^6\right)\cos\left(x\right)^2 + 2\left(a^5b + 2\,a^3b^3 + ab^5\right)\cos\left(x\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b^2*cos(x)^2 - a^3 - 3*a*b^2 - 2*(a^2*b - b^3)*cos(x)*sin(x))/(a^
4*b^2 + 2*a^2*b^4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17752, size = 27, normalized size = 1.8

$$-\frac{2\,b\tan\left(x\right) + a}{2\left(b\tan\left(x\right) + a\right)^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*tan(x) + a)/((b*tan(x) + a)^2*b^2)
```

3.25 $\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=73

$$-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^(3/2)) - (b*
Cos[x] - a*Sin[x])/(2*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])^2)
```

Rubi [A] time = 0.0351555, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3076, 3074, 206}

$$-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-3),x]
```

```
[Out] -ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^(3/2)) - (b*
Cos[x] - a*Sin[x])/(2*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])^2)
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}+\frac{\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{2\left(a^2+b^2\right)}$$
$$= -\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}-\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)}{2\left(a^2+b^2\right)}$$
$$= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)(a\cos(x)+b\sin(x))^2}$$

Mathematica [C] time = 0.153636, size = 101, normalized size = 1.38

$$\frac{\left(a^2+b^2\right)(a\sin(x)-b\cos(x))+2\sqrt{a^2+b^2}(a\cos(x)+b\sin(x))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{2(a-ib)^2(a+ib)^2(a\cos(x)+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[x]) + a*Sin[x]) + 2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*T
an[x/2])/Sqrt[a^2 + b^2]]*(a*Cos[x] + b*Sin[x])^2)/(2*(a - I*b)^2*(a + I*b)
^2*(a*Cos[x] + b*Sin[x])^2)
```

Maple [B] time = 0.117, size = 157, normalized size = 2.2

$$-2\,\frac{1}{\left((\tan(x/2))^2a-2\,b\tan(x/2)-a\right)^2}\left(-1/2\,\frac{\left(a^2+2\,b^2\right)(\tan(x/2))^3}{\left(a^2+b^2\right)a}-1/2\,\frac{b\left(a^2-2\,b^2\right)(\tan(x/2))^2}{\left(a^2+b^2\right)a^2}-1/2\,\frac{\left(a^2-2\,b^2\right)\text{ta}}{\left(a^2+b^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] -2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*x)^3-1/2*b*(a^2-2*b^2)/(a^2+b^2)/a
^2*tan(1/2*x)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*x)+1/2*b/(a^2+b^2))/(ta
n(1/2*x)^2*a-2*b*tan(1/2*x)-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.51828, size = 539, normalized size = 7.38

$$\frac{\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^{2}-b^{2}\right)\cos(x)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^{2}-b^{2}\right)\cos(x)^{2}+b^{2}}\right)-2}{4\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}+\left(a^{6}+a^{4}b^{2}-a^{2}b^{4}-b^{6}\right)\cos\left(x\right)^{2}+2\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)\cos\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)*sqrt(a^2 + b^2)*log
(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 +
b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b
^2)) - 2*(a^2*b + b^3)*cos(x) + 2*(a^3 + a*b^2)*sin(x))/(a^4*b^2 + 2*a^2*b^
4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a^5*b + 2*a^3*b^3 +
 a*b^5)*cos(x)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19978, size = 224, normalized size = 3.07

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3+a^2b\tan\left(\frac{1}{2}\,x\right)^2-2\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right)-2\,a}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + (a^3*tan(1/2*x)^3 + 2*a*b^2*
tan(1/2*x)^3 + a^2*b*tan(1/2*x)^2 - 2*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) - 2
*a*b^2*tan(1/2*x) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x
) - a)^2)
```

3.26 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=59

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}-\frac{\log(a+b\tan(x))}{a^3}+\frac{\log(\tan(x))}{a^3}+\frac{\frac{a}{b^2}+\frac{1}{a}}{2(a+b\tan(x))^2}$$

```
[Out] Log[Tan[x]]/a^3 - Log[a + b*Tan[x]]/a^3 + (a^(-1) + a/b^2)/(2*(a + b*Tan[x]
)^2) + (a^(-2) - b^(-2))/(a + b*Tan[x])
```

Rubi [A] time = 0.0819996, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3087, 894}

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}-\frac{\log(a+b\tan(x))}{a^3}+\frac{\log(\tan(x))}{a^3}+\frac{\frac{a}{b^2}+\frac{1}{a}}{2(a+b\tan(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] Log[Tan[x]]/a^3 - Log[a + b*Tan[x]]/a^3 + (a^(-1) + a/b^2)/(2*(a + b*Tan[x]
)^2) + (a^(-2) - b^(-2))/(a + b*Tan[x])
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{x(a+bx)^3}\,dx, x, \tan(x)\right)\\ &= \text{Subst}\left(\int \left(\frac{1}{a^3x}+\frac{-a^2-b^2}{ab(a+bx)^3}+\frac{a^2-b^2}{a^2b(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx, x, \tan(x)\right)\\ &= \frac{\log(\tan(x))}{a^3}-\frac{\log(a+b\tan(x))}{a^3}+\frac{\frac{1}{a}+\frac{a}{b^2}}{2(a+b\tan(x))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}\end{aligned}$$

Mathematica [A] time = 0.196871, size = 96, normalized size = 1.63

$$\frac{2a^2\cot^2(x)(\log(\sin(x))-\log(a\cos(x)+b\sin(x)))+a^2\csc^2(x)+2b^2(-\log(a\cos(x)+b\sin(x))+\log(\sin(x))-1)+2a}{2a^3(a\cot(x)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (a^2*Csc[x]^2 + 2*a*b*Cot[x]*(-1 + 2*Log[Sin[x]] - 2*Log[a*Cos[x] + b*Sin[x
]]) + 2*b^2*(-1 + Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]) + 2*a^2*Cot[x]^2*
(Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]))/(2*a^3*(b + a*Cot[x])^2)
```

Maple [A] time = 0.12, size = 73, normalized size = 1.2

$$\frac{a}{2\,b^2\,(a+b\tan(x))^2}+\frac{1}{2\,a\,(a+b\tan(x))^2}-\frac{1}{b^2\,(a+b\tan(x))}+\frac{1}{a^2\,(a+b\tan(x))}-\frac{\ln(a+b\tan(x))}{a^3}+\frac{\ln(\tan(x))}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] 1/2*a/b^2/(a+b*tan(x))^2+1/2/a/(a+b*tan(x))^2-1/b^2/(a+b*tan(x))+1/a^2/(a+b
*tan(x))-ln(a+b*tan(x))/a^3+ln(tan(x))/a^3
```

Maxima [B] time = 1.24345, size = 232, normalized size = 3.93

$$-\frac{2\left(\frac{2\,ab\sin(x)}{\cos(x)+1}-\frac{2\,ab\sin(x)^3}{(\cos(x)+1)^3}-\frac{\left(a^2-3\,b^2\right)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5+\frac{4\,a^4b\sin(x)}{\cos(x)+1}-\frac{4\,a^4b\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^5\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\left(a^5-2\,a^3b^2\right)\sin(x)^2}{(\cos(x)+1)^2}}-\frac{\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*a*b*sin(x)/(cos(x) + 1) - 2*a*b*sin(x)^3/(cos(x) + 1)^3 - (a^2 - 3*b^
2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + 4*a^4*b*sin(x)/(cos(x) + 1) - 4*a^4*b*si
n(x)^3/(cos(x) + 1)^3 + a^5*sin(x)^4/(cos(x) + 1)^4 - 2*(a^5 - 2*a^3*b^2)*s
in(x)^2/(cos(x) + 1)^2) - log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(co
s(x) + 1)^2)/a^3 + log(sin(x)/(cos(x) + 1))/a^3
```

Fricas [B] time = 0.553722, size = 524, normalized size = 8.88

$$\frac{4\,a^2b^2\cos(x)^2+a^4-a^2b^2-2\left(a^3b-ab^3\right)\cos(x)\sin(x)-\left(a^2b^2+b^4+\left(a^4-b^4\right)\cos(x)^2+2\left(a^3b+ab^3\right)\cos(x)\sin(x)\right)}{2\left(a^5b^2+a^3b^4+\left(a^7-a^3b^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(x)^2 + a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(x)*sin(x) -
 (a^2*b^2 + b^4 + (a^4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))*l
og(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^2*b^2 + b^4 + (a^
4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))*log(-1/4*cos(x)^2 + 1/
4))/(a^5*b^2 + a^3*b^4 + (a^7 - a^3*b^4)*cos(x)^2 + 2*(a^6*b + a^4*b^3)*cos
(x)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x))**3, x)
```

Giac [A] time = 1.18307, size = 104, normalized size = 1.76

$$-\frac{\log\left(|b\tan\left(x\right)+a|\right)}{a^3}+\frac{\log\left(|\tan\left(x\right)|\right)}{a^3}+\frac{3\,b^4\tan\left(x\right)^2-2\,a^3b\tan\left(x\right)+8\,ab^3\tan\left(x\right)-a^4+6\,a^2b^2}{2\,\left(b\tan\left(x\right)+a\right)^2a^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*tan(x) + a))/a^3 + log(abs(tan(x)))/a^3 + 1/2*(3*b^4*tan(x)^2 -
2*a^3*b*tan(x) + 8*a*b^3*tan(x) - a^4 + 6*a^2*b^2)/((b*tan(x) + a)^2*a^3*b^
2)
```

3.27 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=184

$$-\frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{2b}{a^3(a\cos(x)+b\sin(x))}-2$$

```
[Out] (3*b*ArcTanh[Cos[x]])/a^4 - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/
(2*a^2*Sqrt[a^2 + b^2]) - (2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x]
)/Sqrt[a^2 + b^2]])/a^4 - Csc[x]/a^3 - (b*Cos[x] - a*Sin[x])/(2*a^2*(a*Cos[
x] + b*Sin[x])^2) - (2*b)/(a^3*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.224382, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3105, 3076, 3074, 206, 3103, 3770, 3093}

$$-\frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{2b}{a^3(a\cos(x)+b\sin(x))}-2$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```



```
[Out] (3*b*ArcTanh[Cos[x]])/a^4 - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/
(2*a^2*Sqrt[a^2 + b^2]) - (2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x]
)/Sqrt[a^2 + b^2]])/a^4 - Csc[x]/a^3 - (b*Cos[x] - a*Sin[x])/(2*a^2*(a*Cos[
x] + b*Sin[x])^2) - (2*b)/(a^3*(a*Cos[x] + b*Sin[x]))
```


Rule 3105

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/a^2, Int[Sin[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/a^2, Int
[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*b)/a^2, Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3103

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3093

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2} - \frac{(2b)\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2} + \frac{\left(a^2+b^2\right)\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx}{a^2}$$

$$= -\frac{\csc(x)}{a^3} - \frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2} - \frac{2b}{a^3(a\cos(x)+b\sin(x))} + \frac{\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{2a^2} - \frac{b\int \cdots}{}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\csc(x)}{a^3} - \frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2} - \frac{2b}{a^3(a\cos(x)+b\sin(x))} - \frac{\text{Subst}\cdots}{}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}} - \frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} - \frac{\sqrt{a^2+b^2}\tan\cdots}{}$$

Mathematica [A] time = 0.743807, size = 193, normalized size = 1.05

$$\frac{\csc^3(x)(a\cos(x)+b\sin(x))\left(a\left(a^2+b^2\right)\sin(x)+\frac{6\left(a^2+2b^2\right)(a\cos(x)+b\sin(x))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} - 5ab(a\cos(x)+b\sin(x)) + 6b\,\mathrm{l}\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (Csc[x]^3*(a*Cos[x] + b*Sin[x])*(a*(a^2 + b^2)*Sin[x] - 5*a*b*(a*Cos[x] + b
*Sin[x]) + (6*(a^2 + 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(a*C
os[x] + b*Sin[x])^2)/Sqrt[a^2 + b^2] - a*Cot[x/2]*(a*Cos[x] + b*Sin[x])^2 +
 6*b*Log[Cos[x/2]]*(a*Cos[x] + b*Sin[x])^2 - 6*b*Log[Sin[x/2]]*(a*Cos[x] +
b*Sin[x])^2 - a*(a*Cos[x] + b*Sin[x])^2*Tan[x/2]))/(2*a^4*(b + a*Cot[x])^3)
```

Maple [A] time = 0.162, size = 333, normalized size = 1.8

$$-\frac{1}{2a^3}\tan\left(\frac{x}{2}\right)+\frac{1}{a}\left(\tan\left(\frac{x}{2}\right)\right)^3\left(\left(\tan\left(\frac{x}{2}\right)\right)^2 a-2b\tan(x/2)-a\right)^{-2}+6\frac{(\tan(x/2))^3 b^2}{a^3\left((\tan(x/2))^2 a-2b\tan(x/2)-a\right)^2}+5\frac{\cdots}{a^2\left((\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] -1/2/a^3*tan(1/2*x)+1/a/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)^2*tan(1/2*x)^3+6/
a^3/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)^2*tan(1/2*x)^3*b^2+5/a^2/(tan(1/2*x)^
2*a-2*b*tan(1/2*x)-a)^2*tan(1/2*x)^2*b-10/a^4/(tan(1/2*x)^2*a-2*b*tan(1/2*x
)-a)^2*tan(1/2*x)^2*b^3+1/a/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)^2*tan(1/2*x)-
14/a^3/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)^2*tan(1/2*x)*b^2-5/a^2/(tan(1/2*x)
^2*a-2*b*tan(1/2*x)-a)^2*b+3/a^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x
)-2*b)/(a^2+b^2)^(1/2))+6/a^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2
*b)/(a^2+b^2)^(1/2))*b^2-1/2/a^3/tan(1/2*x)-3/a^4*b*ln(tan(1/2*x))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.721376, size = 1084, normalized size = 5.89

$$-\frac{2a^5 - 10a^3b^2 - 12ab^4 - 6\left(a^5 - a^3b^2 - 2ab^4\right)\cos(x)^2 - 18\left(a^4b + a^2b^3\right)\cos(x)\sin(x) - 3\left(2\left(a^3b + 2ab^3\right)\cos(x)^3 - 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^5 - 10*a^3*b^2 - 12*a*b^4 - 6*(a^5 - a^3*b^2 - 2*a*b^4)*cos(x)^2
- 18*(a^4*b + a^2*b^3)*cos(x)*sin(x) - 3*(2*(a^3*b + 2*a*b^3)*cos(x)^3 - 2*
(a^3*b + 2*a*b^3)*cos(x) - (a^2*b^2 + 2*b^4 + (a^4 + a^2*b^2 - 2*b^4)*cos(x
)^2)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*s
in(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(2*(a^3*b^2 + a*b^4)*cos(x)^3 - 2*
(a^3*b^2 + a*b^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*sin(x))
*log(1/2*cos(x) + 1/2) + 6*(2*(a^3*b^2 + a*b^4)*cos(x)^3 - 2*(a^3*b^2 + a*b
^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*sin(x))*log(-1/2*cos(
x) + 1/2))/(2*(a^7*b + a^5*b^3)*cos(x)^3 - 2*(a^7*b + a^5*b^3)*cos(x) - (a^
```

```
6*b^2 + a^4*b^4 + (a^8 - a^4*b^4)*cos(x)^2)*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x))**3, x)
```

Giac [A] time = 1.27457, size = 286, normalized size = 1.55

$$-\frac{3\,b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a^3}-\frac{3\left(a^2+2\,b^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}a^4}+\frac{6\,b\tan\left(\frac{1}{2}\,x\right)-a}{2\,a^4\tan\left(\frac{1}{2}\,x\right)}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+6\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*b*log(abs(tan(1/2*x)))/a^4 - 1/2*tan(1/2*x)/a^3 - 3/2*(a^2 + 2*b^2)*log(
abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*
sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/2*(6*b*tan(1/2*x) - a)/(a^4*tan
(1/2*x)) + (a^3*tan(1/2*x)^3 + 6*a*b^2*tan(1/2*x)^3 + 5*a^2*b*tan(1/2*x)^2
- 10*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) - 14*a*b^2*tan(1/2*x) - 5*a^2*b)/((a
*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2*a^4)
```

3.28 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=117

$$\frac{(a^2+b^2)^2}{2a^3b^2(a+b\tan(x))^2}-\frac{(a^2-3b^2)(a^2+b^2)}{a^4b^2(a+b\tan(x))}+\frac{2(a^2+3b^2)\log(\tan(x))}{a^5}-\frac{2(a^2+3b^2)\log(a+b\tan(x))}{a^5}+\frac{3b\cot(x)}{a^4}-$$

```
[Out] (3*b*Cot[x])/a^4 - Cot[x]^2/(2*a^3) + (2*(a^2 + 3*b^2)*Log[Tan[x]])/a^5 - (
2*(a^2 + 3*b^2)*Log[a + b*Tan[x]])/a^5 + (a^2 + b^2)^2/(2*a^3*b^2*(a + b*Ta
n[x])^2) - ((a^2 - 3*b^2)*(a^2 + b^2))/(a^4*b^2*(a + b*Tan[x]))
```

Rubi [A] time = 0.136647, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3087, 894}

$$\frac{(a^2+b^2)^2}{2a^3b^2(a+b\tan(x))^2}-\frac{(a^2-3b^2)(a^2+b^2)}{a^4b^2(a+b\tan(x))}+\frac{2(a^2+3b^2)\log(\tan(x))}{a^5}-\frac{2(a^2+3b^2)\log(a+b\tan(x))}{a^5}+\frac{3b\cot(x)}{a^4}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (3*b*Cot[x])/a^4 - Cot[x]^2/(2*a^3) + (2*(a^2 + 3*b^2)*Log[Tan[x]])/a^5 - (
2*(a^2 + 3*b^2)*Log[a + b*Tan[x]])/a^5 + (a^2 + b^2)^2/(2*a^3*b^2*(a + b*Ta
n[x])^2) - ((a^2 - 3*b^2)*(a^2 + b^2))/(a^4*b^2*(a + b*Tan[x]))
```

Rule 3087

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[(x^m*(a + b*x
)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3(a+bx)^3}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{a^3x^3} - \frac{3b}{a^4x^2} + \frac{2\left(a^2+3b^2\right)}{a^5x} - \frac{\left(a^2+b^2\right)^2}{a^3b(a+bx)^3} + \frac{a^4-2a^2b^2-3b^4}{a^4b(a+bx)^2} - \frac{2b\left(a^2+3b^2\right.}{a^5(a+bx)}\right.\right.$$
$$= \frac{3b\cot(x)}{a^4} - \frac{\cot^2(x)}{2a^3} + \frac{2\left(a^2+3b^2\right)\log(\tan(x))}{a^5} - \frac{2\left(a^2+3b^2\right)\log(a+b\tan(x))}{a^5} + \frac{(a}{2a^3b^2(a}$$

Mathematica [A] time = 0.760654, size = 208, normalized size = 1.78

$$\frac{2b^2\left(2\left(a^2+3b^2\right)\log(\sin(x))-2\left(a^2+3b^2\right)\log(a\cos(x)+b\sin(x))-3\left(a^2+b^2\right)\right)+\cot^2(x)\left(4a^2\left(\left(a^2+3b^2\right)\log(\sin(x))\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (6*a^3*b*Cot[x]^3 + a^4*Csc[x]^2 - 2*a*b*Cot[x]*(3*a^2 + a^2*Csc[x]^2 - 4*(
a^2 + 3*b^2)*Log[Sin[x]] + 4*a^2*Log[a*Cos[x] + b*Sin[x]] + 12*b^2*Log[a*Co
s[x] + b*Sin[x]]) + 2*b^2*(-3*(a^2 + b^2) + 2*(a^2 + 3*b^2)*Log[Sin[x]] - 2
*(a^2 + 3*b^2)*Log[a*Cos[x] + b*Sin[x]]) + Cot[x]^2*(-(a^4*Csc[x]^2) + 4*a^
2*(3*b^2 + (a^2 + 3*b^2)*Log[Sin[x]] - (a^2 + 3*b^2)*Log[a*Cos[x] + b*Sin[x
]])))/(2*a^5*(b + a*Cot[x])^2)
```

Maple [A] time = 0.143, size = 151, normalized size = 1.3

$$\frac{a}{2\,b^2\,(a+b\tan(x))^2} + \frac{1}{a\,(a+b\tan(x))^2} + \frac{b^2}{2\,a^3\,(a+b\tan(x))^2} - \frac{1}{b^2\,(a+b\tan(x))} + 2\,\frac{1}{a^2\,(a+b\tan(x))} + 3\,\frac{b^2}{a^4\,(a+b\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x))^3,x)
```

```
[Out] 1/2*a/b^2/(a+b*tan(x))^2+1/a/(a+b*tan(x))^2+1/2/a^3*b^2/(a+b*tan(x))^2-1/b^
2/(a+b*tan(x))+2/a^2/(a+b*tan(x))+3/a^4*b^2/(a+b*tan(x))-2*ln(a+b*tan(x))/a
^3-6/a^5*ln(a+b*tan(x))*b^2-1/2/a^3/tan(x)^2+2*ln(tan(x))/a^3+6/a^5*ln(tan(
x))*b^2+3/a^4*b/tan(x)
```

Maxima [B] time = 1.26935, size = 416, normalized size = 3.56

$$-\frac{a^4 - \frac{8\,a^3 b \sin(x)}{\cos(x)+1} - \frac{2\left(a^4+22\,a^2b^2\right)\sin(x)^2}{(\cos(x)+1)^2} + \frac{4\left(21\,a^3b+4\,ab^3\right)\sin(x)^3}{(\cos(x)+1)^3} - \frac{\left(15\,a^4-144\,a^2b^2-112\,b^4\right)\sin(x)^4}{(\cos(x)+1)^4} - \frac{4\left(19\,a^3b+16\,ab^3\right)\sin(x)^5}{(\cos(x)+1)^5}}{8\left(\frac{a^7\sin(x)^2}{(\cos(x)+1)^2} + \frac{4\,a^6b\sin(x)^3}{(\cos(x)+1)^3} - \frac{4\,a^6b\sin(x)^5}{(\cos(x)+1)^5} + \frac{a^7\sin(x)^6}{(\cos(x)+1)^6} - \frac{2\left(a^7-2\,a^5b^2\right)\sin(x)^4}{(\cos(x)+1)^4}\right)} - \frac{\frac{12\,b\sin(x)}{\cos(x)+1} - \cdots}{8 \cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(a^4 - 8*a^3*b*sin(x)/(cos(x) + 1) - 2*(a^4 + 22*a^2*b^2)*sin(x)^2/(co
s(x) + 1)^2 + 4*(21*a^3*b + 4*a*b^3)*sin(x)^3/(cos(x) + 1)^3 - (15*a^4 - 14
4*a^2*b^2 - 112*b^4)*sin(x)^4/(cos(x) + 1)^4 - 4*(19*a^3*b + 16*a*b^3)*sin(
x)^5/(cos(x) + 1)^5)/(a^7*sin(x)^2/(cos(x) + 1)^2 + 4*a^6*b*sin(x)^3/(cos(x
) + 1)^3 - 4*a^6*b*sin(x)^5/(cos(x) + 1)^5 + a^7*sin(x)^6/(cos(x) + 1)^6 -
2*(a^7 - 2*a^5*b^2)*sin(x)^4/(cos(x) + 1)^4) - 1/8*(12*b*sin(x)/(cos(x) + 1
) + a*sin(x)^2/(cos(x) + 1)^2)/a^4 - 2*(a^2 + 3*b^2)*log(-a - 2*b*sin(x)/(c
os(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^5 + 2*(a^2 + 3*b^2)*log(sin(x)/(c
os(x) + 1))/a^5
```

Fricas [B] time = 0.592269, size = 887, normalized size = 7.58

$$-\frac{24\,a^2b^2\cos(x)^4 - a^4 + 6\,a^2b^2 + 2\left(a^4 - 15\,a^2b^2\right)\cos(x)^2 - 2\left(\left(a^4 + 2\,a^2b^2 - 3\,b^4\right)\cos(x)^4 - a^2b^2 - 3\,b^4 - \left(a^4 + a^2b^2 - \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(24*a^2*b^2*cos(x)^4 - a^4 + 6*a^2*b^2 + 2*(a^4 - 15*a^2*b^2)*cos(x)^2
 - 2*((a^4 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b^2
 - 6*b^4)*cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*cos(
x))*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*((a^4
```

```
 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b^2 - 6*b^4)*
cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*cos(x))*sin(x)
)*log(-1/4*cos(x)^2 + 1/4) - 4*(3*(a^3*b - a*b^3)*cos(x)^3 - (2*a^3*b - 3*a
*b^3)*cos(x))*sin(x))/(a^5*b^2 - (a^7 - a^5*b^2)*cos(x)^4 + (a^7 - 2*a^5*b^
2)*cos(x)^2 - 2*(a^6*b*cos(x)^3 - a^6*b*cos(x))*sin(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x))**3, x)
```

Giac [A] time = 1.19542, size = 197, normalized size = 1.68

$$\frac{2\left(a^2+3\,b^2\right)\log\left(|\tan(x)|\right)}{a^5}-\frac{2\left(a^2b+3\,b^3\right)\log\left(|b\tan(x)+a|\right)}{a^5b}-\frac{2\,a^4b\tan(x)^3-4\,a^2b^3\tan(x)^3-12\,b^5\tan(x)^3+a^5\tan\ldots}{2\left(b\tan(x\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 2*(a^2 + 3*b^2)*log(abs(tan(x)))/a^5 - 2*(a^2*b + 3*b^3)*log(abs(b*tan(x) +
 a))/(a^5*b) - 1/2*(2*a^4*b*tan(x)^3 - 4*a^2*b^3*tan(x)^3 - 12*b^5*tan(x)^3
 + a^5*tan(x)^2 - 6*a^3*b^2*tan(x)^2 - 18*a*b^4*tan(x)^2 - 4*a^2*b^3*tan(x)
 + a^3*b^2)/((b*tan(x)^2 + a*tan(x))^2*a^4*b^2)
```

3.29 $\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=66

$$-\frac{i\sin^{-n}(c+dx)\text{Hypergeometric2F1}\left(1,n,n+1,-\frac{1}{2}i(\cot(c+dx)+i)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

```
[Out] ((-I/2)*Hypergeometric2F1[1, n, 1 + n, (-I/2)*(I + Cot[c + d*x])]*(a*Cos[c
+ d*x] + I*a*Sin[c + d*x])^n)/(d*n*Sin[c + d*x]^n)
```

Rubi [A] time = 0.0631324, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {3083}

$$-\frac{i\sin^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;-\frac{1}{2}i(\cot(c+dx)+i)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

```
[Out] ((-I/2)*Hypergeometric2F1[1, n, 1 + n, (-I/2)*(I + Cot[c + d*x])]*(a*Cos[c
+ d*x] + I*a*Sin[c + d*x])^n)/(d*n*Sin[c + d*x]^n)
```

Rule 3083

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^n*Hypergeometric2F1[1, n, n + 1, (b + a*Cot[c + d*x])/(2*b)])/(2*b
*d*n*Sin[c + d*x]^n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] && E
qQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\,{}_2F_1\left(1,n;1+n;-\frac{1}{2}i(i+\cot(c+dx))\right)\sin^{-n}(c+dx)(a\cos(c+d$$

Mathematica [C] time = 3.45183, size = 367, normalized size = 5.56

$$-\frac{4\sin\left(\frac{1}{2}(c+dx)\right)\cos\left(\frac{1}{2}(c+dx)\right)\sin^{-n}(c+dx)(a(\cos(c+dx)+i\sin(c+dx)))^n\left(\text{Hypergeometri}\right.}{d(n-1)\left(2F_1\left(1-n;-2n,1;2-n;-i\tan\left(\frac{1}{2}(c+dx)\right),i\tan\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\left(-2n(i\sin(c+dx)+\cos(c+dx)-1)F_1\left(2-\right.\right.}{}\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

```
[Out] (-4*Cos[(c + d*x)/2]*(AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c + d*x)/2]
, I*Tan[(c + d*x)/2]] + Hypergeometric2F1[1 - 2*n, 1 - n, 2 - n, (-I)*Tan[(
c + d*x)/2]])*Sin[(c + d*x)/2]*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d*(-
1 + n)*Sin[c + d*x]^n*(2*AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c + d*x)
/2], I*Tan[(c + d*x)/2]] + ((-2*n*AppellF1[2 - n, 1 - 2*n, 1, 3 - n, (-I)*T
an[(c + d*x)/2], I*Tan[(c + d*x)/2]]*(-1 + Cos[c + d*x] + I*Sin[c + d*x]) -
 AppellF1[2 - n, -2*n, 2, 3 - n, (-I)*Tan[(c + d*x)/2], I*Tan[(c + d*x)/2]]
*(-1 + Cos[c + d*x] + I*Sin[c + d*x]) + (-2 + n)*(1 + Cos[c + d*x])*(1 + I*
Tan[(c + d*x)/2])^(2*n))*(1 - I*Tan[(c + d*x)/2]))/(-2 + n)))
```

Maple [F] time = 0.631, size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+ia\sin(dx+c))^n}{(\sin(dx+c))^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

```
[Out] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+i\,a\sin(dx+c))^n\sin(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*sin(d*x + c)^(-n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ae^{(i\,dx+i\,c)}\right)^{n}}{\left(\frac{1}{2}\left(-i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-i\,dx-i\,c)}\right)^{n}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="fric
as")
```

```
[Out] integral((a*e^(I*d*x + I*c))^n/(1/2*(-I*e^(2*I*d*x + 2*I*c) + I)*e^(-I*d*x
 - I*c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(sin(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a\cos\left(dx+c\right)+i\,a\sin\left(dx+c\right)\right)^{n}}{\sin\left(dx+c\right)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="giac
")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/sin(d*x + c)^n, x)
```

3.30 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}$$

```
[Out] (5*a*x)/16 - (b*Cos[c + d*x]^6)/(6*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rubi [A] time = 0.0907608, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 2635, 8, 2565, 30}

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - (b*Cos[c + d*x]^6)/(6*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^6(c+dx)+b\cos^5(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^6(c+dx)\,dx+b\int \cos^5(c+dx)\sin(c+dx)\,dx\\ &= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}(5a)\int \cos^4(c+dx)\,dx-\frac{b\,\mathrm{Subst}\left(\int x\right.}{}\\ &= -\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c}{6d}\\ &= -\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c}{24d}\\ &= \frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)}{24d}\end{aligned}$$

Mathematica [A] time = 0.104173, size = 57, normalized size = 0.66

$$\frac{a(45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+60c+60dx)-32b\cos^6(c+dx)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (-32*b*Cos[c + d*x]^6 + a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c
 + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.042, size = 62, normalized size = 0.7

$$\frac{1}{d}\left(a\left(\frac{\sin\left(dx+c\right)}{6}\left(\left(\cos\left(dx+c\right)\right)^{5}+\frac{5\left(\cos\left(dx+c\right)\right)^{3}}{4}+\frac{15\cos\left(dx+c\right)}{8}\right)+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{b\left(\cos\left(dx+c\right)\right)^{6}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)-1/6*b*cos(d*x+c)^6)
```

Maxima [A] time = 1.22386, size = 84, normalized size = 0.97

$$-\frac{32\,b\cos\left(dx+c\right)^{6}+\left(4\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\sin\left(4\,dx+4\,c\right)-48\sin\left(2\,dx+2\,c\right)\right)a}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/192*(32*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin
(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 0.494747, size = 161, normalized size = 1.85

$$-\frac{8\,b\cos\left(dx+c\right)^{6}-15\,adx-\left(8\,a\cos\left(dx+c\right)^{5}+10\,a\cos\left(dx+c\right)^{3}+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*b*cos(d*x + c)^6 - 15*a*d*x - (8*a*cos(d*x + c)^5 + 10*a*cos(d*x +
 c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 4.50227, size = 219, normalized size = 2.52

$$\begin{cases} \frac{5ax\sin^6(c+dx)}{16} + \frac{15ax\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15ax\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5ax\cos^6(c+dx)}{16} + \frac{5a\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5a\sin^3(c+dx)\cos^3(c+\ldots}{6d} \\ x\left(a\cos(c) + b\sin(c)\right)\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) + b*sin(c + d*x)**6/
(6*d) + b*sin(c + d*x)**4*cos(c + d*x)**2/(2*d) + b*sin(c + d*x)**2*cos(c +
 d*x)**4/(2*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**5, True))
```

Giac [A] time = 1.09659, size = 128, normalized size = 1.47

$$\frac{5}{16}\,ax - \frac{b\cos(6\,dx + 6\,c)}{192\,d} - \frac{b\cos(4\,dx + 4\,c)}{32\,d} - \frac{5\,b\cos(2\,dx + 2\,c)}{64\,d} + \frac{a\sin(6\,dx + 6\,c)}{192\,d} + \frac{3\,a\sin(4\,dx + 4\,c)}{64\,d} + \frac{15\,a\sin(2\ldots}{64\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x - 1/192*b*cos(6*d*x + 6*c)/d - 1/32*b*cos(4*d*x + 4*c)/d - 5/64*b*
cos(2*d*x + 2*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 3/64*a*sin(4*d*x + 4*c)/d
 + 15/64*a*sin(2*d*x + 2*c)/d
```

3.31 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

```
[Out] -(b*Cos[c + d*x]^5)/(5*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0700506, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 2633, 2565, 30}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^5)/(5*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^5(c+dx)+b\cos^4(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^5(c+dx)\,dx+b\int \cos^4(c+dx)\sin(c+dx)\,dx\\ &= -\frac{a\,\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{b\,\mathrm{Subst}\left(\int x^4\,dx,\right.}{d}\\ &= -\frac{b\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0156501, size = 60, normalized size = 1.

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^5)/(5*d) + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.04, size = 46, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a\sin(dx+c)}{5}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+c))^2}{3}\right)-\frac{b\,(\cos(dx+c))^5}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/5*b*cos(d*x+c)^
5)
```

Maxima [A] time = 1.25828, size = 66, normalized size = 1.1

$$-\frac{3\,b\cos\left(dx+c\right)^{5}-\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*a)/d
```

Fricas [A] time = 0.478467, size = 126, normalized size = 2.1

$$-\frac{3\,b\cos\left(dx+c\right)^{5}-\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - (3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c))/d
```

Sympy [A] time = 2.23337, size = 87, normalized size = 1.45

$$\begin{cases} \frac{8a\sin^{5}(c+dx)}{15d}+\frac{4a\sin^{3}(c+dx)\cos^{2}(c+dx)}{3d}+\frac{a\sin(c+dx)\cos^{4}(c+dx)}{d}-\frac{b\cos^{5}(c+dx)}{5d} & \text{for}\ d\neq 0\\ x\left(a\cos(c)+b\sin(c)\right)\cos^{4}(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - b*cos(c + d*x)**5/(5*d), Ne(d,
0)), (x*(a*cos(c) + b*sin(c))*cos(c)**4, True))
```

Giac [A] time = 1.12746, size = 115, normalized size = 1.92

$$-\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80*b*cos(5*d*x + 5*c)/d - 1/16*b*cos(3*d*x + 3*c)/d - 1/8*b*cos(d*x + c)
/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*sin(d*x
+ c)/d
```

3.32 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.0777632, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 2635, 8, 2565, 30}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^4(c+dx)+b\cos^3(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^4(c+dx)\,dx+b\int \cos^3(c+dx)\sin(c+dx)\,dx\\ &= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx-\frac{b\,\mathrm{Subst}\left(\int x\right.}{}\\ &= -\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+}{4d}\\ &= \frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\,\mathrm{s}}{4d}\end{aligned}$$

Mathematica [A] time = 0.0909392, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.04, size = 52, normalized size = 0.8

$$\frac{1}{d}\left(a\left(\frac{\sin(dx+c)}{4}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{(\cos(dx+c))^4 b}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/4*cos
(d*x+c)^4*b)
```

Maxima [A] time = 1.16457, size = 65, normalized size = 1.

$$-\frac{8\,b\cos\left(dx+c\right)^{4}-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/32*(8*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a)/d
```

Fricas [A] time = 0.480382, size = 127, normalized size = 1.95

$$-\frac{2\,b\cos\left(dx+c\right)^{4}-3\,adx-\left(2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [A] time = 1.17358, size = 150, normalized size = 2.31

$$\begin{cases}\frac{3ax\sin^{4}\left(c+dx\right)}{8}+\frac{3ax\sin^{2}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{4}+\frac{3ax\cos^{4}\left(c+dx\right)}{8}+\frac{3a\sin^{3}\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a\sin\left(c+dx\right)\cos^{3}\left(c+dx\right)}{8d}+\frac{b\sin^{4}\left(c+dx\right)}{4d}+\frac{b\sin^{2}\left(c\right.}{} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos^{3}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + b*sin(c + d*x)**4/(4*d) + b*sin(c + d*
x)**2*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**3,
 True))
```

Giac [A] time = 1.11555, size = 88, normalized size = 1.35

$$\frac{3}{8}\,ax - \frac{b\cos\left(4\,dx + 4\,c\right)}{32\,d} - \frac{b\cos\left(2\,dx + 2\,c\right)}{8\,d} + \frac{a\sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{a\sin\left(2\,dx + 2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/32*b*cos(4*d*x + 4*c)/d - 1/8*b*cos(2*d*x + 2*c)/d + 1/32*a*sin
(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

3.33 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0646631, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 2633, 2565, 30}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^3(c+dx)+b\cos^2(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^3(c+dx)\,dx+b\int \cos^2(c+dx)\sin(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{b\,\text{Subst}\left(\int x^2\,dx,x,\cos(\right.}{d}\\ &= -\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0116904, size = 44, normalized size = 1.

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.037, size = 36, normalized size = 0.8

$$\frac{1}{d}\left(\frac{a\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}-\frac{(\cos(dx+c))^3\,b}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)-1/3*cos(d*x+c)^3*b)
```

Maxima [A] time = 1.18483, size = 47, normalized size = 1.07

$$-\frac{b\cos\left(dx+c\right)^{3}+\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Fricas [A] time = 0.47403, size = 90, normalized size = 2.05

$$-\frac{b\cos\left(dx+c\right)^{3}-\left(a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A] time = 0.513386, size = 63, normalized size = 1.43

$$\begin{cases} \frac{2a\sin^{3}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}-\frac{b\cos^{3}\left(c+dx\right)}{3d} & \text{for}\: d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos^{2}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - b
*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**2, True
))
```

Giac [A] time = 1.13003, size = 74, normalized size = 1.68

$$-\frac{b\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{b\cos\left(dx+c\right)}{4\,d}+\frac{a\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*b*cos(3*d*x + 3*c)/d - 1/4*b*cos(d*x + c)/d + 1/12*a*sin(3*d*x + 3*c)
/d + 3/4*a*sin(d*x + c)/d
```

3.34 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin^2(c+dx)}{2d}$$

```
[Out] (a*x)/2 + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0434389, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3090, 2635, 8, 2564, 30}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^2(c+dx)+b\cos(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^2(c+dx)\,dx+b\int \cos(c+dx)\sin(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx+\frac{b\,\text{Subst}(\int x\,dx,x,\sin(c+dx)}{d}\\ &= \frac{ax}{2}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0486597, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}-\frac{b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.033, size = 41, normalized size = 1.

$$\frac{1}{d}\left(a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{(\cos(dx+c))^2 b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/2*cos(d*x+c)^2*b)
```

Maxima [A] time = 1.23586, size = 50, normalized size = 1.16

$$-\frac{2\,b\cos\left(dx+c\right)^{2}-\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*cos(d*x + c)^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 0.467528, size = 86, normalized size = 2.

$$\frac{adx-b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [A] time = 0.276042, size = 73, normalized size = 1.7

$$\begin{cases}\frac{ax\sin^{2}\left(c+dx\right)}{2}+\frac{ax\cos^{2}\left(c+dx\right)}{2}+\frac{a\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{b\cos^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) - b*cos(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*cos(c) + b*si
```

```
n(c))*cos(c), True))
```

Giac [A] time = 1.09206, size = 47, normalized size = 1.09

$$\frac{1}{2}\,ax - \frac{b\cos\left(2\,dx + 2\,c\right)}{4\,d} + \frac{a\sin\left(2\,dx + 2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/4*b*cos(2*d*x + 2*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

3.35 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rubi [A] time = 0.0133799, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2637, 2638}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\,dx+b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0115423, size = 46, normalized size = 1.92

$$\frac{a\sin(c)\cos(dx)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d +
(b*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.017, size = 25, normalized size = 1.

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Maxima [A] time = 1.06214, size = 32, normalized size = 1.33

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Fricas [A] time = 0.467715, size = 51, normalized size = 2.12

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d
```

Sympy [A] time = 0.159322, size = 31, normalized size = 1.29

$$a\left(\begin{cases} \frac{\sin(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\cos(c) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} -\frac{\cos(c+dx)}{d} & \text{for}\ d \neq 0 \\ x\sin(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-c
os(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

Giac [A] time = 1.09333, size = 32, normalized size = 1.33

$$-\frac{b\cos(dx+c)}{d} + \frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

3.36 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=17

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0257225, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3086, 3475}

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int (a+b\tan(c+dx))\,dx\\ &= ax+b\int \tan(c+dx)\,dx\\ &= ax-\frac{b\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.0138661, size = 17, normalized size = 1.

$$ax - \frac{b \log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Maple [A] time = 0.065, size = 24, normalized size = 1.4

$$ax - \frac{b \ln(\cos(dx + c))}{d} + \frac{ac}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] a*x-b*ln(cos(d*x+c))/d+1/d*a*c
```

Maxima [A] time = 1.23574, size = 41, normalized size = 2.41

$$\frac{2(dx + c)a - b \log\left(-\sin(dx + c)^2 + 1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*a - b*log(-sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.489085, size = 46, normalized size = 2.71

$$\frac{adx - b \log(-\cos(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (a*d*x - b*log(-cos(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(c+dx)+b\sin(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [A] time = 1.12282, size = 36, normalized size = 2.12

$$\frac{2\,(dx+c)a+b\log\left(\tan(dx+c)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a + b*log(tan(d*x + c)^2 + 1))/d
```

3.37 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rubi [A] time = 0.0453008, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 3770, 2606, 8}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int (a\sec(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx\\ &= a\int \sec(c+dx)\,dx+b\int \sec(c+dx)\tan(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+dx))}{d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0134372, size = 24, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Maple [A] time = 0.066, size = 34, normalized size = 1.4

$$\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/d*b/cos(d*x+c)
```

Maxima [A] time = 1.12085, size = 54, normalized size = 2.25

$$\frac{a(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+\frac{2b}{\cos(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*b/cos(d*x + c))/
d
```

Fricas [B] time = 0.493424, size = 144, normalized size = 6.

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) + 2\,b}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(c+dx) + b\sin(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.12785, size = 73, normalized size = 3.04

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\,b}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

3.38 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rubi [A] time = 0.0578926, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 3767, 8, 2606, 30}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^2(c+dx)+b\sec^2(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^2(c+dx)\,dx+b\int \sec^2(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\,\mathrm{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d}+\frac{b\,\mathrm{Subst}(\int x\,dx,x,\sec(c+dx))}{d}\\ &= \frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0133589, size = 28, normalized size = 1.

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Maple [A] time = 0.072, size = 25, normalized size = 0.9

$$\frac{1}{d}\left(a\tan(dx+c)+\frac{b}{2\,(\cos(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(a*tan(d*x+c)+1/2*b/cos(d*x+c)^2)
```

Maxima [A] time = 1.05665, size = 41, normalized size = 1.46

$$\frac{2\,a\tan\left(dx+c\right)-\frac{b}{\sin(dx+c)^2-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a*tan(d*x + c) - b/(sin(d*x + c)^2 - 1))/d
```

Fricas [A] time = 0.456184, size = 81, normalized size = 2.89

$$\frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

Giac [A] time = 1.15837, size = 34, normalized size = 1.21

$$\frac{b\tan\left(dx+c\right)^{2}+2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

3.39 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0666957, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 3768, 3770, 2606, 30}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^3(c+dx)+b\sec^3(c+dx)\tan(c+dx)\right)\,dx \\
&= a\int \sec^3(c+dx)\,dx + b\int \sec^3(c+dx)\tan(c+dx)\,dx \\
&= \frac{a\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}a\int \sec(c+dx)\,dx + \frac{b\,\text{Subst}\left(\int x^2\,dx,\right.}{d} \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b\sec^3(c+dx)}{3d} + \frac{a\sec(c+dx)\tan(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.0165684, size = 52, normalized size = 1.

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a\tan(c+dx)\sec(c+dx)}{2d} + \frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.075, size = 54, normalized size = 1.

$$\frac{a\sec(dx+c)\tan(dx+c)}{2d} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{2d} + \frac{b}{3d(\cos(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/2*a*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b/cos
(d*x+c)^3
```

Maxima [A] time = 1.09578, size = 82, normalized size = 1.58

$$-\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{4\,b}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

Fricas [A] time = 0.497882, size = 203, normalized size = 3.9

$$\frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+6\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19135, size = 134, normalized size = 2.58

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 3*
a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.40 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0631878, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 3767, 2606, 30}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^4(c+dx)+b\sec^4(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^4(c+dx)\,dx+b\int \sec^4(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{d}+\frac{b\,\text{Subst}\left(\int x^3\,dx,x,\sec\right.}{d}\\ &= \frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0823192, size = 41, normalized size = 0.93

$$\frac{a\left(\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [A] time = 0.079, size = 38, normalized size = 0.9

$$\frac{1}{d}\left(-a\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)+\frac{b}{4\,(\cos(dx+c))^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/4*b/cos(d*x+c)^4)
```

Maxima [A] time = 1.18939, size = 55, normalized size = 1.25

$$\frac{4\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a+\frac{3\,b}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a + 3*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A] time = 0.458911, size = 116, normalized size = 2.64

$$\frac{4\left(2\,a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+3\,b}{12\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d*
x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13837, size = 65, normalized size = 1.48

$$\frac{3\,b\tan\left(dx+c\right)^{4}+4\,a\tan\left(dx+c\right)^{3}+6\,b\tan\left(dx+c\right)^{2}+12\,a\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

3.41 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.0808342, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 3768, 3770, 2606, 30}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\sec^5(c+dx)+b\sec^5(c+dx)\tan(c+dx)\right)dx$$
$$= a\int \sec^5(c+dx)\,dx + b\int \sec^5(c+dx)\tan(c+dx)\,dx$$
$$= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}(3a)\int \sec^3(c+dx)\,dx + \frac{b\,\mathrm{Subst}\left(\int x\right.}{}$$
$$= \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx)}{8d} + \frac{a\sec^3(c+dx)\tan(c+}{4d}$$
$$= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d} + \frac{b\sec^5(c+dx)}{5d} + \frac{3a\sec(c+dx)\tan(c+dx}{8d}$$

Mathematica [A] time = 0.197467, size = 68, normalized size = 0.92

$$\frac{a\tan(c+dx)\sec^3(c+dx)}{4d} + \frac{3a\left(\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\sec(c+dx)\right)}{8d} + \frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A] time = 0.078, size = 74, normalized size = 1.

$$\frac{a(\sec(dx+c))^3\tan(dx+c)}{4d} + \frac{3a\sec(dx+c)\tan(dx+c)}{8d} + \frac{3a\ln(\sec(dx+c)+\tan(dx+c))}{8d} + \frac{b}{5d(\cos(dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/4*a*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a*ln(se
c(d*x+c)+tan(d*x+c))+1/5/d*b/cos(d*x+c)^5
```

Maxima [A] time = 1.16705, size = 116, normalized size = 1.57

$$-\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{16\,b}{\cos(dx+c)^5}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*b/
cos(d*x + c)^5)/d
```

Fricas [A] time = 0.513423, size = 242, normalized size = 3.27

$$\frac{15\,a\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+10\left(3\,a\cos\left(dx+c\right)^3+2\,a\cos\left(dx+\right.\right.}{80\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(-
sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x + c)
 + 16*b)/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19875, size = 190, normalized size = 2.57

$$\frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)^8
 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2
*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

3.42 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0706185, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3090, 3767, 2606, 30}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
&&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^6(c+dx)+b\sec^6(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^6(c+dx)\,dx+b\int \sec^6(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\,\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx)\right)}{d}+\frac{b\,\mathrm{Subst}\left(\int x^5\,dx,\right.}{d}\\ &= \frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.147887, size = 53, normalized size = 0.88

$$\frac{a\left(\frac{1}{5}\tan^5(c+dx)+\frac{2}{3}\tan^3(c+dx)+\tan(c+dx)\right)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

Maple [A] time = 0.078, size = 48, normalized size = 0.8

$$\frac{1}{d}\left(-a\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4\,(\sec(dx+c))^2}{15}\right)\tan(dx+c)+\frac{b}{6\,(\cos(dx+c))^6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/6*b/cos(d*x
+c)^6)
```

Maxima [A] time = 1.14306, size = 72, normalized size = 1.2

$$\frac{2\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a-\frac{5\,b}{\left(\sin(dx+c)^{2}-1\right)^{3}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 5*b/(s
in(d*x + c)^2 - 1)^3)/d
```

Fricas [A] time = 0.472191, size = 147, normalized size = 2.45

$$\frac{2\left(8\,a\cos\left(dx+c\right)^{5}+4\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5\,b}{30\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*
x + c) + 5*b)/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1932, size = 95, normalized size = 1.58

$$\frac{5\,b\tan\left(dx+c\right)^{6}+6\,a\tan\left(dx+c\right)^{5}+15\,b\tan\left(dx+c\right)^{4}+20\,a\tan\left(dx+c\right)^{3}+15\,b\tan\left(dx+c\right)^{2}+30\,a\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

3.43 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$-\frac{a^2\sin^7(c+dx)}{7d}+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{b^2\sin^7(c+dx)}{7d}-\frac{2b^2\sin^5}{5}$$

```
[Out] (-2*a*b*Cos[c + d*x]^7)/(7*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/d + (b^2*Sin[c + d*x]^3)/(3*d) + (3*a^2*Sin[c + d*x]^5)/(5*d) - (2*b^2*Sin
[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^7)/(7*d
)
```

Rubi [A] time = 0.13825, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2633, 2565, 30, 2564, 270}

$$-\frac{a^2\sin^7(c+dx)}{7d}+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{b^2\sin^7(c+dx)}{7d}-\frac{2b^2\sin^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^7)/(7*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/d + (b^2*Sin[c + d*x]^3)/(3*d) + (3*a^2*Sin[c + d*x]^5)/(5*d) - (2*b^2*Sin
[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^7)/(7*d
)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^7(c+dx)+2ab\cos^6(c+dx)\sin(c+dx)+b^2\cos^5(c+dx)\right.$$

$$= a^2\int \cos^7(c+dx)\,dx+(2ab)\int \cos^6(c+dx)\sin(c+dx)\,dx+b^2\int$$

$$= -\frac{a^2\,\mathrm{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{(2ab)\,\mathrm{Sub}}{}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{3a^2\sin^5(c+}{5d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{b^2\sin^3(c+}{3d}$$

Mathematica [A] time = 0.377821, size = 154, normalized size = 1.12

$$-\frac{-3675a^2\sin(c+dx)-735a^2\sin(3(c+dx))-147a^2\sin(5(c+dx))-15a^2\sin(7(c+dx))+1050ab\cos(c+dx)+630ab}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -(1050*a*b*Cos[c + d*x] + 630*a*b*Cos[3*(c + d*x)] + 210*a*b*Cos[5*(c + d*x
)] + 30*a*b*Cos[7*(c + d*x)] - 3675*a^2*Sin[c + d*x] - 525*b^2*Sin[c + d*x]
 - 735*a^2*Sin[3*(c + d*x)] + 35*b^2*Sin[3*(c + d*x)] - 147*a^2*Sin[5*(c +
d*x)] + 63*b^2*Sin[5*(c + d*x)] - 15*a^2*Sin[7*(c + d*x)] + 15*b^2*Sin[7*(c
 + d*x)])/(6720*d)
```

Maple [A] time = 0.064, size = 108, normalized size = 0.8

$$\frac{1}{d}\left(b^{2}\left(-\frac{\sin(dx+c)(\cos(dx+c))^{6}}{7}+\frac{\sin(dx+c)}{35}\left(\frac{8}{3}+(\cos(dx+c))^{4}+\frac{4(\cos(dx+c))^{2}}{3}\right)\right)-\frac{2ab(\cos(dx+c))^{7}}{7}+\frac{a}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.14375, size = 132, normalized size = 0.96

$$-\frac{30\,ab\cos(dx+c)^{7}+3\left(5\,\sin(dx+c)^{7}-21\,\sin(dx+c)^{5}+35\,\sin(dx+c)^{3}-35\,\sin(dx+c)\right)a^{2}-\left(15\,\sin(dx+c)^{7}-\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

Fricas [A] time = 0.499819, size = 221, normalized size = 1.61

$$-\frac{30\,ab\cos(dx+c)^{7}-\left(15\left(a^{2}-b^{2}\right)\cos(dx+c)^{6}+3\left(6\,a^{2}+b^{2}\right)\cos(dx+c)^{4}+4\left(6\,a^{2}+b^{2}\right)\cos(dx+c)^{2}+48\,a^{2}+8\,b\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2 +
 b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*sin
(d*x + c))/d
```

Sympy [A] time = 7.90145, size = 187, normalized size = 1.36

$$\begin{cases} \frac{16a^2\sin^7(c+dx)}{35d} + \frac{8a^2\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^2\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos^6(c+dx)}{d} - \frac{2ab\cos^7(c+dx)}{7d} + \frac{8b^2\sin^7(c+dx)}{105d} + \frac{4b^2\mathrm{s}}{} \\ x\,(a\cos(c)+b\sin(c))^2\cos^5(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**7/(35*d) + 8*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**2*sin(c + d*x)**3*cos(c + d*x)**4/d + a**2*sin(c + d*x
)*cos(c + d*x)**6/d - 2*a*b*cos(c + d*x)**7/(7*d) + 8*b**2*sin(c + d*x)**7/
(105*d) + 4*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + b**2*sin(c + d*x)
**3*cos(c + d*x)**4/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**5
, True))
```

Giac [A] time = 1.16666, size = 209, normalized size = 1.53

$$-\frac{ab\cos(7\,dx+7\,c)}{224\,d} - \frac{ab\cos(5\,dx+5\,c)}{32\,d} - \frac{3\,ab\cos(3\,dx+3\,c)}{32\,d} - \frac{5\,ab\cos(dx+c)}{32\,d} + \frac{\left(a^2-b^2\right)\sin(7\,dx+7\,c)}{448\,d} + \frac{\left(7\,a^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/224*a*b*cos(7*d*x + 7*c)/d - 1/32*a*b*cos(5*d*x + 5*c)/d - 3/32*a*b*cos(
3*d*x + 3*c)/d - 5/32*a*b*cos(d*x + c)/d + 1/448*(a^2 - b^2)*sin(7*d*x + 7*
c)/d + 1/320*(7*a^2 - 3*b^2)*sin(5*d*x + 5*c)/d + 1/192*(21*a^2 - b^2)*sin(
3*d*x + 3*c)/d + 5/64*(7*a^2 + b^2)*sin(d*x + c)/d
```

3.44 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=174

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}-b^2$$

```
[Out] (5*a^2*x)/16 + (b^2*x)/16 - (a*b*Cos[c + d*x]^6)/(3*d) + (5*a^2*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (b^2*Cos[c + d*x]^5*Sin[c + d
*x])/(6*d)
```

Rubi [A] time = 0.170358, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2635, 8, 2565, 30, 2568}

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}-b^2$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/16 + (b^2*x)/16 - (a*b*Cos[c + d*x]^6)/(3*d) + (5*a^2*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (b^2*Cos[c + d*x]^5*Sin[c + d
*x])/(6*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^6(c+dx)+2ab\cos^5(c+dx)\sin(c+dx)+b^2\cos^4(c+dx)\right.$$

$$= a^2\int \cos^6(c+dx)\,dx+(2ab)\int \cos^5(c+dx)\sin(c+dx)\,dx+b^2\int$$

$$= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{b^2\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}\left(5a^2\right)$$

$$= -\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{b^2\cos^3(c+dx)}{24d}$$

$$= -\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{b^2\cos(c+dx)}{16d}$$

$$= \frac{5a^2x}{16}+\frac{b^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{16d}+$$

Mathematica [A] time = 0.24063, size = 147, normalized size = 0.84

$$\frac{(5a^2+b^2)(c+dx)}{16d}+\frac{(15a^2+b^2)\sin(2(c+dx))}{64d}+\frac{(3a^2-b^2)\sin(4(c+dx))}{64d}+\frac{(a^2-b^2)\sin(6(c+dx))}{192d}-\frac{5ab\cos(2(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((5*a^2 + b^2)*(c + d*x))/(16*d) - (5*a*b*Cos[2*(c + d*x)])/(32*d) - (a*b*C
os[4*(c + d*x)])/(16*d) - (a*b*Cos[6*(c + d*x)])/(96*d) + ((15*a^2 + b^2)*S
in[2*(c + d*x)])/(64*d) + ((3*a^2 - b^2)*Sin[4*(c + d*x)])/(64*d) + ((a^2 -
 b^2)*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.061, size = 118, normalized size = 0.7

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^5}{6}+\frac{\sin(dx+c)}{24}\left((\cos(dx+c))^3+\frac{3\cos(dx+c)}{2}\right)+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ab(\cos(dx+c))^6}{3}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*s
in(d*x+c)+1/16*d*x+1/16*c)-1/3*a*b*cos(d*x+c)^6+a^2*(1/6*(cos(d*x+c)^5+5/4*
cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.22801, size = 138, normalized size = 0.79

$$-\frac{64\,ab\cos(dx+c)^6+\left(4\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\sin(4\,dx+4\,c)-48\sin(2\,dx+2\,c)\right)a^2-\left(4\sin(2\,dx+2\,c\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/192*(64*a*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*s
in(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - (4*sin(2*d*x + 2*c)^3 + 12*d*x
```

```
 + 12*c - 3*sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A] time = 0.503055, size = 223, normalized size = 1.28

$$-\frac{16\,ab\cos\left(dx+c\right)^{6}-3\left(5\,a^{2}+b^{2}\right)dx-\left(8\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{5}+2\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{3}+3\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a*b*cos(d*x + c)^6 - 3*(5*a^2 + b^2)*d*x - (8*(a^2 - b^2)*cos(d*x
 + c)^5 + 2*(5*a^2 + b^2)*cos(d*x + c)^3 + 3*(5*a^2 + b^2)*cos(d*x + c))*si
n(d*x + c))/d
```

Sympy [A] time = 4.89803, size = 384, normalized size = 2.21

$$\begin{cases}\frac{5a^{2}x\sin^{6}\left(c+dx\right)}{16}+\frac{15a^{2}x\sin^{4}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{16}+\frac{15a^{2}x\sin^{2}\left(c+dx\right)\cos^{4}\left(c+dx\right)}{16}+\frac{5a^{2}x\cos^{6}\left(c+dx\right)}{16}+\frac{5a^{2}\sin^{5}\left(c+dx\right)\cos\left(c+dx\right)}{16d}+\frac{5a^{2}\sin^{3}\left(c+dx\right)\cos}{6d} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^{2}\cos^{4}\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**2*x*cos(c
+ d*x)**6/16 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 + a*b*sin(c + d*x)**6/(3*d) + a*b*sin(c + d*x)**4*cos(c + d*x)**2/d + a*b*
sin(c + d*x)**2*cos(c + d*x)**4/d + b**2*x*sin(c + d*x)**6/16 + 3*b**2*x*si
n(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4
/16 + b**2*x*cos(c + d*x)**6/16 + b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d)
+ b**2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**2*sin(c + d*x)*cos(c + d*
x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**4, True))
```

Giac [A] time = 1.15153, size = 178, normalized size = 1.02

$$\frac{1}{16}\left(5\,a^{2}+b^{2}\right)x-\frac{ab\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{ab\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{5\,ab\cos\left(2\,dx+2\,c\right)}{32\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{\left(3\,a^{2}-b^{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(5*a^2 + b^2)*x - 1/96*a*b*cos(6*d*x + 6*c)/d - 1/16*a*b*cos(4*d*x + 4
*c)/d - 5/32*a*b*cos(2*d*x + 2*c)/d + 1/192*(a^2 - b^2)*sin(6*d*x + 6*c)/d
+ 1/64*(3*a^2 - b^2)*sin(4*d*x + 4*c)/d + 1/64*(15*a^2 + b^2)*sin(2*d*x + 2
*c)/d
```

3.45 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

```
[Out] (-2*a*b*Cos[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c + d*x]^
3)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d) - (b^2*S
in[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.121948, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2633, 2565, 30, 2564, 14}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c + d*x]^
3)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d) - (b^2*S
in[c + d*x]^5)/(5*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^5(c+dx)+2ab\cos^4(c+dx)\sin(c+dx)+b^2\cos^3(c+dx\right.$$

$$= a^2\int \cos^5(c+dx)\,dx+(2ab)\int \cos^4(c+dx)\sin(c+dx)\,dx+b^2\int$$

$$= -\frac{a^2\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, -\sin(c+dx)\right)}{d}-\frac{(2ab)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^5(c}{5d}$$

$$= -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c}{3d}$$

Mathematica [A] time = 0.171228, size = 116, normalized size = 1.13

$$\frac{150a^2\sin(c+dx)+25a^2\sin(3(c+dx))+3a^2\sin(5(c+dx))-60ab\cos(c+dx)-30ab\cos(3(c+dx))-6ab\cos(5(c+}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-60*a*b*Cos[c + d*x] - 30*a*b*Cos[3*(c + d*x)] - 6*a*b*Cos[5*(c + d*x)] +
150*a^2*Sin[c + d*x] + 30*b^2*Sin[c + d*x] + 25*a^2*Sin[3*(c + d*x)] - 5*b^
2*Sin[3*(c + d*x)] + 3*a^2*Sin[5*(c + d*x)] - 3*b^2*Sin[5*(c + d*x)])/(240*
d)
```

Maple [A] time = 0.059, size = 88, normalized size = 0.9

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)-\frac{2\,ab(\cos(dx+c))^5}{5}+\frac{a^2\sin(dx+c)}{5}\left(\frac{8}{3}+(\mathrm{co}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2/
5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.09447, size = 104, normalized size = 1.01

$$-\frac{6\,ab\cos(dx+c)^5-\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)a^2+\left(3\,\sin(dx+c)^5-5\,\sin(dx+c)^3\right)b^2}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

Fricas [A] time = 0.486424, size = 169, normalized size = 1.64

$$-\frac{6\,ab\cos(dx+c)^5-\left(3\left(a^2-b^2\right)\cos(dx+c)^4+\left(4\,a^2+b^2\right)\cos(dx+c)^2+8\,a^2+2\,b^2\right)\sin(dx+c)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2)
*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.3534, size = 138, normalized size = 1.34

$$\begin{cases} \frac{8a^2\sin^5(c+dx)}{15d} + \frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^4(c+dx)}{d} - \frac{2ab\cos^5(c+dx)}{5d} + \frac{2b^2\sin^5(c+dx)}{15d} + \frac{b^2\sin^3(c+dx)\cos^2(c+dx)}{3d} & \text{for } d \\ x\left(a\cos(c) + b\sin(c)\right)^2\cos^3(c) & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d - 2*a*b*cos(c + d*x)**5/
(5*d) + 2*b**2*sin(c + d*x)**5/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x)**
2/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**3, True))
```

Giac [A] time = 1.16224, size = 154, normalized size = 1.5

$$-\frac{ab\cos(5\,dx + 5\,c)}{40\,d} - \frac{ab\cos(3\,dx + 3\,c)}{8\,d} - \frac{ab\cos(dx + c)}{4\,d} + \frac{\left(a^2 - b^2\right)\sin(5\,dx + 5\,c)}{80\,d} + \frac{\left(5\,a^2 - b^2\right)\sin(3\,dx + 3\,c)}{48\,d} + $$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40*a*b*cos(5*d*x + 5*c)/d - 1/8*a*b*cos(3*d*x + 3*c)/d - 1/4*a*b*cos(d*x
 + c)/d + 1/80*(a^2 - b^2)*sin(5*d*x + 5*c)/d + 1/48*(5*a^2 - b^2)*sin(3*d*
x + 3*c)/d + 1/8*(5*a^2 + b^2)*sin(d*x + c)/d
```

3.46 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3a^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b^2\text{si}}{}$$

```
[Out] (3*a^2*x)/8 + (b^2*x)/8 - (a*b*Cos[c + d*x]^4)/(2*d) + (3*a^2*Cos[c + d*x]*
Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c +
d*x]^3*Sin[c + d*x])/(4*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rubi [A] time = 0.134596, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2635, 8, 2565, 30, 2568}

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3a^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b^2\text{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/8 + (b^2*x)/8 - (a*b*Cos[c + d*x]^4)/(2*d) + (3*a^2*Cos[c + d*x]*
Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c +
d*x]^3*Sin[c + d*x])/(4*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)+2ab\cos^3(c+dx)\sin(c+dx)+b^2\cos^2(c+dx\right.$$

$$= a^2\int \cos^4(c+dx)\,dx + (2ab)\int \cos^3(c+dx)\sin(c+dx)\,dx + b^2\int$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{b^2\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{1}{4}\left(3a^2\right.$$

$$= -\frac{ab\cos^4(c+dx)}{2d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{b^2\cos(c+dx)\,\mathrm{si}}{8d}$$

$$= \frac{3a^2x}{8} + \frac{b^2x}{8} - \frac{ab\cos^4(c+dx)}{2d} + \frac{3a^2\cos(c+dx)\sin(c+dx)}{8d} + \frac{b^2\,\mathrm{c}}{}$$

Mathematica [A] time = 0.225099, size = 98, normalized size = 0.78

$$\frac{\left(3a^2+b^2\right)(c+dx)}{8d}+\frac{\left(a^2-b^2\right)\sin(4(c+dx))}{32d}+\frac{a^2\sin(2(c+dx))}{4d}-\frac{ab\cos(2(c+dx))}{4d}-\frac{ab\cos(4(c+dx))}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + b^2)*(c + d*x))/(8*d) - (a*b*Cos[2*(c + d*x)])/(4*d) - (a*b*Cos[4
*(c + d*x)])/(16*d) + (a^2*Sin[2*(c + d*x)])/(4*d) + ((a^2 - b^2)*Sin[4*(c
+ d*x)])/(32*d)
```

Maple [A] time = 0.058, size = 97, normalized size = 0.8

$$\frac{1}{d}\left(b^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab(\cos(dx+c))^4}{2}+a^2\left(\frac{\sin(dx+c)}{4}\left((\cos(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.12365, size = 101, normalized size = 0.8

$$-\frac{16\,ab\cos(dx+c)^4-(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^2-(4\,dx+4\,c-\sin(4\,dx+4\,c))b^2}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32*(16*a*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*
d*x + 2*c))*a^2 - (4*d*x + 4*c - sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A] time = 0.482353, size = 170, normalized size = 1.35

$$-\frac{4\,ab\cos(dx+c)^4-\left(3\,a^2+b^2\right)dx-\left(2\left(a^2-b^2\right)\cos(dx+c)^3+\left(3\,a^2+b^2\right)\cos(dx+c)\right)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x + c
)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.36889, size = 260, normalized size = 2.06

$$\begin{cases} \frac{3a^2x\sin^4(c+dx)}{8} + \frac{3a^2x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^2x\cos^4(c+dx)}{8} + \frac{3a^2\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^2\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{ab\sin^4(c+dx)}{2d} + \frac{a \\ x\,(a\cos(c) + b\sin(c))^2\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + a*b*sin(c + d*x)**4/(2*
d) + a*b*sin(c + d*x)**2*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**4/8 + b**
2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + b**2*x*cos(c + d*x)**4/8 + b**2*sin
(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d),
Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**2, True))
```

Giac [A] time = 1.17283, size = 115, normalized size = 0.91

$$\frac{1}{8}\left(3\,a^2 + b^2\right)x - \frac{ab\cos(4\,dx + 4\,c)}{16\,d} - \frac{ab\cos(2\,dx + 2\,c)}{4\,d} + \frac{a^2\sin(2\,dx + 2\,c)}{4\,d} + \frac{\left(a^2 - b^2\right)\sin(4\,dx + 4\,c)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*a^2 + b^2)*x - 1/16*a*b*cos(4*d*x + 4*c)/d - 1/4*a*b*cos(2*d*x + 2*c
)/d + 1/4*a^2*sin(2*d*x + 2*c)/d + 1/32*(a^2 - b^2)*sin(4*d*x + 4*c)/d
```

3.47 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

```
[Out] (-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0908333, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3090, 2633, 2565, 30, 2564}

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\cos^3(c+dx)+2ab\cos^2(c+dx)\sin(c+dx)+b^2\cos(c+dx)\mathrm{s}\right.\\ &= a^2\int \cos^3(c+dx)\,dx+(2ab)\int \cos^2(c+dx)\sin(c+dx)\,dx+b^2\int \\ &= -\frac{a^2\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{(2ab)\,\mathrm{Subst}\left(\int x^2\,dx,\right.}{d}\\ &= -\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+}{3d}\end{aligned}$$

Mathematica [A] time = 0.388145, size = 64, normalized size = 0.96

$$\frac{\sin(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+5a^2+b^2\right)-3ab\cos(c+dx)-ab\cos(3(c+dx))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x] - a*b*Cos[3*(c + d*x)] + (5*a^2 + b^2 + (a^2 - b^2)*Co
s[2*(c + d*x)])*Sin[c + d*x])/(6*d)
```

Maple [A] time = 0.051, size = 52, normalized size = 0.8

$$\frac{1}{d}\left(\frac{b^2\left(\sin\left(dx+c\right)\right)^3}{3}-\frac{2\,ab\left(\cos\left(dx+c\right)\right)^3}{3}+\frac{a^2\left(2+\left(\cos\left(dx+c\right)\right)^2\right)\sin\left(dx+c\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3-2/3*a*b*cos(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*sin
(d*x+c))
```

Maxima [A] time = 1.124, size = 70, normalized size = 1.04

$$-\frac{2\,ab\cos\left(dx+c\right)^{3}-b^{2}\sin\left(dx+c\right)^{3}+\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d
*x + c))*a^2)/d
```

Fricas [A] time = 0.472204, size = 120, normalized size = 1.79

$$-\frac{2\,ab\cos\left(dx+c\right)^{3}-\left(\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+2\,a^{2}+b^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*sin
(d*x + c))/d
```

Sympy [A] time = 0.545085, size = 85, normalized size = 1.27

$$\begin{cases}\frac{2a^{2}\sin^{3}\left(c+dx\right)}{3d}+\frac{a^{2}\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}-\frac{2ab\cos^{3}\left(c+dx\right)}{3d}+\frac{b^{2}\sin^{3}\left(c+dx\right)}{3d} & \text{for}\ d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^{2}\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d - 2*a*b*cos(c + d*x)**3/(3*d) + b**2*sin(c + d*x)**3/(3*d), Ne(d, 0)), (
x*(a*cos(c) + b*sin(c))**2*cos(c), True))
```

Giac [A] time = 1.15405, size = 99, normalized size = 1.48

$$-\frac{ab\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{ab\cos\left(dx+c\right)}{2\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(3\,a^{2}+b^{2}\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*a*b*cos(3*d*x + 3*c)/d - 1/2*a*b*cos(d*x + c)/d + 1/12*(a^2 - b^2)*sin
(3*d*x + 3*c)/d + 1/4*(3*a^2 + b^2)*sin(d*x + c)/d
```

3.48 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rubi [A] time = 0.0200204, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}+\frac{1}{2}\left(a^2+b^2\right)\int 1\\ &= \frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}\end{aligned}$$

Mathematica [A] time = 0.100649, size = 52, normalized size = 0.95

$$\frac{2\left(a^2+b^2\right)(c+dx)+\left(a^2-b^2\right)\sin(2(c+dx))-2ab\cos(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
d*x)])/(4*d)
```

Maple [A] time = 0.058, size = 70, normalized size = 1.3

$$\frac{1}{d}\left(b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-(\cos(dx+c))^2ab+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-cos(d*x+c)^2*a*b+a^2*(1
/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.17941, size = 92, normalized size = 1.67

$$-\frac{ab\cos(dx+c)^2}{d}+\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))a^2}{4\,d}+\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(2
*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

Fricas [A] time = 0.479537, size = 120, normalized size = 2.18

$$-\frac{2\,ab\cos\left(dx+c\right)^{2}-\left(a^{2}+b^{2}\right)dx-\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

Sympy [A] time = 0.321601, size = 128, normalized size = 2.33

$$\begin{cases}\frac{a^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{a^{2}x\cos^{2}\left(c+dx\right)}{2}+\frac{a^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}-\frac{ab\cos^{2}\left(c+dx\right)}{d}+\frac{b^{2}x\sin^{2}\left(c+dx\right)}{2}+\frac{b^{2}x\cos^{2}\left(c+dx\right)}{2}-\frac{b^{2}\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d} & \text{for} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^{2} & \text{oth}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) - a*b*cos(c + d*x)**2/d + b**2*x*sin(c + d*x)**2
/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

Giac [A] time = 1.11429, size = 68, normalized size = 1.24

$$\frac{1}{2}\left(a^{2}+b^{2}\right)x-\frac{ab\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
+ 2*c)/d
```

3.49 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + (a^2*Sin[c + d*x])
/d - (b^2*Sin[c + d*x])/d
```

Rubi [A] time = 0.07073, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3090, 2637, 2638, 2592, 321, 206}

$$\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + (a^2*Sin[c + d*x])
/d - (b^2*Sin[c + d*x])/d
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\cos(c+dx)+2ab\sin(c+dx)+b^2\sin(c+dx)\tan(c+dx)\right)\,dx\\ &= a^2\int\cos(c+dx)\,dx+(2ab)\int\sin(c+dx)\,dx+b^2\int\sin(c+dx)\tan(\\ &= -\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}+\frac{b^2\operatorname{Subst}\left(\int\frac{x^2}{1-x^2}\,dx,x,\sin(c+d\right.}{d}\\ &= -\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+\frac{b^2\operatorname{Subst}\left(\int\frac{1}{1-}\right.}{}\\ &= \frac{b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{b^2\sin(c}{d}\end{aligned}$$

Mathematica [A] time = 0.141852, size = 84, normalized size = 1.53

$$\frac{\left(a^2-b^2\right)\sin(c+dx)-2ab\cos(c+dx)+b^2\left(\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

Maple [A] time = 0.085, size = 63, normalized size = 1.2

$$-2\frac{ab\cos(dx+c)}{d}+\frac{a^2\sin(dx+c)}{d}-\frac{b^2\sin(dx+c)}{d}+\frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -2*a*b*cos(d*x+c)/d+a^2*sin(d*x+c)/d-b^2*sin(d*x+c)/d+1/d*b^2*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 1.18698, size = 81, normalized size = 1.47

$$\frac{b^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-4ab\cos(dx+c)+2a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) -
 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

Fricas [A] time = 0.505515, size = 155, normalized size = 2.82

$$-\frac{4ab\cos(dx+c)-b^2\log(\sin(dx+c)+1)+b^2\log(-\sin(dx+c)+1)-2\left(a^2-b^2\right)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x + c
) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x), x)
```

Giac [A] time = 1.17296, size = 120, normalized size = 2.18

$$\frac{b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c) - 2*a*b)/(ta
n(1/2*d*x + 1/2*c)^2 + 1))/d
```

3.50 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=39

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] (a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.0564281, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {3086, 3477, 3475}

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3477

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[(b^2*Tan[c + d*x])/d,
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int (a+b\tan(c+dx))^2\,dx\\ &= \left(a^2-b^2\right)x+\frac{b^2\tan(c+dx)}{d}+(2ab)\int \tan(c+dx)\,dx\\ &= \left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.123585, size = 69, normalized size = 1.77

$$\frac{2b^2\tan(c+dx)-i\left((a+ib)^2\log(-\tan(c+dx)+i)-(a-ib)^2\log(\tan(c+dx)+i)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-I)*((a + I*b)^2*Log[I - Tan[c + d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]
]) + 2*b^2*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.095, size = 57, normalized size = 1.5

$$a^2x-b^2x+\frac{b^2\tan(dx+c)}{d}-2\,\frac{ab\ln(\cos(dx+c))}{d}+\frac{a^2c}{d}-\frac{b^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] a^2*x-b^2*x+b^2*tan(d*x+c)/d-2*a*b*ln(cos(d*x+c))/d+1/d*a^2*c-1/d*b^2*c
```

Maxima [A] time = 1.69376, size = 66, normalized size = 1.69

$$\frac{(dx+c)a^2-(dx+c-\tan(dx+c))b^2-ab\log\left(-\sin(dx+c)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a^2 - (d*x + c - tan(d*x + c))*b^2 - a*b*log(-sin(d*x + c)^2 + 1
))/d
```

Fricas [A] time = 0.493956, size = 146, normalized size = 3.74

$$\frac{\left(a^2 - b^2\right)dx\cos\left(dx + c\right) - 2\,ab\cos\left(dx + c\right)\log\left(-\cos\left(dx + c\right)\right) + b^2\sin\left(dx + c\right)}{d\cos\left(dx + c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - b^2)*d*x*cos(d*x + c) - 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) + b^2
*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\cos\left(c + dx\right) + b\sin\left(c + dx\right)\right)^2 \sec^2\left(c + dx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [A] time = 1.15995, size = 59, normalized size = 1.51

$$\frac{ab\log\left(\tan\left(dx + c\right)^2 + 1\right) + b^2\tan\left(dx + c\right) + \left(a^2 - b^2\right)\left(dx + c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*b*log(tan(d*x + c)^2 + 1) + b^2*tan(d*x + c) + (a^2 - b^2)*(d*x + c))/d
```

3.51 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0936005, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3090, 3770, 2606, 8, 2611}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\sec(c+dx)+2ab\sec(c+dx)\tan(c+dx)+b^2\sec(c+dx)\,\mathrm{ta}\right.$$

$$= a^2\int \sec(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan(c+dx)\,dx+b^2\int \mathrm{s}$$

$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}b^2\int \sec($$

$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\sec(c+dx}{d}$$

Mathematica [A] time = 0.0469392, size = 67, normalized size = 1.

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.102, size = 98, normalized size = 1.5

$$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+2\frac{ab}{d\cos(dx+c)}+\frac{b^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2}+\frac{b^2\sin(dx+c)}{2d}-\frac{b^2\ln(\sec(dx+c)+\tan(}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/d*a*b/cos(d*x+c)+1/2/d*b^2*sin(d*x+c)^3
/cos(d*x+c)^2+1/2*b^2*sin(d*x+c)/d-1/2/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.19866, size = 120, normalized size = 1.79

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-2a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-\frac{}{\text{co}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 8*a*b/cos(d*x + c))/d
```

Fricas [A] time = 0.498405, size = 234, normalized size = 3.49

$$\frac{(2a^2-b^2)\cos(dx+c)^2\log(\sin(dx+c)+1)-(2a^2-b^2)\cos(dx+c)^2\log(-\sin(dx+c)+1)+8ab\cos(dx+c)+2b^2}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18976, size = 165, normalized size = 2.46

$$\frac{\left(2\,a^{2}-b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,a^{2}-b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+b^{2}\,\mathrm{t}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^2)/d
```

3.52 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$\frac{\tan^3(c+dx)(a\cot(c+dx)+b)^3}{3bd}$$

```
[Out] ((b + a*Cot[c + d*x])^3*Tan[c + d*x]^3)/(3*b*d)
```

Rubi [A] time = 0.0464568, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$\frac{\tan^3(c+dx)(a\cot(c+dx)+b)^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((b + a*Cot[c + d*x])^3*Tan[c + d*x]^3)/(3*b*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^4}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{(b+a\cot(c+dx))^3\tan^3(c+dx)}{3bd}$$

Mathematica [A] time = 0.0404834, size = 46, normalized size = 1.53

$$\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + (b^2*Tan[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.103, size = 48, normalized size = 1.6

$$\frac{1}{d}\left(a^2\tan(dx+c)+\frac{ab}{(\cos(dx+c))^2}+\frac{b^2(\sin(dx+c))^3}{3(\cos(dx+c))^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+a*b/cos(d*x+c)^2+1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A] time = 1.0484, size = 61, normalized size = 2.03

$$\frac{b^2\tan(dx+c)^3+3a^2\tan(dx+c)-\frac{3ab}{\sin(dx+c)^2-1}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 3*a*b/(sin(d*x + c)^2 - 1))/
d
```

Fricas [A] time = 0.463329, size = 131, normalized size = 4.37

$$\frac{3\,ab\cos\left(dx+c\right)+\left(\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c)
)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16747, size = 55, normalized size = 1.83

$$\frac{b^2\tan\left(dx+c\right)^3+3\,ab\tan\left(dx+c\right)^2+3\,a^2\tan\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

3.53 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=120

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\sec}{4d}$$

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.141966, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 3768, 3770, 2606, 30, 2611}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\sec}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\sec^3(c+dx)+2ab\sec^3(c+dx)\tan(c+dx)+b^2\sec^3(c+dx)\right. \\ &= a^2\int \sec^3(c+dx)\,dx+(2ab)\int \sec^3(c+dx)\tan(c+dx)\,dx+b^2\int \\ &= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{2}a^2\int \\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+}{2d} \\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\sec^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0762503, size = 120, normalized size = 1.

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\sec^3(}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.105, size = 143, normalized size = 1.2

$$\frac{a^2\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{2ab}{3d(\cos(dx+c))^3}+\frac{b^2(\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{b^2(\sin(d}{8d(\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/2*a^2*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+2/3/d*a
*b/cos(d*x+c)^3+1/4/d*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/8/d*b^2*sin(d*x+c)^3/
cos(d*x+c)^2+1/8*b^2*sin(d*x+c)/d-1/8/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.09274, size = 174, normalized size = 1.45

$$\frac{3b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-12a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)\right.}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
- 1)) + 32*a*b/cos(d*x + c)^3)/d
```

Fricas [A] time = 0.525434, size = 289, normalized size = 2.41

$$\frac{3\left(4a^2-b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(4a^2-b^2\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)+32ab\cos(dx+c)}{48d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2 -
 b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19329, size = 336, normalized size = 2.8

$$3\left(4\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1/2*
d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1/2*c
)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*a*
b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x
+ 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.54 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=85

$$\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^4(c+dx)}{2d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((a^2 + b^2)*Tan[c + d*x]^3
)/(3*d) + (a*b*Tan[c + d*x]^4)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0766075, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^4(c+dx)}{2d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((a^2 + b^2)*Tan[c + d*x]^3
)/(3*d) + (a*b*Tan[c + d*x]^4)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^2(1+x^2)}{x^6}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^6}+\frac{2ab}{x^5}+\frac{a^2+b^2}{x^4}+\frac{2ab}{x^3}+\frac{a^2}{x^2}\right) dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{ab\tan}{}$$

Mathematica [A] time = 0.184587, size = 54, normalized size = 0.64

$$\frac{(a+b\tan(c+dx))^3\left(a^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx)+10b^2\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
d*x]^2))/(30*b^3*d)
```

Maple [A] time = 0.111, size = 82, normalized size = 1.

$$\frac{1}{d}\left(-a^2\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)+\frac{ab}{2(\cos(dx+c))^4}+b^2\left(\frac{(\sin(dx+c))^3}{5(\cos(dx+c))^5}+\frac{2(\sin(dx+c))^3}{15(\cos(dx+c))^3}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/2*a*b/cos(d*x+c)^4+b^2*(1/5*
sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A] time = 1.14459, size = 95, normalized size = 1.12

$$\frac{10\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^2+2\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)b^2+\frac{15\,ab}{\left(\sin(dx+c)^2-1\right)^2}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 + 5*ta
n(d*x + c)^3)*b^2 + 15*a*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A] time = 0.474388, size = 184, normalized size = 2.16

$$\frac{15\,ab\cos\left(dx+c\right)+2\left(2\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^4+\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^2+3\,b^2\right)\sin\left(dx+c\right)}{30\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b^
2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15193, size = 108, normalized size = 1.27

$$\frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+10\,a^2\tan\left(dx+c\right)^3+10\,b^2\tan\left(dx+c\right)^3+30\,ab\tan\left(dx+c\right)^2+30\,a^2\tan\left(dx\right.}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
+ 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

3.55 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{b^2\tanh^{-1}(\mathrm{s}}{16}$$

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
(2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (b
^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Tan[c
+ d*x])/(6*d)
```

Rubi [A] time = 0.169557, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 3768, 3770, 2606, 30, 2611}

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{b^2\tanh^{-1}(\mathrm{s}}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```


```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
(2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (b
^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Tan[c
+ d*x])/(6*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\sec^5(c+dx)+2ab\sec^5(c+dx)\tan(c+dx)+b^2\sec^5(c+dx)\right.\\
&= a^2\int \sec^5(c+dx)\,dx+(2ab)\int \sec^5(c+dx)\tan(c+dx)\,dx+b^2\int \mathrm{s}\\
&= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{b^2\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{1}{4}\left(3a^2\right)\\
&= \frac{2ab\sec^5(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2\sec^3(c+dx)\tan}{4d}\\
&= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\sec^5(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c}{8d}\\
&= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{2ab\sec^5(c+dx}{5d}\end{aligned}$$

Mathematica [A] time = 0.555151, size = 104, normalized size = 0.62

$$\frac{15\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))+10\left(6a^2-b^2\right)\tan(c+dx)\sec^3(c+dx)+15\left(6a^2-b^2\right)\tan(c+dx)\sec(c+dx)+8b}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (15*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]] + 15*(6*a^2 - b^2)*Sec[c + d*x]*Tan
[c + d*x] + 10*(6*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x] + 8*b*Sec[c + d*x]
^5*(12*a + 5*b*Tan[c + d*x]))/(240*d)
```

Maple [A] time = 0.11, size = 189, normalized size = 1.1

$$\frac{a^2(\sec(dx+c))^3\tan(dx+c)}{4d}+\frac{3a^2\sec(dx+c)\tan(dx+c)}{8d}+\frac{3a^2\ln(\sec(dx+c)+\tan(dx+c))}{8d}+\frac{2ab}{5d(\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^2*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^2
*ln(sec(d*x+c)+tan(d*x+c))+2/5/d*a*b/cos(d*x+c)^5+1/6/d*b^2*sin(d*x+c)^3/co
s(d*x+c)^6+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1/16/d*b^2*sin(d*x+c)^3/cos(
d*x+c)^2+1/16*b^2*sin(d*x+c)/d-1/16/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.09373, size = 243, normalized size = 1.45

$$\frac{5b^2\left(\frac{2\left(3\sin(dx+c)^5-8\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-30a^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(d}{\sin(dx+c)^4-2\sin(dx+c)}\right.}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin
(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c)
```

```
 + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x
+ c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d
```

Fricas [A] time = 0.525928, size = 343, normalized size = 2.04

$$\frac{15\left(6a^2-b^2\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(6a^2-b^2\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+192\,ab\cos\left(dx+c\right.}{480\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3*(
6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20809, size = 463, normalized size = 2.76

$$\frac{15\left(6a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(6a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(150a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+15b^2\tan\left(\frac{1}{2}dx\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^11
+ 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210*a^
2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan(1/2
*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x + 1/2
*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^5 + 39
0*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan
(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1/2*d*x +
 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 96
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.56 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^6(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{d}+\frac{ab\tan^2(c+}{d}$$

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((2*a^2 + b^2)*Tan[c + d*x]
^3)/(3*d) + (a*b*Tan[c + d*x]^4)/d + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) +
 (a*b*Tan[c + d*x]^6)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.104404, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^6(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{d}+\frac{ab\tan^2(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```


```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((2*a^2 + b^2)*Tan[c + d*x]
^3)/(3*d) + (a*b*Tan[c + d*x]^4)/d + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) +
 (a*b*Tan[c + d*x]^6)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^2(1+x^2)^2}{x^8}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^8}+\frac{2ab}{x^7}+\frac{a^2+2b^2}{x^6}+\frac{4ab}{x^5}+\frac{2a^2+b^2}{x^4}+\frac{2ab}{x^3}+\frac{a^2}{x^2}\right)dx, x, \text{cot}$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{\left(2a^2+b^2\right)\tan^3(c+dx)}{3d}+\frac{ab\,\text{t}}{}$$

Mathematica [A] time = 0.654685, size = 104, normalized size = 0.83

$$\frac{\tan(c+dx)\left(21\left(a^2+2b^2\right)\tan^4(c+dx)+35\left(2a^2+b^2\right)\tan^2(c+dx)+105a^2+35ab\tan^5(c+dx)+105ab\tan^3(c+dx\right.}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d*
x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*Ta
n[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

Maple [A] time = 0.114, size = 110, normalized size = 0.9

$$\frac{1}{d}\left(-a^2\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4(\sec(dx+c))^2}{15}\right)\tan(dx+c)+\frac{ab}{3(\cos(dx+c))^6}+b^2\left(\frac{(\sin(dx+c))^3}{7(\cos(dx+c))^7}+\frac{4(\sin(d}{35(\cos(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*b/cos
(d*x+c)^6+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5
+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A] time = 1.12142, size = 123, normalized size = 0.98

$$\frac{7\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{2}+\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)b^{2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/105*(7*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (15
*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*b^2 - 35*a*b/(sin(
d*x + c)^2 - 1)^3)/d
```

Fricas [A] time = 0.487539, size = 231, normalized size = 1.85

$$\frac{35\,ab\cos\left(dx+c\right)+\left(8\left(7\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+4\left(7\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{4}+3\left(7\,a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+15\,b^{2}\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 - b
^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c))
/(d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17727, size = 159, normalized size = 1.27

$$\frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5+42\,b^2\tan\left(dx+c\right)^5+105\,ab\tan\left(dx+c\right)^4+70\,a^2\tan\left(
}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)^
5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^3
+ 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))/d
```

3.57 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=265

$$-\frac{3a^2b\cos^8(c+dx)}{8d}+\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^3}{}$$

```
[Out] (35*a^3*x)/128 + (15*a*b^2*x)/128 - (b^3*Cos[c + d*x]^6)/(6*d) - (3*a^2*b*C
os[c + d*x]^8)/(8*d) + (b^3*Cos[c + d*x]^8)/(8*d) + (35*a^3*Cos[c + d*x]*Si
n[c + d*x])/(128*d) + (15*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^
3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (5*a*b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(64*d) + (7*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a*b^2*Cos[c + d*
x]^5*Sin[c + d*x])/(16*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3*a*
b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rubi [A] time = 0.245691, antiderivative size = 265, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14}

$$-\frac{3a^2b\cos^8(c+dx)}{8d}+\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (35*a^3*x)/128 + (15*a*b^2*x)/128 - (b^3*Cos[c + d*x]^6)/(6*d) - (3*a^2*b*C
os[c + d*x]^8)/(8*d) + (b^3*Cos[c + d*x]^8)/(8*d) + (35*a^3*Cos[c + d*x]*Si
n[c + d*x])/(128*d) + (15*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^
3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (5*a*b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(64*d) + (7*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a*b^2*Cos[c + d*
x]^5*Sin[c + d*x])/(16*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3*a*
b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^8(c+dx)+3a^2b\cos^7(c+dx)\sin(c+dx)+3ab^2\cos^6(c+
$$
$$= a^3\int \cos^8(c+dx)\,dx + \left(3a^2b\right)\int \cos^7(c+dx)\sin(c+dx)\,dx + \left(3ab^2\right.$$
$$= \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3ab^2\cos^7(c+dx)\sin(c+dx)}{8d} + \frac{1}{8}\left(7a^3\right.$$
$$= -\frac{3a^2b\cos^8(c+dx)}{8d} + \frac{7a^3\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{ab^2\cos^5(c+d}{16}$$
$$= -\frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + \frac{b^3\cos^8(c+dx)}{8d} + \frac{35a^3\cos^3}{}$$
$$= -\frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + \frac{b^3\cos^8(c+dx)}{8d} + \frac{35a^3\cos(}{}$$
$$= \frac{35a^3x}{128} + \frac{15}{128}ab^2x - \frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + \frac{b^3\cos^8(}{8d}$$

Mathematica [A] time = 0.474988, size = 235, normalized size = 0.89

$$\frac{5a\left(7a^2+3b^2\right)(c+dx)}{128d} + \frac{a\left(14a^2+3b^2\right)\sin(2(c+dx))}{64d} + \frac{a\left(7a^2-3b^2\right)\sin(4(c+dx))}{128d} + \frac{a\left(2a^2-3b^2\right)\sin(6(c+dx))}{192d} + $$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a*(7*a^2 + 3*b^2)*(c + d*x))/(128*d) - (3*b*(7*a^2 + b^2)*Cos[2*(c + d*x
)])/(128*d) - (b*(21*a^2 + b^2)*Cos[4*(c + d*x)])/(256*d) - (b*(9*a^2 - b^2
)*Cos[6*(c + d*x)])/(384*d) - (b*(3*a^2 - b^2)*Cos[8*(c + d*x)])/(1024*d) +
 (a*(14*a^2 + 3*b^2)*Sin[2*(c + d*x)])/(64*d) + (a*(7*a^2 - 3*b^2)*Sin[4*(c
 + d*x)])/(128*d) + (a*(2*a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*d) + (a*(a^2
- 3*b^2)*Sin[8*(c + d*x)])/(1024*d)
```

Maple [A] time = 0.076, size = 175, normalized size = 0.7

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{8} - \frac{(\cos(dx+c))^6}{24}\right) + 3\,ab^2\left(-1/8\,\sin(dx+c)(\cos(dx+c))^7 + 1/48\left((\cos(dx+c))\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a*b^2*(-1/8*s
in(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))
*sin(d*x+c)+5/128*d*x+5/128*c)-3/8*a^2*b*cos(d*x+c)^8+a^3*(1/8*(cos(d*x+c)^
7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d
*x+35/128*c))
```

Maxima [A] time = 1.09768, size = 220, normalized size = 0.83

$$-\frac{1152\,a^2b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\,c\right)-768\,\sin\left(2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3072*(1152*a^2*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3
 - 3*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin
(4*d*x + 4*c))*a*b^2 - 128*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin(d*x
 + c)^4)*b^3)/d
```

Fricas [A] time = 0.533803, size = 350, normalized size = 1.32

$$-\frac{64\,b^3\cos\left(dx+c\right)^6+48\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^8-15\left(7\,a^3+3\,ab^2\right)dx-\left(48\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^7+8\left(7\,a^3+3\,a\right.\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/384*(64*b^3*cos(d*x + c)^6 + 48*(3*a^2*b - b^3)*cos(d*x + c)^8 - 15*(7*a
^3 + 3*a*b^2)*d*x - (48*(a^3 - 3*a*b^2)*cos(d*x + c)^7 + 8*(7*a^3 + 3*a*b^2
)*cos(d*x + c)^5 + 10*(7*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 15*(7*a^3 + 3*a*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 22.5692, size = 508, normalized size = 1.92

$$\begin{cases}\frac{35a^3x\sin^8(c+dx)}{128}+\frac{35a^3x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{105a^3x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{35a^3x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{35a^3x\cos^8(c+dx)}{128}+\frac{35a^3\sin}{} \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos^5(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((35*a**3*x*sin(c + d*x)**8/128 + 35*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**3*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**3*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**3*x*cos(c + d*x)**8/128 + 35*a**3*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**3*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**3*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - 3*a**2*b*cos(c + d*x)**8/(8*d) + 15*a
*b**2*x*sin(c + d*x)**8/128 + 15*a*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 45*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**2*x*sin(c + d*
x)**2*cos(c + d*x)**6/32 + 15*a*b**2*x*cos(c + d*x)**8/128 + 15*a*b**2*sin(
c + d*x)**7*cos(c + d*x)/(128*d) + 55*a*b**2*sin(c + d*x)**5*cos(c + d*x)**
3/(128*d) + 73*a*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 15*a*b**2*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**6/
(6*d) - b**3*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3
*cos(c)**5, True))
```

Giac [A] time = 1.18124, size = 294, normalized size = 1.11

$$\frac{5}{128}\left(7a^3+3ab^2\right)x-\frac{\left(3a^2b-b^3\right)\cos\left(8dx+8c\right)}{1024d}-\frac{\left(9a^2b-b^3\right)\cos\left(6dx+6c\right)}{384d}-\frac{\left(21a^2b+b^3\right)\cos\left(4dx+4c\right)}{256d}-\frac{3\left(7a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/128*(7*a^3 + 3*a*b^2)*x - 1/1024*(3*a^2*b - b^3)*cos(8*d*x + 8*c)/d - 1/3
84*(9*a^2*b - b^3)*cos(6*d*x + 6*c)/d - 1/256*(21*a^2*b + b^3)*cos(4*d*x +
4*c)/d - 3/128*(7*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/1024*(a^3 - 3*a*b^2)*
sin(8*d*x + 8*c)/d + 1/192*(2*a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 1/128*(7*
a^3 - 3*a*b^2)*sin(4*d*x + 4*c)/d + 1/64*(14*a^3 + 3*a*b^2)*sin(2*d*x + 2*c
)/d
```

3.58 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=175

$$-\frac{3a^2b\cos^7(c+dx)}{7d}-\frac{a^3\sin^7(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{3ab^2\sin^7(c+dx)}{7d}-\frac{6ab^2}{}$$

```
[Out] -(b^3*Cos[c + d*x]^5)/(5*d) - (3*a^2*b*Cos[c + d*x]^7)/(7*d) + (b^3*Cos[c +
 d*x]^7)/(7*d) + (a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*x]^3)/d + (a*b^2*Sin
[c + d*x]^3)/d + (3*a^3*Sin[c + d*x]^5)/(5*d) - (6*a*b^2*Sin[c + d*x]^5)/(5
*d) - (a^3*Sin[c + d*x]^7)/(7*d) + (3*a*b^2*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.183704, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{3a^2b\cos^7(c+dx)}{7d}-\frac{a^3\sin^7(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{3ab^2\sin^7(c+dx)}{7d}-\frac{6ab^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*Cos[c + d*x]^5)/(5*d) - (3*a^2*b*Cos[c + d*x]^7)/(7*d) + (b^3*Cos[c +
 d*x]^7)/(7*d) + (a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*x]^3)/d + (a*b^2*Sin
[c + d*x]^3)/d + (3*a^3*Sin[c + d*x]^5)/(5*d) - (6*a*b^2*Sin[c + d*x]^5)/(5
*d) - (a^3*Sin[c + d*x]^7)/(7*d) + (3*a*b^2*Sin[c + d*x]^7)/(7*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^7(c+dx)+3a^2b\cos^6(c+dx)\sin(c+dx)+3ab^2\cos^5(c+
$$

$$= a^3\int \cos^7(c+dx)\,dx+\left(3a^2b\right)\int \cos^6(c+dx)\sin(c+dx)\,dx+\left(3ab^2
$$

$$= -\frac{a^3\,\text{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\text{Su}
$$

$$= -\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{3a^3\sin^5(c
}{5d}
$$

$$= -\frac{b^3\cos^5(c+dx)}{5d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{b^3\cos^7(c+dx)}{7d}+\frac{a^3\sin(c+
}{d}
$$

Mathematica [A] time = 0.412475, size = 204, normalized size = 1.17

$$\frac{-105b\left(5a^2+b^2\right)\cos(c+dx)-35\left(9a^2b+b^3\right)\cos(3(c+dx))-105a^2b\cos(5(c+dx))-15a^2b\cos(7(c+dx))+1225a^3\sin \cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-105*b*(5*a^2 + b^2)*Cos[c + d*x] - 35*(9*a^2*b + b^3)*Cos[3*(c + d*x)] -
105*a^2*b*Cos[5*(c + d*x)] + 7*b^3*Cos[5*(c + d*x)] - 15*a^2*b*Cos[7*(c + d
*x)] + 5*b^3*Cos[7*(c + d*x)] + 1225*a^3*Sin[c + d*x] + 525*a*b^2*Sin[c + d
*x] + 245*a^3*Sin[3*(c + d*x)] - 35*a*b^2*Sin[3*(c + d*x)] + 49*a^3*Sin[5*(
c + d*x)] - 63*a*b^2*Sin[5*(c + d*x)] + 5*a^3*Sin[7*(c + d*x)] - 15*a*b^2*S
in[7*(c + d*x)])/(2240*d)
```

Maple [A] time = 0.081, size = 145, normalized size = 0.8

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^5}{7}-\frac{2\,(\cos(dx+c))^5}{35}\right)+3\,ab^2\left(-1/7\,\sin(dx+c)(\cos(dx+c))^6+1/35\,\left(8/3+(\cos \cdots\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/7*s
in(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-3/7*a^2*b*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.12117, size = 170, normalized size = 0.97

$$-\frac{15\,a^2b\cos(dx+c)^7+\left(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c)\right)a^3-\left(15\,\sin(dx+c)^7-\cdots\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x +
c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3
)/d
```

Fricas [A] time = 0.506159, size = 278, normalized size = 1.59

$$-\frac{7\,b^3\cos\left(dx+c\right)^5+5\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+3\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^4+16\,a^3+8\,\cdots}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a*b
^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 8.9804, size = 233, normalized size = 1.33

$$\begin{cases}\frac{16a^3\sin^7(c+dx)}{35d}+\frac{8a^3\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^3\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^6(c+dx)}{d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{8ab^2\sin^7(c+dx)}{35d}+\frac{4ab\cdots}{\cdots} \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((16*a**3*sin(c + d*x)**7/(35*d) + 8*a**3*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + a**3*sin(c + d*x
)*cos(c + d*x)**6/d - 3*a**2*b*cos(c + d*x)**7/(7*d) + 8*a*b**2*sin(c + d*x
)**7/(35*d) + 4*a*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + a*b**2*sin(c
 + d*x)**3*cos(c + d*x)**4/d - b**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) -
 2*b**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(
c)**4, True))
```

Giac [A] time = 1.1788, size = 266, normalized size = 1.52

$$-\frac{\left(3\,a^2b-b^3\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(15\,a^2b-b^3\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(9\,a^2b+b^3\right)\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\left(5\,a^2b+b^3\right)\cos\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/448*(3*a^2*b - b^3)*cos(7*d*x + 7*c)/d - 1/320*(15*a^2*b - b^3)*cos(5*d*
x + 5*c)/d - 1/64*(9*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/64*(5*a^2*b + b^3)
*cos(d*x + c)/d + 1/448*(a^3 - 3*a*b^2)*sin(7*d*x + 7*c)/d + 1/320*(7*a^3 -
 9*a*b^2)*sin(5*d*x + 5*c)/d + 1/64*(7*a^3 - a*b^2)*sin(3*d*x + 3*c)/d + 5/
64*(7*a^3 + 3*a*b^2)*sin(d*x + c)/d
```

3.59 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=216

$$-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}-$$

```
[Out] (5*a^3*x)/16 + (3*a*b^2*x)/16 - (a^2*b*Cos[c + d*x]^6)/(2*d) + (5*a^3*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + (3*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
 (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*b^2*Cos[c + d*x]^3*Sin[c +
 d*x])/(8*d) + (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (a*b^2*Cos[c + d*x
]^5*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]^4)/(4*d) - (b^3*Sin[c + d*x]^6)
/(6*d)
```

Rubi [A] time = 0.212641, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2635, 8, 2565, 30, 2568, 2564, 14}

$$-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (5*a^3*x)/16 + (3*a*b^2*x)/16 - (a^2*b*Cos[c + d*x]^6)/(2*d) + (5*a^3*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + (3*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
 (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*b^2*Cos[c + d*x]^3*Sin[c +
 d*x])/(8*d) + (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (a*b^2*Cos[c + d*x
]^5*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]^4)/(4*d) - (b^3*Sin[c + d*x]^6)
/(6*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)+3a^2b\cos^5(c+dx)\sin(c+dx)+3ab^2\cos^4(c+d\right.$$
$$= a^3\int \cos^6(c+dx)\,dx + \left(3a^2b\right)\int \cos^5(c+dx)\sin(c+dx)\,dx + \left(3ab^2\right.$$
$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{ab^2\cos^5(c+dx)\sin(c+dx)}{2d} + \frac{1}{6}\left(5a^3\right)$$
$$= -\frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{ab^2\cos^3(c+dx)}{8d}$$
$$= -\frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{3ab^2\cos(c+dx)}{16d}$$
$$= \frac{5a^3x}{16} + \frac{3}{16}ab^2x - \frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} +$$

Mathematica [A] time = 0.30174, size = 171, normalized size = 0.79

$$\frac{a\left(5a^2+3b^2\right)(c+dx)}{16d} + \frac{3a\left(5a^2+b^2\right)\sin(2(c+dx))}{64d} + \frac{3a\left(a^2-b^2\right)\sin(4(c+dx))}{64d} + \frac{a\left(a^2-3b^2\right)\sin(6(c+dx))}{192d} - \frac{3b\left(5a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(5*a^2 + 3*b^2)*(c + d*x))/(16*d) - (3*b*(5*a^2 + b^2)*Cos[2*(c + d*x)])
/(64*d) - (3*a^2*b*Cos[4*(c + d*x)])/(32*d) - (b*(3*a^2 - b^2)*Cos[6*(c + d
*x)])/(192*d) + (3*a*(5*a^2 + b^2)*Sin[2*(c + d*x)])/(64*d) + (3*a*(a^2 - b
^2)*Sin[4*(c + d*x)])/(64*d) + (a*(a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.078, size = 155, normalized size = 0.7

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6} - \frac{(\cos(dx+c))^4}{12}\right) + 3\,ab^2\left(-1/6\,\sin(dx+c)(\cos(dx+c))^5 + 1/24\,\left((\cos(dx+c))^3\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/6*s
in(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d
```

```
*x+1/16*c)-1/2*a^2*b*cos(d*x+c)^6+a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+1
5/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.22564, size = 177, normalized size = 0.82

$$-\frac{96\,a^2b\cos(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c)\right)a^3-3\left(4\,\sin(2\,dx+\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(96*a^2*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 3*(4*sin(2*d*x + 2*c)^3 + 12
*d*x + 12*c - 3*sin(4*d*x + 4*c))*a*b^2 + 16*(2*sin(d*x + c)^6 - 3*sin(d*x
+ c)^4)*b^3)/d
```

Fricas [A] time = 0.515885, size = 292, normalized size = 1.35

$$-\frac{12\,b^3\cos(dx+c)^4+8\left(3\,a^2b-b^3\right)\cos(dx+c)^6-3\left(5\,a^3+3\,ab^2\right)dx-\left(8\left(a^3-3\,ab^2\right)\cos(dx+c)^5+2\left(5\,a^3+3\,ab^2\right)\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*b^3*cos(d*x + c)^4 + 8*(3*a^2*b - b^3)*cos(d*x + c)^6 - 3*(5*a^3
+ 3*a*b^2)*d*x - (8*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 2*(5*a^3 + 3*a*b^2)*co
s(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 6.82375, size = 454, normalized size = 2.1

$$\begin{cases}\frac{5a^3x\sin^6(c+dx)}{16}+\frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^3x\cos^6(c+dx)}{16}+\frac{5a^3\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a^3\sin^3(c+dx)}{6d} \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos^3(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**3*x*cos(c
+ d*x)**6/16 + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**3*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 + a**2*b*sin(c + d*x)**6/(2*d) + 3*a**2*b*sin(c + d*x)**4*cos(c + d*x)**2/
(2*d) + 3*a**2*b*sin(c + d*x)**2*cos(c + d*x)**4/(2*d) + 3*a*b**2*x*sin(c +
 d*x)**6/16 + 9*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*si
n(c + d*x)**2*cos(c + d*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2
*sin(c + d*x)**5*cos(c + d*x)/(16*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)*
*3/(2*d) - 3*a*b**2*sin(c + d*x)*cos(c + d*x)**5/(16*d) + b**3*sin(c + d*x)
**6/(12*d) + b**3*sin(c + d*x)**4*cos(c + d*x)**2/(4*d), Ne(d, 0)), (x*(a*c
os(c) + b*sin(c))**3*cos(c)**3, True))
```

Giac [A] time = 1.18411, size = 212, normalized size = 0.98

$$-\frac{3\,a^2b\cos\left(4\,dx+4\,c\right)}{32\,d}+\frac{1}{16}\left(5\,a^3+3\,ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{3\left(5\,a^2b+b^3\right)\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{\left(a^3-3\,ab^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -3/32*a^2*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^3 + 3*a*b^2)*x - 1/192*(3*a^2*b
- b^3)*cos(6*d*x + 6*c)/d - 3/64*(5*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/192
*(a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 3/64*(a^3 - a*b^2)*sin(4*d*x + 4*c)/d
 + 3/64*(5*a^3 + a*b^2)*sin(2*d*x + 2*c)/d
```

3.60 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=140

$$-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\cdots}{}$$

```
[Out] -(b^3*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^5)/(5*d) + (b^3*Cos[c +
 d*x]^5)/(5*d) + (a^3*Sin[c + d*x])/d - (2*a^3*Sin[c + d*x]^3)/(3*d) + (a*b
^2*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^5)/(5*d) - (3*a*b^2*Sin[c + d*x]^5
)/(5*d)
```

Rubi [A] time = 0.163265, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 2633, 2565, 30, 2564, 14}

$$-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^3*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^5)/(5*d) + (b^3*Cos[c +
 d*x]^5)/(5*d) + (a^3*Sin[c + d*x])/d - (2*a^3*Sin[c + d*x]^3)/(3*d) + (a*b
^2*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^5)/(5*d) - (3*a*b^2*Sin[c + d*x]^5
)/(5*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^5(c+dx)+3a^2b\cos^4(c+dx)\sin(c+dx)+3ab^2\cos^3(c+
$$

$$= a^3\int \cos^5(c+dx)\,dx+\left(3a^2b\right)\int \cos^4(c+dx)\sin(c+dx)\,dx+\left(3ab^2\right.$$

$$= -\frac{a^3\,\text{Subst}\left(\int\left(1-2x^2+x^4\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{a^3\sin(c+dx)}{d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin^5(c}{5d}$$

$$= -\frac{b^3\cos^3(c+dx)}{3d}-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{b^3\cos^5(c+dx)}{5d}+\frac{a^3\sin(c+}{d}$$

Mathematica [A] time = 0.28896, size = 150, normalized size = 1.07

$$\frac{-30b\left(3a^2+b^2\right)\cos(c+dx)-5\left(9a^2b+b^3\right)\cos(3(c+dx))-9a^2b\cos(5(c+dx))+150a^3\sin(c+dx)+25a^3\sin(3(c+dx}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-30*b*(3*a^2 + b^2)*Cos[c + d*x] - 5*(9*a^2*b + b^3)*Cos[3*(c + d*x)] - 9*
a^2*b*Cos[5*(c + d*x)] + 3*b^3*Cos[5*(c + d*x)] + 150*a^3*Sin[c + d*x] + 90
*a*b^2*Sin[c + d*x] + 25*a^3*Sin[3*(c + d*x)] - 15*a*b^2*Sin[3*(c + d*x)] +
 3*a^3*Sin[5*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)])/(240*d)
```

Maple [A] time = 0.07, size = 125, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^3}{5}-\frac{2\,(\cos(dx+c))^3}{15}\right)+3\,ab^2\left(-1/5\,\sin(dx+c)(\cos(dx+c))^4+1/15\,\left(2+(\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*a^2*b*cos(d*x+
c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.23966, size = 144, normalized size = 1.03

$$-\frac{9\,a^2b\cos(dx+c)^5-\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)a^3+3\left(3\,\sin(dx+c)^5-5\,\sin(dx+c)^3\right)ab^2}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*
sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*cos(
d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

Fricas [A] time = 0.494217, size = 230, normalized size = 1.64

$$-\frac{5\,b^3\cos(dx+c)^3+3\left(3\,a^2b-b^3\right)\cos(dx+c)^5-\left(3\left(a^3-3\,ab^2\right)\cos(dx+c)^4+8\,a^3+6\,ab^2+\left(4\,a^3+3\,ab^2\right)\cos(dx+\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/d
```

Sympy [A] time = 2.78219, size = 182, normalized size = 1.3

$$\begin{cases} \frac{8a^3\sin^5(c+dx)}{15d} + \frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^4(c+dx)}{d} - \frac{3a^2b\cos^5(c+dx)}{5d} + \frac{2ab^2\sin^5(c+dx)}{5d} + \frac{ab^2\sin^3(c+dx)\cos^2(c+dx)}{d} - \frac{b^3\mathrm{si}}{} \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - 3*a**2*b*cos(c + d*x)*
*5/(5*d) + 2*a*b**2*sin(c + d*x)**5/(5*d) + a*b**2*sin(c + d*x)**3*cos(c +
d*x)**2/d - b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b**3*cos(c + d*x
)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c)**2, True))
```

Giac [A] time = 1.1482, size = 196, normalized size = 1.4

$$-\frac{\left(3\,a^2b-b^3\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{\left(9\,a^2b+b^3\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{\left(3\,a^2b+b^3\right)\cos\left(dx+c\right)}{8\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(5\,dx+5\,c\right)}{80\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/80*(3*a^2*b - b^3)*cos(5*d*x + 5*c)/d - 1/48*(9*a^2*b + b^3)*cos(3*d*x +
 3*c)/d - 1/8*(3*a^2*b + b^3)*cos(d*x + c)/d + 1/80*(a^3 - 3*a*b^2)*sin(5*d
*x + 5*c)/d + 1/48*(5*a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 1/8*(5*a^3 + 3*a*
b^2)*sin(d*x + c)/d
```

3.61 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=78

$$\frac{3}{8}ax\left(a^2+b^2\right)+\frac{\sin^4(c+dx)(a\cot(c+dx)+b)^3}{4d}+\frac{3a\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{8d}$$

```
[Out] (3*a*(a^2 + b^2)*x)/8 + (3*a*(b + a*Cot[c + d*x])*(a - b*Cot[c + d*x])*Sin[
c + d*x]^2)/(8*d) + ((b + a*Cot[c + d*x])^3*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0635502, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3088, 805, 723, 203}

$$\frac{3}{8}ax\left(a^2+b^2\right)+\frac{\sin^4(c+dx)(a\cot(c+dx)+b)^3}{4d}+\frac{3a\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (3*a*(a^2 + b^2)*x)/8 + (3*a*(b + a*Cot[c + d*x])*(a - b*Cot[c + d*x])*Sin[
c + d*x]^2)/(8*d) + ((b + a*Cot[c + d*x])^3*Sin[c + d*x]^4)/(4*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax)^3}{\left(1+x^2\right)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(b+a\cot(c+dx))^3\sin^4(c+dx)}{4d} - \frac{(3a)\,\text{Subst}\left(\int \frac{(b+ax)^2}{\left(1+x^2\right)^2}\,dx, x, \cot(c+\right.}{4d}$$

$$= \frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+dx)}{8d} + \frac{(b+a\cot(c+}{}$$

$$= \frac{3}{8}a\left(a^2+b^2\right)x + \frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+dx)}{8d}$$

Mathematica [A] time = 0.398133, size = 94, normalized size = 1.21

$$\frac{12a\left(a^2+b^2\right)(c+dx)+a\left(a^2-3b^2\right)\sin(4(c+dx))-4\left(3a^2b+b^3\right)\cos(2(c+dx))+\left(b^3-3a^2b\right)\cos(4(c+dx))+8a^3\sin(}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (12*a*(a^2 + b^2)*(c + d*x) - 4*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + (-3*a^2*
b + b^3)*Cos[4*(c + d*x)] + 8*a^3*Sin[2*(c + d*x)] + a*(a^2 - 3*b^2)*Sin[4*
(c + d*x)])/(32*d)
```

Maple [A] time = 0.066, size = 114, normalized size = 1.5

$$\frac{1}{d}\left(\frac{b^{3}\left(\sin\left(dx+c\right)\right)^{4}}{4}+3\,ab^{2}\left(-1/4\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{3}+1/8\,\cos\left(dx+c\right)\sin\left(dx+c\right)+1/8\,dx+c/8\right)-\frac{3\,a^{2}b\left(\mathrm{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4+3*a*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x
+c)*sin(d*x+c)+1/8*d*x+1/8*c)-3/4*a^2*b*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.19002, size = 123, normalized size = 1.58

$$-\frac{24\,a^{2}b\cos\left(dx+c\right)^{4}-8\,b^{3}\sin\left(dx+c\right)^{4}-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^{3}-3\left(4\,dx+4\,c-\sin\left(4\right.\right.}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(24*a^2*b*cos(d*x + c)^4 - 8*b^3*sin(d*x + c)^4 - (12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c
))*a*b^2)/d
```

Fricas [A] time = 0.495729, size = 228, normalized size = 2.92

$$-\frac{4\,b^{3}\cos\left(dx+c\right)^{2}+2\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{4}-3\left(a^{3}+ab^{2}\right)dx-\left(2\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}+3\left(a^{3}+ab^{2}\right)\cos\left(dx\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a*
b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [A] time = 1.58976, size = 299, normalized size = 3.83

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8} + \frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3a^3x\cos^4(c+dx)}{8} + \frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d} + \frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{3a^2b\sin^4(c+dx)}{4d} + 3 \\ x(a\cos(c)+b\sin(c))^3\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 3*a**2*b*sin(c + d*x)**
4/(4*d) + 3*a**2*b*sin(c + d*x)**2*cos(c + d*x)**2/(2*d) + 3*a*b**2*x*sin(c
 + d*x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*a*b**2*x*co
s(c + d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*b**2*si
n(c + d*x)*cos(c + d*x)**3/(8*d) + b**3*sin(c + d*x)**4/(4*d), Ne(d, 0)), (
x*(a*cos(c) + b*sin(c))**3*cos(c), True))
```

Giac [A] time = 1.15493, size = 140, normalized size = 1.79

$$\frac{a^3\sin(2\,dx+2\,c)}{4\,d} + \frac{3}{8}\left(a^3+ab^2\right)x - \frac{\left(3\,a^2b-b^3\right)\cos(4\,dx+4\,c)}{32\,d} - \frac{\left(3\,a^2b+b^3\right)\cos(2\,dx+2\,c)}{8\,d} + \frac{\left(a^3-3\,ab^2\right)\sin(4\,dx+}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*a^3*sin(2*d*x + 2*c)/d + 3/8*(a^3 + a*b^2)*x - 1/32*(3*a^2*b - b^3)*cos
(4*d*x + 4*c)/d - 1/8*(3*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^3 - 3*a*
b^2)*sin(4*d*x + 4*c)/d
```

3.62 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=58

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rubi [A] time = 0.0232823, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3072}

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}\end{aligned}$$

Mathematica [A] time = 0.332719, size = 81, normalized size = 1.4

$$\frac{-9b\left(a^2+b^2\right)\cos(c+dx)+\left(b^3-3a^2b\right)\cos(3(c+dx))+2a\sin(c+dx)\left(\left(a^2-3b^2\right)\cos(2(c+dx))+5a^2+3b^2\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)
```

Maple [A] time = 0.066, size = 75, normalized size = 1.3

$$\frac{1}{d}\left(-\frac{b^3\left(2+(\sin(dx+c))^2\right)\cos(dx+c)}{3}+ab^2(\sin(dx+c))^3-(\cos(dx+c))^3a^2b+\frac{a^3\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-cos(d*x+c)^3*a
^2*b+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.20716, size = 113, normalized size = 1.95

$$-\frac{a^2b\cos(dx+c)^3}{d}+\frac{ab^2\sin(dx+c)^3}{d}-\frac{\left(\sin(dx+c)^3-3\sin(dx+c)\right)a^3}{3d}+\frac{\left(\cos(dx+c)^3-3\cos(dx+c)\right)b^3}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3*
sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d
```

Fricas [A] time = 0.480274, size = 173, normalized size = 2.98

$$-\frac{3\,b^{3}\cos\left(dx+c\right)+\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{3}-\left(2\,a^{3}+3\,ab^{2}+\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^
2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 0.752623, size = 117, normalized size = 2.02

$$\begin{cases} \frac{2a^{3}\sin^{3}(c+dx)}{3d}+\frac{a^{3}\sin(c+dx)\cos^{2}(c+dx)}{d}-\frac{a^{2}b\cos^{3}(c+dx)}{d}+\frac{ab^{2}\sin^{3}(c+dx)}{d}-\frac{b^{3}\sin^{2}(c+dx)\cos(c+dx)}{d}-\frac{2b^{3}\cos^{3}(c+dx)}{3d} & \text{for}\ d\neq 0 \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d*x
)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**3, True))
```

Giac [A] time = 1.13038, size = 123, normalized size = 2.12

$$-\frac{\left(3\,a^{2}b-b^{3}\right)\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{3\left(a^{2}b+b^{3}\right)\cos\left(dx+c\right)}{4\,d}+\frac{\left(a^{3}-3\,ab^{2}\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{3\left(a^{3}+ab^{2}\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/d
 + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x + c)
/d
```

3.63 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{\sin^2(c+dx)\left(a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)\right)}{2d}+\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{b^3\log(\sin(c+dx))}{d}+\frac{b^3\log(\tan(c+dx))}{d}$$

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Sin[c + d*x]])/d + (b^3*Log[Tan[c + d*x]])
/d + ((b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d
)
```

Rubi [A] time = 0.119388, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3088, 1805, 801, 635, 203, 260}

$$\frac{\sin^2(c+dx)\left(a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)\right)}{2d}+\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{b^3\log(\sin(c+dx))}{d}+\frac{b^3\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Sin[c + d*x]])/d + (b^3*Log[Tan[c + d*x]])
/d + ((b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d
)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d} + \frac{\text{Subst}\left(\int \frac{-2b}{}\right.}{}$$
$$= \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d} + \frac{\text{Subst}\left(\int \left(-\right.\right.}{}$$
$$= \frac{b^3\log(\tan(c+dx))}{d} + \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx)\right)\sin^2(c}{2d}$$
$$= \frac{b^3\log(\tan(c+dx))}{d} + \frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx)\right)\sin^2(c}{2d}$$
$$= \frac{1}{2}a\left(a^2+3b^2\right)x - \frac{b^3\log(\sin(c+dx))}{d} + \frac{b^3\log(\tan(c+dx))}{d} + \frac{\left(b\left(3a^2\right.\right.}{}$$

Mathematica [B] time = 0.734223, size = 401, normalized size = 4.41

$$\frac{ab\left(-2a^2b^2+a^4-3b^4\right)\sin(2(c+dx))+\left(-2a^2b^4-3a^4b^2+b^6\right)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2a^2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d*
x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] - b
*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3*(-
b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt[-b^
2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*b^6*L
og[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3*(-b^2)
^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin
[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

Maple [A] time = 0.109, size = 123, normalized size = 1.4

$$\frac{a^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^3x}{2}+\frac{a^3c}{2d}-\frac{3a^2b(\cos(dx+c))^2}{2d}-\frac{3ab^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{3ab^2x}{2}+\frac{3ab^2c}{2d}-\frac{(s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3*c-3/2*a^2*b*cos(d*x+c)^
2/d-3/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d+3/2*a*b^2*x+3/2/d*a*b^2*c-1/2/d*sin(d
*x+c)^2*b^3-b^3*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.24556, size = 123, normalized size = 1.35

$$\frac{6a^2b\sin(dx+c)^2+(2dx+2c+\sin(2dx+2c))a^3+3(2dx+2c-\sin(2dx+2c))ab^2-2\left(\sin(dx+c)^2+\log\left(\sin(\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(6*a^2*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 3*(2*d
*x + 2*c - sin(2*d*x + 2*c))*a*b^2 - 2*(sin(d*x + c)^2 + log(sin(d*x + c)^2
 - 1))*b^3)/d
```

Fricas [A] time = 0.515797, size = 181, normalized size = 1.99

$$-\frac{2b^3\log(-\cos(dx+c))-\left(a^3+3ab^2\right)dx+\left(3a^2b-b^3\right)\cos(dx+c)^2-\left(a^3-3ab^2\right)\cos(dx+c)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos(
d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20132, size = 126, normalized size = 1.38

$$\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)+\left(a^3+3\,ab^2\right)(dx+c)-\frac{b^3\tan(dx+c)^2-a^3\tan(dx+c)+3\,ab^2\tan(dx+c)+3\,a^2b}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (b^3*tan(d*x
 + c)^2 - a^3*tan(d*x + c) + 3*a*b^2*tan(d*x + c) + 3*a^2*b)/(tan(d*x + c)^
2 + 1))/d
```

3.64 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=86

$$-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

```
[Out] (3*a*b^2*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b*Cos[c + d*x])/d + (b^3*Cos[c +
 d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d
*x])/d
```

Rubi [A] time = 0.111642, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2637, 2638, 2592, 321, 206, 2590, 14}

$$-\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b*Cos[c + d*x])/d + (b^3*Cos[c +
 d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d
*x])/d
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos(c+dx)+3a^2b\sin(c+dx)+3ab^2\sin(c+dx)\tan(c+dx\right.$$
$$= a^3\int\cos(c+dx)\,dx+\left(3a^2b\right)\int\sin(c+dx)\,dx+\left(3ab^2\right)\int\sin(c+$$
$$= -\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}+\frac{\left(3ab^2\right)\text{Subst}\left(\int\frac{x^2}{1-x^2}\,dx,x,\text{s}\right.}{d}$$
$$= -\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}+\frac{\left(3ab^2\right)\text{Su}}{}$$
$$= \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b\cos(c+dx)}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b}{}$$

Mathematica [A] time = 1.06423, size = 131, normalized size = 1.52

$$\frac{\sec(c+dx)\left(\left(b^3-3a^2b\right)\cos(2(c+dx))-3a^2b+a^3\sin(2(c+dx))-3ab^2\sin(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*b
^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d*x)
]))/(2*d)
```

Maple [A] time = 0.119, size = 126, normalized size = 1.5

$$\frac{a^3\sin(dx+c)}{d}-3\frac{a^2b\cos(dx+c)}{d}-3\frac{ab^2\sin(dx+c)}{d}+3\frac{ab^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^3(\sin(dx+c))^4}{d\cos(dx+c)}+\frac{\text{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*sin(d*x+c)/d-3*a^2*b*cos(d*x+c)/d-3*a*b^2*sin(d*x+c)/d+3/d*a*b^2*ln(sec
(d*x+c)+tan(d*x+c))+1/d*b^3*sin(d*x+c)^4/cos(d*x+c)+1/d*cos(d*x+c)*sin(d*x+
c)^2*b^3+2*b^3*cos(d*x+c)/d
```

Maxima [A] time = 1.22597, size = 113, normalized size = 1.31

$$\frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+3\,ab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))-6\,a^2b\cos(dx+c)+}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3*s
in(d*x + c))/d
```

Fricas [A] time = 0.512028, size = 273, normalized size = 3.17

$$\frac{3\,ab^2\cos(dx+c)\log(\sin(dx+c)+1)-3\,ab^2\cos(dx+c)\log(-\sin(dx+c)+1)+2\,b^3-2\left(3\,a^2b-b^3\right)\cos(dx+c)^2+}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 - 3*
a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19763, size = 203, normalized size = 2.36

$$\frac{3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,dx+\ldots\right.}{\tan\left(\frac{1}{2}\,dx+\ldots\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^2*tan(1/2*d*x + 1/2*c
)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*t
an(1/2*d*x + 1/2*c) + 3*a^2*b - 2*b^3)/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

3.65 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=72

$$-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

```
[Out] a*(a^2 - 3*b^2)*x - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (2*a*b^2*Tan[c
+ d*x])/d + (b*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rubi [A] time = 0.0925312, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3086, 3482, 3525, 3475}

$$-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] a*(a^2 - 3*b^2)*x - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (2*a*b^2*Tan[c
+ d*x])/d + (b*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
```

```
f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \int (a+b\tan(c+dx))^3\,dx\\ &= \frac{b(a+b\tan(c+dx))^2}{2d}+\int (a+b\tan(c+dx))\left(a^2-b^2+2ab\tan(c\right.\\ &= a\left(a^2-3b^2\right)x+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}+\left(b\left(3a\right.\right.\\ &= a\left(a^2-3b^2\right)x-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{2ab^2\tan(c+dx)}{d}+\end{aligned}$$

Mathematica [C] time = 0.267994, size = 79, normalized size = 1.1

$$\frac{6ab^2\tan(c+dx)+(-b+ia)^3\log(-\tan(c+dx)+i)-(b+ia)^3\log(\tan(c+dx)+i)+b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.122, size = 93, normalized size = 1.3

$$a^3x+\frac{a^3c}{d}-3\,\frac{a^2b\ln(\cos(dx+c))}{d}-3\,ab^2x+3\,\frac{ab^2\tan(dx+c)}{d}-3\,\frac{ab^2c}{d}+\frac{b^3(\tan(dx+c))^2}{2\,d}+\frac{b^3\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] a^3*x+1/d*a^3*c-3*a^2*b*ln(cos(d*x+c))/d-3*a*b^2*x+3*a*b^2*tan(d*x+c)/d-3/d
*a*b^2*c+1/2/d*b^3*tan(d*x+c)^2+b^3*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.7547, size = 115, normalized size = 1.6

$$\frac{2\,(dx+c)a^3 - 6\,(dx+c-\tan(dx+c))ab^2 - b^3\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin(dx+c)^2-1\right)\right) - 3\,a^2b\log\left(-\sin(dx+c)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 - 6*(d*x + c - tan(d*x + c))*a*b^2 - b^3*(1/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 3*a^2*b*log(-sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.505707, size = 215, normalized size = 2.99

$$\frac{2\left(a^3-3\,ab^2\right)dx\cos(dx+c)^2 + 6\,ab^2\cos(dx+c)\sin(dx+c) - 2\left(3\,a^2b-b^3\right)\cos(dx+c)^2\log(-\cos(dx+c)) + b^3}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*d*x*cos(d*x + c)^2 + 6*a*b^2*cos(d*x + c)*sin(d*x +
c) - 2*(3*a^2*b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + b^3)/(d*cos(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20417, size = 96, normalized size = 1.33

$$\frac{b^3 \tan(dx+c)^2 + 6ab^2 \tan(dx+c) + 2\left(a^3 - 3ab^2\right)(dx+c) + \left(3a^2b - b^3\right)\log\left(\tan(dx+c)^2 + 1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + 2*(a^3 - 3*a*b^2)*(d*x + c
) + (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1))/d
```

3.66 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=103

$$\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{b^3\sec^3(c+dx)}{3d}$$

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/d - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (3*
a^2*b*Sec[c + d*x])/d - (b^3*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^3)/(3*d) +
 (3*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.122525, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3090, 3770, 2606, 8, 2611}

$$\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{b^3\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/d - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (3*
a^2*b*Sec[c + d*x])/d - (b^3*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^3)/(3*d) +
 (3*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec(c+dx)+3a^2b\sec(c+dx)\tan(c+dx)+3ab^2\sec(c+dx\right.$$

$$= a^3\int \sec(c+dx)\,dx + \left(3a^2b\right)\int \sec(c+dx)\tan(c+dx)\,dx + \left(3ab^2\right.$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} + \frac{3ab^2\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\left(3ab^2\right)$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d} + \frac{3a^2b\sec(c+}{d}$$

Mathematica [B] time = 1.5871, size = 293, normalized size = 2.84

$$-6a\left(2a^2-3b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b\sin^2\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(\left(18a^2-5b^2\right)\cos(2(c+dx))\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2 - b^
2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*
```

```
Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^
3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

Maple [A] time = 0.124, size = 187, normalized size = 1.8

$$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\frac{a^2b}{d\cos(dx+c)}+\frac{3ab^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2}+\frac{3ab^2\sin(dx+c)}{2d}-\frac{3ab^2\ln(\sec(dx+c)+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/d*a^2*b/cos(d*x+c)+3/2/d*a*b^2*sin(d*x+
c)^3/cos(d*x+c)^2+3/2*a*b^2*sin(d*x+c)/d-3/2/d*a*b^2*ln(sec(d*x+c)+tan(d*x+
c))+1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1
/3/d*cos(d*x+c)*sin(d*x+c)^2*b^3-2/3*b^3*cos(d*x+c)/d
```

Maxima [A] time = 1.24014, size = 159, normalized size = 1.54

$$-\frac{9ab^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-6a^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) - 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) - 36*a^2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^
3)/d
```

Fricas [A] time = 0.510289, size = 304, normalized size = 2.95

$$\frac{3\left(2a^3-3ab^2\right)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\left(2a^3-3ab^2\right)\cos(dx+c)^3\log(-\sin(dx+c)+1)+18ab^2\cos(dx}{12d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 -
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*sin
(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20761, size = 231, normalized size = 2.24

$$\frac{3\left(2a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(2a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(9ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18a^2b\tan\right.}{}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12
*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 4*b
^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.67 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=30

$$\frac{\tan^4(c+dx)(a\cot(c+dx)+b)^4}{4bd}$$

```
[Out] ((b + a*Cot[c + d*x])^4*Tan[c + d*x]^4)/(4*b*d)
```

Rubi [A] time = 0.0474499, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$\frac{\tan^4(c+dx)(a\cot(c+dx)+b)^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((b + a*Cot[c + d*x])^4*Tan[c + d*x]^4)/(4*b*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\operatorname{Subst}\left(\int \frac{(b+ax)^3}{x^5}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{(b+a\cot(c+dx))^4\tan^4(c+dx)}{4bd}$$

Mathematica [A] time = 0.169052, size = 57, normalized size = 1.9

$$\frac{\tan(c+dx)\left(6a^2b\tan(c+dx)+4a^3+4ab^2\tan^2(c+dx)+b^3\tan^3(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Tan[c + d*x]*(4*a^3 + 6*a^2*b*Tan[c + d*x] + 4*a*b^2*Tan[c + d*x]^2 + b^3*
Tan[c + d*x]^3))/(4*d)
```

Maple [B] time = 0.121, size = 72, normalized size = 2.4

$$\frac{1}{d}\left(a^3\tan(dx+c)+\frac{3a^2b}{2(\cos(dx+c))^2}+\frac{ab^2(\sin(dx+c))^3}{(\cos(dx+c))^3}+\frac{b^3(\sin(dx+c))^4}{4(\cos(dx+c))^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3/2*a^2*b/cos(d*x+c)^2+a*b^2*sin(d*x+c)^3/cos(d*x+c)^3+
1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [B] time = 1.18469, size = 117, normalized size = 3.9

$$\frac{4ab^2\tan(dx+c)^3+4a^3\tan(dx+c)+\frac{(2\sin(dx+c)^2-1)b^3}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-\frac{6a^2b}{\sin(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + (2*sin(d*x + c)^2 - 1)*b
^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 6*a^2*b/(sin(d*x + c)^2 - 1))/
d
```

Fricas [B] time = 0.467081, size = 181, normalized size = 6.03

$$\frac{b^3 + 2\left(3\,a^2b - b^3\right)\cos\left(dx + c\right)^2 + 4\left(ab^2\cos\left(dx + c\right) + \left(a^3 - ab^2\right)\cos\left(dx + c\right)^3\right)\sin\left(dx + c\right)}{4\,d\cos\left(dx + c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
- a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23172, size = 77, normalized size = 2.57

$$\frac{b^3\tan\left(dx + c\right)^4 + 4\,ab^2\tan\left(dx + c\right)^3 + 6\,a^2b\tan\left(dx + c\right)^2 + 4\,a^3\tan\left(dx + c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 +
 4*a^3*tan(d*x + c))/d
```

3.68 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=158

$$\frac{a^2b\sec^3(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3ab^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(8*d) +
 (a^2*b*Sec[c + d*x]^3)/d - (b^3*Sec[c + d*x]^3)/(3*d) + (b^3*Sec[c + d*x]^
5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (3*a*b^2*Sec[c + d*x]*Ta
n[c + d*x])/(8*d) + (3*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rubi [A] time = 0.180935, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{a^2b\sec^3(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3ab^2\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(8*d) +
 (a^2*b*Sec[c + d*x]^3)/d - (b^3*Sec[c + d*x]^3)/(3*d) + (b^3*Sec[c + d*x]^
5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (3*a*b^2*Sec[c + d*x]*Ta
n[c + d*x])/(8*d) + (3*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \int \left(a^3\sec^3(c+dx)+3a^2b\sec^3(c+dx)\tan(c+dx)+3ab^2\sec^3(c+\right.\\
&= a^3\int \sec^3(c+dx)\,dx+\left(3a^2b\right)\int \sec^3(c+dx)\tan(c+dx)\,dx+\left(3a\right.\\
&= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{3ab^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{2}a^3\\
&= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2b\sec^3(c+dx)}{d}+\frac{a^3\sec(c+dx)\tan(c-}{2d}\\
&= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2b\sec^3(c+}{d}
\end{aligned}$$

Mathematica [B] time = 1.31819, size = 464, normalized size = 2.94

$$\frac{\sec^5(c+dx)\left(320\left(3a^2b-b^3\right)\cos(2(c+dx))-150a\left(4a^2-3b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
- 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225*a
*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*Cos
[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^2)*Co
s[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24
0*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Sin[4*(c + d*
x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

Maple [A] time = 0.131, size = 256, normalized size = 1.6

$$\frac{a^3\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{a^2b}{d(\cos(dx+c))^3}+\frac{3ab^2(\sin(dx+c))^3}{4d(\cos(dx+c))^4}+\frac{3ab^2(\sin(}{8d(\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+1/d*a^2
*b/cos(d*x+c)^3+3/4/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a*b^2*sin(d*x+c
)^3/cos(d*x+c)^2+3/8*a*b^2*sin(d*x+c)/d-3/8/d*a*b^2*ln(sec(d*x+c)+tan(d*x+c
))+1/5/d*b^3*sin(d*x+c)^4/cos(d*x+c)^5+1/15/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3
-1/15/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1/15/d*cos(d*x+c)*sin(d*x+c)^2*b^3-2/15
*b^3*cos(d*x+c)/d
```

Maxima [A] time = 1.2544, size = 212, normalized size = 1.34

$$\frac{45\,ab^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right.\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

Fricas [A] time = 0.520492, size = 362, normalized size = 2.29

$$\frac{15\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(4\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+48\,b^3+80}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b -
 b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(d*x
 + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23838, size = 450, normalized size = 2.85

$$\frac{15\left(4a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4a^3-3ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(60a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+45ab^2\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
- 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^8
- 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*a
^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*x + 1/2
*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 45*a*
b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

3.69 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=120

$$\frac{b\left(3a^2+b^2\right)\tan^4(c+dx)}{4d}+\frac{a\left(a^2+3b^2\right)\tan^3(c+dx)}{3d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a^3\tan(c+dx)}{d}+\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{b^3}{}$$

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan
[c + d*x]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(6*d)
```

Rubi [A] time = 0.0979102, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{b\left(3a^2+b^2\right)\tan^4(c+dx)}{4d}+\frac{a\left(a^2+3b^2\right)\tan^3(c+dx)}{3d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a^3\tan(c+dx)}{d}+\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan
[c + d*x]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(6*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3(1+x^2)}{x^7}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^7}+\frac{3ab^2}{x^6}+\frac{3a^2b+b^3}{x^5}+\frac{a^3+3ab^2}{x^4}+\frac{3a^2b}{x^3}+\frac{a^3}{x^2}\right)dx, x, \cot(c+\right.}{d}$$

$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a\left(a^2+3b^2\right)\tan^3(c+dx)}{3d}+\ldots$$

<u>Mathematica</u> [A] time = 0.372622, size = 54, normalized size = 0.45

$$\frac{(a+b\tan(c+dx))^4\left(a^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx)+15b^2\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c +
 d*x]^2))/(60*b^3*d)
```

<u>Maple</u> [A] time = 0.13, size = 127, normalized size = 1.1

$$\frac{1}{d}\left(-a^3\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)+\frac{3a^2b}{4(\cos(dx+c))^4}+3ab^2\left(1/5\frac{(\sin(dx+c))^3}{(\cos(dx+c))^5}+2/15\frac{(\sin(dx+c))^3}{(\cos(dx+c))^3}\right)+b^3\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3/4*a^2*b/cos(d*x+c)^4+3*a*b^2
*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/6*si
n(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A] time = 1.18704, size = 165, normalized size = 1.38

$$\frac{20\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{3}+12\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)ab^{2}-\frac{5\left(3\,\sin(dx+c)^{2}-1\right)b^{3}}{\sin(dx+c)^{6}-3\,\sin(dx+c)^{4}+3\,\sin(dx+c)^{2}-1}+}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(20*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 + 12*(3*tan(d*x + c)^5 + 5*t
an(d*x + c)^3)*a*b^2 - 5*(3*sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 45*a^2*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A] time = 0.510117, size = 246, normalized size = 2.05

$$\frac{10\,b^{3}+15\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}+4\left(2\left(5\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{5}+9\,ab^{2}\cos\left(dx+c\right)+\left(5\,a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}}{60\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*c
os(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2001, size = 151, normalized size = 1.26

$$\frac{10\,b^3\tan\left(dx+c\right)^6+36\,ab^2\tan\left(dx+c\right)^5+45\,a^2b\tan\left(dx+c\right)^4+15\,b^3\tan\left(dx+c\right)^4+20\,a^3\tan\left(dx+c\right)^3+60\,ab^2\tan\left(\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x + c
)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

3.70 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=210

$$\frac{3a^2b\sec^5(c+dx)}{5d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{8d}-\frac{3ab^2\tanh}{}$$

```
[Out] (3*a^3*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(16*d
) + (3*a^2*b*Sec[c + d*x]^5)/(5*d) - (b^3*Sec[c + d*x]^5)/(5*d) + (b^3*Sec[
c + d*x]^7)/(7*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a*b^2*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) - (
a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (a*b^2*Sec[c + d*x]^5*Tan[c + d*
x])/(2*d)
```

Rubi [A] time = 0.219708, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{3a^2b\sec^5(c+dx)}{5d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{8d}-\frac{3ab^2\tanh}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (3*a^3*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(16*d
) + (3*a^2*b*Sec[c + d*x]^5)/(5*d) - (b^3*Sec[c + d*x]^5)/(5*d) + (b^3*Sec[
c + d*x]^7)/(7*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a*b^2*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) - (
a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (a*b^2*Sec[c + d*x]^5*Tan[c + d*
x])/(2*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec^5(c+dx)+3a^2b\sec^5(c+dx)\tan(c+dx)+3ab^2\sec^5(c+$$

$$= a^3\int \sec^5(c+dx)\,dx + \left(3a^2b\right)\int \sec^5(c+dx)\tan(c+dx)\,dx + \left(3a$$

$$= \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{ab^2\sec^5(c+dx)\tan(c+dx)}{2d} + \frac{1}{4}\left(3a\right.$$

$$= \frac{3a^2b\sec^5(c+dx)}{5d} + \frac{3a^3\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^3\sec^3(c+dx)}{4d}$$

$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d} + \frac{3a^2b\sec^5(c+dx)}{5d} - \frac{b^3\sec^5(c+dx)}{5d} + \cdots$$

$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d} - \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{16d} + \frac{3a^2b\sec^5(c}{5d}$$

Mathematica [B] time = 2.02354, size = 637, normalized size = 3.03

$$\frac{\sec^7(c+dx)\left(3584\left(3a^2b-b^3\right)\cos(2(c+dx))-3675a\left(2a^2-b^2\right)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c + d
*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 147
0*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a*b^2
*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*Cos[7*
(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7*(c +
d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2)*Cos[c
 + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 105*a*b
^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4340*a^3*Sin
[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4*(c + d*x)] - 1
400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210*a*b^2*Sin[6*(c
+ d*x)]))/(35840*d)
```

Maple [A] time = 0.133, size = 328, normalized size = 1.6

$$\frac{a^3 (\sec (dx+c))^3 \tan (dx+c)}{4d}+\frac{3a^3 \sec (dx+c) \tan (dx+c)}{8d}+\frac{3a^3 \ln (\sec (dx+c)+\tan (dx+c))}{8d}+\frac{3a^2b}{5d (\cos (dx+c))^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4*a^3*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^3*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^3
*ln(sec(d*x+c)+tan(d*x+c))+3/5/d*a^2*b/cos(d*x+c)^5+1/2/d*a*b^2*sin(d*x+c)^
3/cos(d*x+c)^6+3/8/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/16/d*a*b^2*sin(d*x+c
)^3/cos(d*x+c)^2+3/16*a*b^2*sin(d*x+c)/d-3/16/d*a*b^2*ln(sec(d*x+c)+tan(d*x
+c))+1/7/d*b^3*sin(d*x+c)^4/cos(d*x+c)^7+3/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)
^5+1/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/35/d*b^3*sin(d*x+c)^4/cos(d*x+c)-
1/35/d*cos(d*x+c)*sin(d*x+c)^2*b^3-2/35*b^3*cos(d*x+c)/d
```

Maxima [A] time = 1.22831, size = 281, normalized size = 1.34

$$\frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)-70\,a^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}\right.}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/
(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x + c)
^2 - 5)*b^3/cos(d*x + c)^7)/d
```

Fricas [A] time = 0.554504, size = 410, normalized size = 1.95

$$\frac{105\left(2\,a^3-ab^2\right)\cos(dx+c)^7\log(\sin(dx+c)+1)-105\left(2\,a^3-ab^2\right)\cos(dx+c)^7\log(-\sin(dx+c)+1)+160\,b^3+224}{1120\,d\,\mathrm{co}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*a
^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*b
- b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*cos(
d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^
7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20132, size = 628, normalized size = 2.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
- a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/2*
c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)
^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)^11
+ 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^10 + 6
30*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5040*a^
2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*a^2*b*t
an(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*tan(1/2*d
*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan(1/2*d*x
+ 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x + 1/2*c)^
3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
```

```
224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) - 105*a*b^2*t
an(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

3.71 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=174

$$\frac{b(3a^2+2b^2)\tan^6(c+dx)}{6d}+\frac{a(a^2+6b^2)\tan^5(c+dx)}{5d}+\frac{b(6a^2+b^2)\tan^4(c+dx)}{4d}+\frac{a(2a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{3a^2}{}$$

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(2*a^2 + 3*b^2)*
Tan[c + d*x]^3)/(3*d) + (b*(6*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (a*(a^2 +
6*b^2)*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Tan[c + d*x]^6)/(6*d) + (
3*a*b^2*Tan[c + d*x]^7)/(7*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.13956, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{b(3a^2+2b^2)\tan^6(c+dx)}{6d}+\frac{a(a^2+6b^2)\tan^5(c+dx)}{5d}+\frac{b(6a^2+b^2)\tan^4(c+dx)}{4d}+\frac{a(2a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{3a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(2*a^2 + 3*b^2)*
Tan[c + d*x]^3)/(3*d) + (b*(6*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (a*(a^2 +
6*b^2)*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Tan[c + d*x]^6)/(6*d) + (
3*a*b^2*Tan[c + d*x]^7)/(7*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
```

```
& EqQ[d, 0]))
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3\left(1+x^2\right)^2}{x^9}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^9}+\frac{3ab^2}{x^8}+\frac{3a^2b+2b^3}{x^7}+\frac{a^3+6ab^2}{x^6}+\frac{6a^2b+b^3}{x^5}+\frac{2a^3+3ab^2}{x^4}+\frac{3a^2b}{x^3}\right.\right.}{d}$$
$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a\left(2a^2+3b^2\right)\tan^3(c+dx)}{3d}+$$

Mathematica [A] time = 0.597798, size = 115, normalized size = 0.66

$$\frac{\frac{1}{3}\left(3a^2+b^2\right)(a+b\tan(c+dx))^6-\frac{4}{5}a\left(a^2+b^2\right)(a+b\tan(c+dx))^5+\frac{1}{4}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^4+\frac{1}{8}(a+b\tan(c+}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[c
+ d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

Maple [A] time = 0.128, size = 173, normalized size = 1.

$$\frac{1}{d}\left(-a^3\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4(\sec(dx+c))^2}{15}\right)\tan(dx+c)+\frac{a^2b}{2(\cos(dx+c))^6}+3ab^2\left(1/7\frac{(\sin(dx+c))^3}{(\cos(dx+c))^7}+\frac{4(\sin}{35(\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/2*a^2*b/c
os(d*x+c)^6+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*
x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+
```

```
1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A] time = 1.28866, size = 208, normalized size = 1.2

$$\frac{56\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+24\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+c\right.}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(56*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a*b^2 + 35*(4*
sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)
^4 - 4*sin(d*x + c)^2 + 1) - 420*a^2*b/(sin(d*x + c)^2 - 1)^3)/d
```

Fricas [A] time = 0.512588, size = 304, normalized size = 1.75

$$\frac{105\,b^3+140\left(3\,a^2b-b^3\right)\cos(dx+c)^2+8\left(8\left(7\,a^3-3\,ab^2\right)\cos(dx+c)^7+4\left(7\,a^3-3\,ab^2\right)\cos(dx+c)^5+45\,ab^2\cos(d\right.}{840\,d\cos(dx+c)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^2
)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x +
c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

[Out] Timed out

Giac [A] time = 1.18352, size = 224, normalized size = 1.29

$$\frac{105\,b^3\tan(dx+c)^8+360\,ab^2\tan(dx+c)^7+420\,a^2b\tan(dx+c)^6+280\,b^3\tan(dx+c)^6+168\,a^3\tan(dx+c)^5+1008}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d*
x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*tan
(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a^3*
tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 + 840
*a^3*tan(d*x + c))/d
```

3.72 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=259

$$\frac{3a^2b\sec^7(c+dx)}{7d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^3\tan(c$$

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a*b^2*ArcTanh[Sin[c + d*x]])/(12
8*d) + (3*a^2*b*Sec[c + d*x]^7)/(7*d) - (b^3*Sec[c + d*x]^7)/(7*d) + (b^3*S
ec[c + d*x]^9)/(9*d) + (5*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a*b^2
*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) - (5*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (a^3*Sec[c + d*x]^5*
Tan[c + d*x])/(6*d) - (a*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (3*a*b^2
*Sec[c + d*x]^7*Tan[c + d*x])/(8*d)
```

Rubi [A] time = 0.267958, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{3a^2b\sec^7(c+dx)}{7d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^3\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a*b^2*ArcTanh[Sin[c + d*x]])/(12
8*d) + (3*a^2*b*Sec[c + d*x]^7)/(7*d) - (b^3*Sec[c + d*x]^7)/(7*d) + (b^3*S
ec[c + d*x]^9)/(9*d) + (5*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a*b^2
*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) - (5*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (a^3*Sec[c + d*x]^5*
Tan[c + d*x])/(6*d) - (a*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (3*a*b^2
*Sec[c + d*x]^7*Tan[c + d*x])/(8*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \Big(a^3\sec^7(c+dx)+3a^2b\sec^7(c+dx)\tan(c+dx)+3ab^2\sec^7(c$$

$$= a^3\int \sec^7(c+dx)\,dx+\left(3a^2b\right)\int \sec^7(c+dx)\tan(c+dx)\,dx+\Big(3$$

$$= \frac{a^3\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{3ab^2\sec^7(c+dx)\tan(c+dx)}{8d}+\frac{1}{6}\Big($$

$$= \frac{3a^2b\sec^7(c+dx)}{7d}+\frac{5a^3\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^3\sec^5(c+dx}{6}$$

$$= \frac{3a^2b\sec^7(c+dx)}{7d}-\frac{b^3\sec^7(c+dx)}{7d}+\frac{b^3\sec^9(c+dx)}{9d}+\frac{5a^3\sec(c}{}$$

$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3a^2b\sec^7(c+dx)}{7d}-\frac{b^3\sec^7(c+dx)}{7d}+$$

$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15ab^2\tanh^{-1}(\sin(c+dx))}{128d}+\frac{3a^2b\sec^7}{7}$$

Mathematica [B] time = 4.10923, size = 810, normalized size = 3.13

$$\frac{\sec^9(c+dx)\left(-211680\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)a^3-90720\cos(5(c+dx))\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^9*(442368*a^2*b + 81920*b^3 + 147456*(3*a^2*b - b^3)*Cos[2*(c
 + d*x)] - 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3
4020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2268
0*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 8505*a*b^
2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2520*a^3*Cos[
9*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 945*a*b^2*Cos[9*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39690*a*(8*a^2 - 3*b^2)*
Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]) + 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]] - 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] - 34020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 22680*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 8505*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 2520*a^3*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 945*
```

```
a*b^2*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 223776*a^
3*Sin[2*(c + d*x)] + 303156*a*b^2*Sin[2*(c + d*x)] + 167328*a^3*Sin[4*(c +
d*x)] - 62748*a*b^2*Sin[4*(c + d*x)] + 43680*a^3*Sin[6*(c + d*x)] - 16380*a
*b^2*Sin[6*(c + d*x)] + 5040*a^3*Sin[8*(c + d*x)] - 1890*a*b^2*Sin[8*(c + d
*x)]))/(2064384*d)
```

Maple [A] time = 0.139, size = 399, normalized size = 1.5

$$\frac{a^3 (\sec(dx+c))^5 \tan(dx+c)}{6d} + \frac{5a^3 (\sec(dx+c))^3 \tan(dx+c)}{24d} + \frac{5a^3 \sec(dx+c)\tan(dx+c)}{16d} + \frac{5a^3 \ln(\sec(dx+c)+}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/6*a^3*sec(d*x+c)^5*tan(d*x+c)/d+5/24*a^3*sec(d*x+c)^3*tan(d*x+c)/d+5/16*a
^3*sec(d*x+c)*tan(d*x+c)/d+5/16/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3/7/d*a^2*b
/cos(d*x+c)^7+3/8/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^8+5/16/d*a*b^2*sin(d*x+c)
^3/cos(d*x+c)^6+15/64/d*a*b^2*sin(d*x+c)^3/cos(d*x+c)^4+15/128/d*a*b^2*sin(
d*x+c)^3/cos(d*x+c)^2+15/128*a*b^2*sin(d*x+c)/d-15/128/d*a*b^2*ln(sec(d*x+c
)+tan(d*x+c))+1/9/d*b^3*sin(d*x+c)^4/cos(d*x+c)^9+5/63/d*b^3*sin(d*x+c)^4/c
os(d*x+c)^7+1/21/d*b^3*sin(d*x+c)^4/cos(d*x+c)^5+1/63/d*b^3*sin(d*x+c)^4/co
s(d*x+c)^3-1/63/d*b^3*sin(d*x+c)^4/cos(d*x+c)-1/63/d*cos(d*x+c)*sin(d*x+c)^
2*b^3-2/63*b^3*cos(d*x+c)/d
```

Maxima [A] time = 1.21562, size = 335, normalized size = 1.29

$$\frac{63\,ab^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1} - 15\,\log(\sin(dx+c)+1) + 15\,\log(\sin(dx+c)-1)\right) - 168\,a^3\Big(}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] 1/16128*(63*a*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x +
c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)
^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
```

```
 - 1)) - 168*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c
))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(
d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6912*a^2*b/cos(d*x + c)^7 - 256
*(9*cos(d*x + c)^2 - 7)*b^3/cos(d*x + c)^9)/d
```

Fricas [A] time = 0.581469, size = 481, normalized size = 1.86

$$\frac{315\left(8a^3 - 3ab^2\right)\cos(dx+c)^9\log(\sin(dx+c)+1) - 315\left(8a^3 - 3ab^2\right)\cos(dx+c)^9\log(-\sin(dx+c)+1) + 1792b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/16128*(315*(8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(sin(d*x + c) + 1) - 315*(
8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(-sin(d*x + c) + 1) + 1792*b^3 + 2304*(3
*a^2*b - b^3)*cos(d*x + c)^2 + 42*(15*(8*a^3 - 3*a*b^2)*cos(d*x + c)^7 + 10
*(8*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 144*a*b^2*cos(d*x + c) + 8*(8*a^3 - 3*a
*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.21515, size = 806, normalized size = 3.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8064*(315*(8*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 315*(8*a
^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5544*a^3*tan(1/2*d*x
+ 1/2*c)^17 + 945*a*b^2*tan(1/2*d*x + 1/2*c)^17 - 24192*a^2*b*tan(1/2*d*x +
 1/2*c)^16 - 15792*a^3*tan(1/2*d*x + 1/2*c)^15 + 24066*a*b^2*tan(1/2*d*x +
1/2*c)^15 + 48384*a^2*b*tan(1/2*d*x + 1/2*c)^14 - 16128*b^3*tan(1/2*d*x + 1
/2*c)^14 + 29232*a^3*tan(1/2*d*x + 1/2*c)^13 + 31374*a*b^2*tan(1/2*d*x + 1/
2*c)^13 - 145152*a^2*b*tan(1/2*d*x + 1/2*c)^12 - 26880*b^3*tan(1/2*d*x + 1/
2*c)^12 - 33264*a^3*tan(1/2*d*x + 1/2*c)^11 + 54810*a*b^2*tan(1/2*d*x + 1/2
*c)^11 + 241920*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 80640*b^3*tan(1/2*d*x + 1/2
*c)^10 - 193536*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 48384*b^3*tan(1/2*d*x + 1/2*
c)^8 + 33264*a^3*tan(1/2*d*x + 1/2*c)^7 - 54810*a*b^2*tan(1/2*d*x + 1/2*c)^
7 + 145152*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 48384*b^3*tan(1/2*d*x + 1/2*c)^6
- 29232*a^3*tan(1/2*d*x + 1/2*c)^5 - 31374*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 7
6032*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 6912*b^3*tan(1/2*d*x + 1/2*c)^4 + 15792
*a^3*tan(1/2*d*x + 1/2*c)^3 - 24066*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6912*a^2
*b*tan(1/2*d*x + 1/2*c)^2 - 2304*b^3*tan(1/2*d*x + 1/2*c)^2 - 5544*a^3*tan(
1/2*d*x + 1/2*c) - 945*a*b^2*tan(1/2*d*x + 1/2*c) - 3456*a^2*b + 256*b^3)/(
tan(1/2*d*x + 1/2*c)^2 - 1)^9)/d
```

3.73 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=213

$$\frac{3b\left(a^2+b^2\right)\tan^8(c+dx)}{8d}+\frac{a\left(a^2+9b^2\right)\tan^7(c+dx)}{7d}+\frac{b\left(3a^2+b^2\right)\tan^6(c+dx)}{2d}+\frac{3a\left(a^2+3b^2\right)\tan^5(c+dx)}{5d}+\frac{b\left(9a\right.}{}$$

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b*(9*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)
*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^6)/(2*d) + (a*(a^2 +
 9*b^2)*Tan[c + d*x]^7)/(7*d) + (3*b*(a^2 + b^2)*Tan[c + d*x]^8)/(8*d) + (a
*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*Tan[c + d*x]^10)/(10*d)
```

Rubi [A] time = 0.178643, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{3b\left(a^2+b^2\right)\tan^8(c+dx)}{8d}+\frac{a\left(a^2+9b^2\right)\tan^7(c+dx)}{7d}+\frac{b\left(3a^2+b^2\right)\tan^6(c+dx)}{2d}+\frac{3a\left(a^2+3b^2\right)\tan^5(c+dx)}{5d}+\frac{b\left(9a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b*(9*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)
*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^6)/(2*d) + (a*(a^2 +
 9*b^2)*Tan[c + d*x]^7)/(7*d) + (3*b*(a^2 + b^2)*Tan[c + d*x]^8)/(8*d) + (a
*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*Tan[c + d*x]^10)/(10*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3(1+x^2)^3}{x^{11}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^{11}}+\frac{3ab^2}{x^{10}}+\frac{3b(a^2+b^2)}{x^9}+\frac{a^3+9ab^2}{x^8}+\frac{3(3a^2b+b^3)}{x^7}+\frac{3(a^3+3ab^2)}{x^6}\right.}{d}$$

$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a\left(a^2+b^2\right)\tan^3(c+dx)}{d}+\frac{b}{-}$$

Mathematica [A] time = 1.98851, size = 177, normalized size = 0.83

$$\frac{\frac{3}{8}\left(5a^2+b^2\right)(a+b\tan(c+dx))^8-\frac{4}{7}a\left(5a^2+3b^2\right)(a+b\tan(c+dx))^7+\frac{1}{2}\left(a^2+b^2\right)\left(5a^2+b^2\right)(a+b\tan(c+dx))^6-\frac{6}{5}a}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[c
 + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4*a
*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan[c
+ d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10/10)
/(b^7*d)
```

Maple [A] time = 0.129, size = 219, normalized size = 1.

$$\frac{1}{d}\left(-a^3\left(-\frac{16}{35}-\frac{(\sec(dx+c))^6}{7}-\frac{6\,(\sec(dx+c))^4}{35}-\frac{8\,(\sec(dx+c))^2}{35}\right)\tan(dx+c)+\frac{3\,a^2b}{8\,(\cos(dx+c))^8}+3\,ab^2\left(1/9\,\frac{(\sin}{(\cos}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+3/8*a^2*b/cos(d*x+c)^8+3*a*b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*
sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)
^3/cos(d*x+c)^3)+b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos
(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A] time = 1.29145, size = 248, normalized size = 1.16

$$\frac{24\left(5\tan\left(dx+c\right)^{7}+21\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}+35\tan\left(dx+c\right)\right)a^{3}+8\left(35\tan\left(dx+c\right)^{9}+135\tan\left(dx+c\right.\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] 1/840*(24*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*ta
n(d*x + c))*a^3 + 8*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x +
 c)^5 + 105*tan(d*x + c)^3)*a*b^2 - 21*(5*sin(d*x + c)^2 - 1)*b^3/(sin(d*x
+ c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(
d*x + c)^2 - 1) + 315*a^2*b/(sin(d*x + c)^2 - 1)^4)/d
```

Fricas [A] time = 0.55323, size = 344, normalized size = 1.62

$$\frac{84\,b^{3}+105\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}+8\left(16\left(3\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{9}+8\left(3\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{7}+6\left(3\,a^{3}-ab^{2}\right)\mathrm{co}\right.}{840\,d\cos\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)*
cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos(d
*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d
*x + c))/(d*cos(d*x + c)^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20083, size = 297, normalized size = 1.39

$$\frac{84\,b^3\tan(dx+c)^{10} + 280\,ab^2\tan(dx+c)^9 + 315\,a^2b\tan(dx+c)^8 + 315\,b^3\tan(dx+c)^8 + 120\,a^3\tan(dx+c)^7 + 1080\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d*
x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*tan
(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a^3*
tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4 + 21
0*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 +
1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d
```

3.74 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=279

$$-\frac{2a^2b^2\sin^9(c+dx)}{3d}+\frac{18a^2b^2\sin^7(c+dx)}{7d}-\frac{18a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{a^4\sin^9(c}{9d}$$

```
[Out] (-4*a*b^3*Cos[c + d*x]^7)/(7*d) - (4*a^3*b*Cos[c + d*x]^9)/(9*d) + (4*a*b^3
*Cos[c + d*x]^9)/(9*d) + (a^4*Sin[c + d*x])/d - (4*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (6*a^4*Sin[c + d*x]^5)/(5*d) - (18*a^2*b
^2*Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (4*a^4*Sin[c + d*x]
^7)/(7*d) + (18*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (2*b^4*Sin[c + d*x]^7)/(7*d
) + (a^4*Sin[c + d*x]^9)/(9*d) - (2*a^2*b^2*Sin[c + d*x]^9)/(3*d) + (b^4*Si
n[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.258005, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{2a^2b^2\sin^9(c+dx)}{3d}+\frac{18a^2b^2\sin^7(c+dx)}{7d}-\frac{18a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{a^4\sin^9(c}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```



```
[Out] (-4*a*b^3*Cos[c + d*x]^7)/(7*d) - (4*a^3*b*Cos[c + d*x]^9)/(9*d) + (4*a*b^3
*Cos[c + d*x]^9)/(9*d) + (a^4*Sin[c + d*x])/d - (4*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (6*a^4*Sin[c + d*x]^5)/(5*d) - (18*a^2*b
^2*Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (4*a^4*Sin[c + d*x]
^7)/(7*d) + (18*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (2*b^4*Sin[c + d*x]^7)/(7*d
) + (a^4*Sin[c + d*x]^9)/(9*d) - (2*a^2*b^2*Sin[c + d*x]^9)/(3*d) + (b^4*Si
n[c + d*x]^9)/(9*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^9(c+dx)+4a^3b\cos^8(c+dx)\sin(c+dx)+6a^2b^2\cos^7(c\right.$$
$$= a^4\int \cos^9(c+dx)\,dx+\left(4a^3b\right)\int \cos^8(c+dx)\sin(c+dx)\,dx+\left(6a\right.$$
$$= -\frac{a^4\,\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{(4}{}$$
$$= -\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{6a^4\sin^5}{5}$$
$$= -\frac{4ab^3\cos^7(c+dx)}{7d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{4ab^3\cos^9(c+dx)}{9d}+\frac{a^4}{}$$

Mathematica [A] time = 0.718227, size = 237, normalized size = 0.85

$$\frac{1890\left(14a^2b^2+21a^4+b^4\right)\sin(c+dx)+420\left(21a^4-b^4\right)\sin(3(c+dx))+252\left(-12a^2b^2+9a^4-b^4\right)\sin(5(c+dx))+45}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-2520*a*b*(7*a^2 + 3*b^2)*Cos[c + d*x] - 1680*a*b*(7*a^2 + 2*b^2)*Cos[3*(c
 + d*x)] - 5040*a^3*b*Cos[5*(c + d*x)] - 180*a*b*(7*a^2 - 3*b^2)*Cos[7*(c +
 d*x)] - 140*a*b*(a^2 - b^2)*Cos[9*(c + d*x)] + 1890*(21*a^4 + 14*a^2*b^2 +
 b^4)*Sin[c + d*x] + 420*(21*a^4 - b^4)*Sin[3*(c + d*x)] + 252*(9*a^4 - 12*
a^2*b^2 - b^4)*Sin[5*(c + d*x)] + 45*(9*a^4 - 30*a^2*b^2 + b^4)*Sin[7*(c +
d*x)] + 35*(a^4 - 6*a^2*b^2 + b^4)*Sin[9*(c + d*x)])/(80640*d)
```

Maple [A] time = 0.084, size = 236, normalized size = 0.9

$$\frac{1}{d}\left(b^4\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^6}{9}-\frac{\sin(dx+c)(\cos(dx+c))^6}{21}+\frac{\sin(dx+c)}{105}\left(\frac{8}{3}+(\cos(dx+c))^4+\frac{4\,(\cos(dx+}{3}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+4*a*b^3*(-1/9*sin(d*x+c)^2
```

```
*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+6*a^2*b^2*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/
63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-4/9*a^
3*b*cos(d*x+c)^9+1/9*a^4*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*
x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.56097, size = 251, normalized size = 0.9

$$-\frac{140\,a^3b\cos(dx+c)^9-\left(35\,\sin(dx+c)^9-180\,\sin(dx+c)^7+378\,\sin(dx+c)^5-420\,\sin(dx+c)^3+315\,\sin(dx+c)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/315*(140*a^3*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
+ 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4 + 6*(35*s
in(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^
3)*a^2*b^2 - 20*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a*b^3 - (35*sin(d*x +
 c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*b^4)/d
```

Fricas [A] time = 0.55069, size = 413, normalized size = 1.48

$$-\frac{180\,ab^3\cos(dx+c)^7+140\left(a^3b-ab^3\right)\cos(dx+c)^9-\left(35\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^8+10\left(4\,a^4+3\,a^2b^2-5\,b^4\right)\text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/315*(180*a*b^3*cos(d*x + c)^7 + 140*(a^3*b - a*b^3)*cos(d*x + c)^9 - (35
*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^8 + 10*(4*a^4 + 3*a^2*b^2 - 5*b^4)*co
s(d*x + c)^6 + 3*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^4 + 128*a^4 + 96*
a^2*b^2 + 8*b^4 + 4*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c
))/d
```

Sympy [A] time = 21.7535, size = 367, normalized size = 1.32

$$\begin{cases}\frac{128a^4\sin^9(c+dx)}{315d}+\frac{64a^4\sin^7(c+dx)\cos^2(c+dx)}{35d}+\frac{16a^4\sin^5(c+dx)\cos^4(c+dx)}{5d}+\frac{8a^4\sin^3(c+dx)\cos^6(c+dx)}{3d}+\frac{a^4\sin(c+dx)\cos^8(c+dx)}{d}-\frac{4a^3b\cos^9}{9d} \\ x\left(a\cos(c)+b\sin(c)\right)^4\cos^5(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((128*a**4*sin(c + d*x)**9/(315*d) + 64*a**4*sin(c + d*x)**7*cos(c
 + d*x)**2/(35*d) + 16*a**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**4*
sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**8/d
 - 4*a**3*b*cos(c + d*x)**9/(9*d) + 32*a**2*b**2*sin(c + d*x)**9/(105*d) +
48*a**2*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 12*a**2*b**2*sin(c +
d*x)**5*cos(c + d*x)**4/(5*d) + 2*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**6
/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 8*a*b**3*cos(c + d*x)
**9/(63*d) + 8*b**4*sin(c + d*x)**9/(315*d) + 4*b**4*sin(c + d*x)**7*cos(c
+ d*x)**2/(35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d), Ne(d, 0)), (
x*(a*cos(c) + b*sin(c))**4*cos(c)**5, True))
```

Giac [A] time = 1.21216, size = 363, normalized size = 1.3

$$-\frac{a^3b\cos(5\,dx+5\,c)}{16\,d}-\frac{\left(a^3b-ab^3\right)\cos(9\,dx+9\,c)}{576\,d}-\frac{\left(7\,a^3b-3\,ab^3\right)\cos(7\,dx+7\,c)}{448\,d}-\frac{\left(7\,a^3b+2\,ab^3\right)\cos(3\,dx+3\,c)}{48\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/16*a^3*b*cos(5*d*x + 5*c)/d - 1/576*(a^3*b - a*b^3)*cos(9*d*x + 9*c)/d -
 1/448*(7*a^3*b - 3*a*b^3)*cos(7*d*x + 7*c)/d - 1/48*(7*a^3*b + 2*a*b^3)*co
s(3*d*x + 3*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(d*x + c)/d + 1/2304*(a^4 -
6*a^2*b^2 + b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^4 - 30*a^2*b^2 + b^4)*sin
(7*d*x + 7*c)/d + 1/320*(9*a^4 - 12*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d + 1/1
92*(21*a^4 - b^4)*sin(3*d*x + 3*c)/d + 3/128*(21*a^4 + 14*a^2*b^2 + b^4)*si
n(d*x + c)/d
```

3.75 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=381

$$-\frac{3a^2b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{5a^2b^2\sin(c+dx)\cos^3(c+dx)}{32d}+\frac{15a^2b^2\sin(c+dx)\cos}{64d}$$

```
[Out] (35*a^4*x)/128 + (15*a^2*b^2*x)/64 + (3*b^4*x)/128 - (2*a*b^3*Cos[c + d*x]^
6)/(3*d) - (a^3*b*Cos[c + d*x]^8)/(2*d) + (a*b^3*Cos[c + d*x]^8)/(2*d) + (3
5*a^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (15*a^2*b^2*Cos[c + d*x]*Sin[c +
 d*x])/(64*d) + (3*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^4*Cos[c +
 d*x]^3*Sin[c + d*x])/(192*d) + (5*a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(32
*d) + (b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^4*Cos[c + d*x]^5*Sin[
c + d*x])/(48*d) + (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/(8*d) - (b^4*Cos[c
 + d*x]^5*Sin[c + d*x])/(16*d) + (a^4*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) -
(3*a^2*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (b^4*Cos[c + d*x]^5*Sin[c +
 d*x]^3)/(8*d)
```

Rubi [A] time = 0.388978, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14}

$$-\frac{3a^2b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{5a^2b^2\sin(c+dx)\cos^3(c+dx)}{32d}+\frac{15a^2b^2\sin(c+dx)\cos}{64d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (35*a^4*x)/128 + (15*a^2*b^2*x)/64 + (3*b^4*x)/128 - (2*a*b^3*Cos[c + d*x]^
6)/(3*d) - (a^3*b*Cos[c + d*x]^8)/(2*d) + (a*b^3*Cos[c + d*x]^8)/(2*d) + (3
5*a^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (15*a^2*b^2*Cos[c + d*x]*Sin[c +
 d*x])/(64*d) + (3*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^4*Cos[c +
 d*x]^3*Sin[c + d*x])/(192*d) + (5*a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(32
*d) + (b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^4*Cos[c + d*x]^5*Sin[
c + d*x])/(48*d) + (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/(8*d) - (b^4*Cos[c
 + d*x]^5*Sin[c + d*x])/(16*d) + (a^4*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) -
(3*a^2*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (b^4*Cos[c + d*x]^5*Sin[c +
 d*x]^3)/(8*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^8(c+dx)+4a^3b\cos^7(c+dx)\sin(c+dx)+6a^2b^2\cos^6(c+\right.$$
$$= a^4\int \cos^8(c+dx)\,dx + \left(4a^3b\right)\int \cos^7(c+dx)\sin(c+dx)\,dx + \left(6a^2b\right.$$
$$= \frac{a^4\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3a^2b^2\cos^7(c+dx)\sin(c+dx)}{4d} - \frac{b^4\text{co}}{}$$
$$= -\frac{a^3b\cos^8(c+dx)}{2d} + \frac{7a^4\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{a^2b^2\cos^5(c+dx}{8d}$$
$$= -\frac{2ab^3\cos^6(c+dx)}{3d} - \frac{a^3b\cos^8(c+dx)}{2d} + \frac{ab^3\cos^8(c+dx)}{2d} + \frac{35a^4\text{c}}{}$$
$$= -\frac{2ab^3\cos^6(c+dx)}{3d} - \frac{a^3b\cos^8(c+dx)}{2d} + \frac{ab^3\cos^8(c+dx)}{2d} + \frac{35a^4\text{c}}{}$$
$$= \frac{35a^4x}{128} + \frac{15}{64}a^2b^2x + \frac{3b^4x}{128} - \frac{2ab^3\cos^6(c+dx)}{3d} - \frac{a^3b\cos^8(c+dx)}{2d} +$$

Mathematica [A] time = 0.600107, size = 222, normalized size = 0.58

$$\frac{24\left(30a^2b^2+35a^4+3b^4\right)(c+dx)+96a^2\left(7a^2+3b^2\right)\sin(2(c+dx))+32a^2\left(a^2-3b^2\right)\sin(6(c+dx))+24\left(-6a^2b^2+7a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (24*(35*a^4 + 30*a^2*b^2 + 3*b^4)*(c + d*x) - 96*a*b*(7*a^2 + 3*b^2)*Cos[2*
(c + d*x)] - 48*a*b*(7*a^2 + b^2)*Cos[4*(c + d*x)] - 32*a*b*(3*a^2 - b^2)*C
os[6*(c + d*x)] - 12*a*b*(a^2 - b^2)*Cos[8*(c + d*x)] + 96*a^2*(7*a^2 + 3*b
^2)*Sin[2*(c + d*x)] + 24*(7*a^4 - 6*a^2*b^2 - b^4)*Sin[4*(c + d*x)] + 32*a
^2*(a^2 - 3*b^2)*Sin[6*(c + d*x)] + 3*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c + d*
x)])/(3072*d)
```

Maple [A] time = 0.086, size = 250, normalized size = 0.7

$$\frac{1}{d}\left(b^4\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^5}{8} - \frac{\sin(dx+c)(\cos(dx+c))^5}{16} + \frac{\sin(dx+c)}{64}\left((\cos(dx+c))^3 + \frac{3\cos(dx+c)}{2}\right) + \frac{3}{1}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+4*a*b^3*(-1/8*s
in(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+6*a^2*b^2*(-1/8*sin(d*x+c)*cos(
d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
128*d*x+5/128*c)-1/2*a^3*b*cos(d*x+c)^8+a^4*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+
c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A] time = 1.14786, size = 269, normalized size = 0.71

$$-\frac{1536\,a^3b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\,c\right)-768\,\sin(2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3072*(1536*a^3*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^4
 - 6*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin
(4*d*x + 4*c))*a^2*b^2 - 512*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin(d
*x + c)^4)*a*b^3 - 3*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c)
)*b^4)/d
```

Fricas [A] time = 0.539061, size = 424, normalized size = 1.11

$$-\frac{256\,ab^3\cos\left(dx+c\right)^6+192\left(a^3b-ab^3\right)\cos\left(dx+c\right)^8-3\left(35\,a^4+30\,a^2b^2+3\,b^4\right)dx-\left(48\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/384*(256*a*b^3*cos(d*x + c)^6 + 192*(a^3*b - a*b^3)*cos(d*x + c)^8 - 3*(
35*a^4 + 30*a^2*b^2 + 3*b^4)*d*x - (48*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)
^7 + 8*(7*a^4 + 6*a^2*b^2 - 9*b^4)*cos(d*x + c)^5 + 2*(35*a^4 + 30*a^2*b^2
+ 3*b^4)*cos(d*x + c)^3 + 3*(35*a^4 + 30*a^2*b^2 + 3*b^4)*cos(d*x + c))*sin
(d*x + c))/d
```

Sympy [A] time = 15.7511, size = 736, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((35*a**4*x*sin(c + d*x)**8/128 + 35*a**4*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**4*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**4*x*cos(c + d*x)**8/128 + 35*a**4*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**4*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**4*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - a**3*b*cos(c + d*x)**8/(2*d) + 15*a**
2*b**2*x*sin(c + d*x)**8/64 + 15*a**2*b**2*x*sin(c + d*x)**6*cos(c + d*x)**
2/16 + 45*a**2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 15*a**2*b**2*x*s
in(c + d*x)**2*cos(c + d*x)**6/16 + 15*a**2*b**2*x*cos(c + d*x)**8/64 + 15*
a**2*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*a**2*b**2*sin(c + d*x)**
5*cos(c + d*x)**3/(64*d) + 73*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(64
*d) - 15*a**2*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) - 2*a*b**3*sin(c + d
*x)**2*cos(c + d*x)**6/(3*d) - a*b**3*cos(c + d*x)**8/(6*d) + 3*b**4*x*sin(
c + d*x)**8/128 + 3*b**4*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**4*x*si
n(c + d*x)**4*cos(c + d*x)**4/64 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**6
/32 + 3*b**4*x*cos(c + d*x)**8/128 + 3*b**4*sin(c + d*x)**7*cos(c + d*x)/(1
28*d) + 11*b**4*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**4*sin(c + d
*x)**3*cos(c + d*x)**5/(128*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**7/(128*d
), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**4*cos(c)**4, True))
```

Giac [A] time = 1.2205, size = 331, normalized size = 0.87

$$\frac{1}{128}\left(35\,a^4+30\,a^2b^2+3\,b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(8\,dx+8\,c\right)}{256\,d}-\frac{\left(3\,a^3b-ab^3\right)\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{\left(7\,a^3b+ab^3\right)\cos\left(4\,dx+4\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/128*(35*a^4 + 30*a^2*b^2 + 3*b^4)*x - 1/256*(a^3*b - a*b^3)*cos(8*d*x + 8
*c)/d - 1/96*(3*a^3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/64*(7*a^3*b + a*b^3)*
cos(4*d*x + 4*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(2*d*x + 2*c)/d + 1/1024*(
a^4 - 6*a^2*b^2 + b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^4 - 3*a^2*b^2)*sin(6*d*
x + 6*c)/d + 1/128*(7*a^4 - 6*a^2*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/32*(7*a
```

```
^4 + 3*a^2*b^2)*sin(2*d*x + 2*c)/d
```

3.76 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=220

$$\frac{6a^2b^2\sin^7(c+dx)}{7d}-\frac{12a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^7(c+dx)}{7d}-\frac{a^4\sin^7(c+dx)}{7d}+\frac{3a^4\sin^5(c+dx)}{5d}$$

```
[Out] (-4*a*b^3*Cos[c + d*x]^5)/(5*d) - (4*a^3*b*Cos[c + d*x]^7)/(7*d) + (4*a*b^3
*Cos[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/d + (2
*a^2*b^2*Sin[c + d*x]^3)/d + (3*a^4*Sin[c + d*x]^5)/(5*d) - (12*a^2*b^2*Sin
[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d
) + (6*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (b^4*Sin[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.234498, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$\frac{6a^2b^2\sin^7(c+dx)}{7d}-\frac{12a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^7(c+dx)}{7d}-\frac{a^4\sin^7(c+dx)}{7d}+\frac{3a^4\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a*b^3*Cos[c + d*x]^5)/(5*d) - (4*a^3*b*Cos[c + d*x]^7)/(7*d) + (4*a*b^3
*Cos[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/d + (2
*a^2*b^2*Sin[c + d*x]^3)/d + (3*a^4*Sin[c + d*x]^5)/(5*d) - (12*a^2*b^2*Sin
[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d
) + (6*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (b^4*Sin[c + d*x]^7)/(7*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^7(c+dx)+4a^3b\cos^6(c+dx)\sin(c+dx)+6a^2b^2\cos^5(c+\right.$$
$$= a^4\int \cos^7(c+dx)\,dx+\left(4a^3b\right)\int \cos^6(c+dx)\sin(c+dx)\,dx+\left(6a^2\right.$$
$$= -\frac{a^4\,\mathrm{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(4a^3b\right)\mathrm{Su}}{}$$
$$= -\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{a^4\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{d}+\frac{3a^4\sin^5(c}{5d}$$
$$= -\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{4ab^3\cos^7(c+dx)}{7d}+\frac{a^4\,\mathrm{si}}{}$$

Mathematica [A] time = 0.520488, size = 204, normalized size = 0.93

$$\frac{35\left(30a^2b^2+35a^4+3b^4\right)\sin(c+dx)+35\left(-2a^2b^2+7a^4-b^4\right)\sin(3(c+dx))+7\left(-18a^2b^2+7a^4-b^4\right)\sin(5(c+dx))+5}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-140*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x] - 140*a*b*(3*a^2 + b^2)*Cos[3*(c + d
*x)] - 28*a*b*(5*a^2 - b^2)*Cos[5*(c + d*x)] - 20*a*b*(a^2 - b^2)*Cos[7*(c
+ d*x)] + 35*(35*a^4 + 30*a^2*b^2 + 3*b^4)*Sin[c + d*x] + 35*(7*a^4 - 2*a^2
*b^2 - b^4)*Sin[3*(c + d*x)] + 7*(7*a^4 - 18*a^2*b^2 - b^4)*Sin[5*(c + d*x)
] + 5*(a^4 - 6*a^2*b^2 + b^4)*Sin[7*(c + d*x)])/(2240*d)
```

Maple [A] time = 0.081, size = 206, normalized size = 0.9

$$\frac{1}{d}\left(b^4\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^4}{7}-\frac{3\sin(dx+c)(\cos(dx+c))^4}{35}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{35}\right)+4ab^3\left(-1/7\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(2+cos(d*x+c)^2)*sin(d*x+c))+4*a*b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*c
os(d*x+c)^5)+6*a^2*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4
```

```
+4/3*cos(d*x+c)^2)*sin(d*x+c))-4/7*a^3*b*cos(d*x+c)^7+1/7*a^4*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.14551, size = 208, normalized size = 0.95

$$-\frac{20\,a^3b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^4-2\left(15\,\sin\left(dx+c\right)^7\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(20*a^3*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4 - 2*(15*sin(d*x + c)^7 - 42*sin(d*x
+ c)^5 + 35*sin(d*x + c)^3)*a^2*b^2 - 4*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^
5)*a*b^3 + (5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*b^4)/d
```

Fricas [A] time = 0.525996, size = 336, normalized size = 1.53

$$-\frac{28\,ab^3\cos\left(dx+c\right)^5+20\left(a^3b-ab^3\right)\cos\left(dx+c\right)^7-\left(5\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6+2\left(3\,a^4+3\,a^2b^2-4\,b^4\right)\cos\left(\right.\right.}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(28*a*b^3*cos(d*x + c)^5 + 20*(a^3*b - a*b^3)*cos(d*x + c)^7 - (5*(a^
4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(3*a^4 + 3*a^2*b^2 - 4*b^4)*cos(d*x
 + c)^4 + 16*a^4 + 16*a^2*b^2 + 2*b^4 + (8*a^4 + 8*a^2*b^2 + b^4)*cos(d*x +
 c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 8.20488, size = 286, normalized size = 1.3

$$\begin{cases}\frac{16a^4\sin^7(c+dx)}{35d}+\frac{8a^4\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^4\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^4\sin(c+dx)\cos^6(c+dx)}{d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{16a^2b^2\sin^7(c+dx)}{35d}+\\x\left(a\cos(c)+b\sin(c)\right)^4\cos^3(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((16*a**4*sin(c + d*x)**7/(35*d) + 8*a**4*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**4*sin(c + d*x)**3*cos(c + d*x)**4/d + a**4*sin(c + d*x
)*cos(c + d*x)**6/d - 4*a**3*b*cos(c + d*x)**7/(7*d) + 16*a**2*b**2*sin(c +
 d*x)**7/(35*d) + 8*a**2*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*a**
2*b**2*sin(c + d*x)**3*cos(c + d*x)**4/d - 4*a*b**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 8*a*b**3*cos(c + d*x)**7/(35*d) + 2*b**4*sin(c + d*x)**7/(
35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**4*cos(c)**3, True))
```

Giac [A] time = 1.21194, size = 309, normalized size = 1.4

$$-\frac{\left(a^3b-ab^3\right)\cos\left(7\,dx+7\,c\right)}{112\,d}-\frac{\left(5\,a^3b-ab^3\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{\left(3\,a^3b+ab^3\right)\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{\left(5\,a^3b+3\,ab^3\right)\cos\left(dx+c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/112*(a^3*b - a*b^3)*cos(7*d*x + 7*c)/d - 1/80*(5*a^3*b - a*b^3)*cos(5*d*
x + 5*c)/d - 1/16*(3*a^3*b + a*b^3)*cos(3*d*x + 3*c)/d - 1/16*(5*a^3*b + 3*
a*b^3)*cos(d*x + c)/d + 1/448*(a^4 - 6*a^2*b^2 + b^4)*sin(7*d*x + 7*c)/d +
1/320*(7*a^4 - 18*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d + 1/64*(7*a^4 - 2*a^2*b
^2 - b^4)*sin(3*d*x + 3*c)/d + 1/64*(35*a^4 + 30*a^2*b^2 + 3*b^4)*sin(d*x +
 c)/d
```

3.77 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=301

$$-\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{d}+\frac{a^2b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2b^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}a^2b^2x-\frac{2a^3b\cos^6(c}{3d}$$

```
[Out] (5*a^4*x)/16 + (3*a^2*b^2*x)/8 + (b^4*x)/16 - (2*a^3*b*Cos[c + d*x]^6)/(3*d
) + (5*a^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (3*a^2*b^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (b^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^4*Cos[c + d
*x]^3*Sin[c + d*x])/(24*d) + (a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) -
(b^4*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*x]^5*Sin[c + d*x])
/(6*d) - (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/d - (b^4*Cos[c + d*x]^3*Sin[
c + d*x]^3)/(6*d) + (a*b^3*Sin[c + d*x]^4)/d - (2*a*b^3*Sin[c + d*x]^6)/(3*
d)
```

Rubi [A] time = 0.303332, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2635, 8, 2565, 30, 2568, 2564, 14}

$$-\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{d}+\frac{a^2b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2b^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}a^2b^2x-\frac{2a^3b\cos^6(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (5*a^4*x)/16 + (3*a^2*b^2*x)/8 + (b^4*x)/16 - (2*a^3*b*Cos[c + d*x]^6)/(3*d
) + (5*a^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (3*a^2*b^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (b^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^4*Cos[c + d
*x]^3*Sin[c + d*x])/(24*d) + (a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) -
(b^4*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*x]^5*Sin[c + d*x])
/(6*d) - (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/d - (b^4*Cos[c + d*x]^3*Sin[
c + d*x]^3)/(6*d) + (a*b^3*Sin[c + d*x]^4)/d - (2*a*b^3*Sin[c + d*x]^6)/(3*
d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^6(c+dx)+4a^3b\cos^5(c+dx)\sin(c+dx)+6a^2b^2\cos^4(c\right.$$

$$= a^4\int \cos^6(c+dx)\,dx + \left(4a^3b\right)\int \cos^5(c+dx)\sin(c+dx)\,dx + \left(6a\right.$$

$$= \frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{a^2b^2\cos^5(c+dx)\sin(c+dx)}{d} - \frac{b^4\,\mathrm{co}}{}$$

$$= -\frac{2a^3b\cos^6(c+dx)}{3d} + \frac{5a^4\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{a^2b^2\cos^3(c+}{}$$

$$= -\frac{2a^3b\cos^6(c+dx)}{3d} + \frac{5a^4\cos(c+dx)\sin(c+dx)}{16d} + \frac{3a^2b^2\cos(c+}{8}$$

$$= \frac{5a^4x}{16} + \frac{3}{8}a^2b^2x + \frac{b^4x}{16} - \frac{2a^3b\cos^6(c+dx)}{3d} + \frac{5a^4\cos(c+dx)\sin(c}{16d}$$

Mathematica [C] time = 0.427276, size = 178, normalized size = 0.59

$$\frac{12(a-ib)(a+ib)\left(5a^2+b^2\right)(c+dx)+3\left(6a^2b^2+15a^4-b^4\right)\sin(2(c+dx))+3\left(-6a^2b^2+3a^4-b^4\right)\sin(4(c+dx))+\left(-\right.}{19}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a - I*b)*(a + I*b)*(5*a^2 + b^2)*(c + d*x) - 12*a*b*(5*a^2 + 3*b^2)*Co
s[2*(c + d*x)] - 24*a^3*b*Cos[4*(c + d*x)] - 4*a*b*(a^2 - b^2)*Cos[6*(c + d
*x)] + 3*(15*a^4 + 6*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2
 - b^4)*Sin[4*(c + d*x)] + (a^4 - 6*a^2*b^2 + b^4)*Sin[6*(c + d*x)])/(192*d
)
```

Maple [A] time = 0.08, size = 219, normalized size = 0.7

$$\frac{1}{d}\left(b^4\left(-\frac{(\sin(dx+c))^3(\cos(dx+c))^3}{6} - \frac{\sin(dx+c)(\cos(dx+c))^3}{8} + \frac{\cos(dx+c)\sin(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) + 4\,ab^3\left(-1\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+4*a*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)
^4-1/12*cos(d*x+c)^4)+6*a^2*b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-2/3*a^3*b*cos(d*x+c)^6+a^
4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+
5/16*c))
```

Maxima [A] time = 1.24684, size = 230, normalized size = 0.76

$$-\frac{128\,a^3b\cos\left(dx+c\right)^6+\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^4-6\left(4\,\sin\left(2\,dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/192*(128*a^3*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c -
9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 6*(4*sin(2*d*x + 2*c)^3 + 1
2*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2*b^2 + 64*(2*sin(d*x + c)^6 - 3*sin(d
*x + c)^4)*a*b^3 + (4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*
c))*b^4)/d
```

Fricas [A] time = 0.53733, size = 342, normalized size = 1.14

$$-\frac{48\,ab^3\cos\left(dx+c\right)^4+32\left(a^3b-ab^3\right)\cos\left(dx+c\right)^6-3\left(5\,a^4+6\,a^2b^2+b^4\right)dx-\left(8\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^5+2\left(5\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/48*(48*a*b^3*cos(d*x + c)^4 + 32*(a^3*b - a*b^3)*cos(d*x + c)^6 - 3*(5*a
^4 + 6*a^2*b^2 + b^4)*d*x - (8*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^5 + 2*(
5*a^4 + 6*a^2*b^2 - 7*b^4)*cos(d*x + c)^3 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*cos
(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 5.41183, size = 614, normalized size = 2.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**4*x*cos(c
+ d*x)**6/16 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 + 2*a**3*b*sin(c + d*x)**6/(3*d) + 2*a**3*b*sin(c + d*x)**4*cos(c + d*x)**
2/d + 2*a**3*b*sin(c + d*x)**2*cos(c + d*x)**4/d + 3*a**2*b**2*x*sin(c + d*
x)**6/8 + 9*a**2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 9*a**2*b**2*x*s
in(c + d*x)**2*cos(c + d*x)**4/8 + 3*a**2*b**2*x*cos(c + d*x)**6/8 + 3*a**2
*b**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + a**2*b**2*sin(c + d*x)**3*cos(c
+ d*x)**3/d - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) + a*b**3*sin(c
 + d*x)**6/(3*d) + a*b**3*sin(c + d*x)**4*cos(c + d*x)**2/d + b**4*x*sin(c
+ d*x)**6/16 + 3*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**4*x*sin(c
 + d*x)**2*cos(c + d*x)**4/16 + b**4*x*cos(c + d*x)**6/16 + b**4*sin(c + d*
x)**5*cos(c + d*x)/(16*d) - b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b*
*4*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))
**4*cos(c)**2, True))
```

Giac [A] time = 1.23918, size = 252, normalized size = 0.84

$$-\frac{a^3b\cos(4\,dx+4\,c)}{8\,d}+\frac{1}{16}\left(5\,a^4+6\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos(6\,dx+6\,c)}{48\,d}-\frac{\left(5\,a^3b+3\,ab^3\right)\cos(2\,dx+2\,c)}{16\,d}+\frac{\left(a^4-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*a^3*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^4 + 6*a^2*b^2 + b^4)*x - 1/48*(a^
3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/16*(5*a^3*b + 3*a*b^3)*cos(2*d*x + 2*c)
/d + 1/192*(a^4 - 6*a^2*b^2 + b^4)*sin(6*d*x + 6*c)/d + 1/64*(3*a^4 - 6*a^2
*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/64*(15*a^4 + 6*a^2*b^2 - b^4)*sin(2*d*x
+ 2*c)/d
```

3.78 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=165

$$-\frac{6a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{a^4\sin^5(c+dx)}{5d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}+\frac{4}{}$$

```
[Out] (-4*a*b^3*Cos[c + d*x]^3)/(3*d) - (4*a^3*b*Cos[c + d*x]^5)/(5*d) + (4*a*b^3
*Cos[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*x])/d - (2*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (a^4*Sin[c + d*x]^5)/(5*d) - (6*a^2*b^2*
Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.181317, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3090, 2633, 2565, 30, 2564, 14}

$$-\frac{6a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{a^4\sin^5(c+dx)}{5d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}+\frac{4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (-4*a*b^3*Cos[c + d*x]^3)/(3*d) - (4*a^3*b*Cos[c + d*x]^5)/(5*d) + (4*a*b^3
*Cos[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*x])/d - (2*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (a^4*Sin[c + d*x]^5)/(5*d) - (6*a^2*b^2*
Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^5(c+dx)+4a^3b\cos^4(c+dx)\sin(c+dx)+6a^2b^2\cos^3(c+\right.$$
$$= a^4\int \cos^5(c+dx)\,dx+\left(4a^3b\right)\int \cos^4(c+dx)\sin(c+dx)\,dx+\left(6a^2\right.$$
$$= -\frac{a^4\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(4a^3b\right)\text{Subst}\left(\right.}{}$$
$$= -\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{a^4\sin(c+dx)}{d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin^5(c}{5d}$$
$$= -\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{4ab^3\cos^5(c+dx)}{5d}+\frac{a^4\,\text{s}}{}$$

Mathematica [A] time = 0.379479, size = 146, normalized size = 0.88

$$\frac{30\left(6a^2b^2+5a^4+b^4\right)\sin(c+dx)+5\left(-6a^2b^2+5a^4-3b^4\right)\sin(3(c+dx))+3\left(-6a^2b^2+a^4+b^4\right)\sin(5(c+dx))-120a}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-120*a*b*(a^2 + b^2)*Cos[c + d*x] - 20*a*b*(3*a^2 + b^2)*Cos[3*(c + d*x)]
- 12*a*b*(a^2 - b^2)*Cos[5*(c + d*x)] + 30*(5*a^4 + 6*a^2*b^2 + b^4)*Sin[c
+ d*x] + 5*(5*a^4 - 6*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*(a^4 - 6*a^2*b^
2 + b^4)*Sin[5*(c + d*x)])/(240*d)
```

Maple [A] time = 0.069, size = 142, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b^4(\sin(dx+c))^5}{5}+4\,ab^3\left(-1/5\,(\sin(dx+c))^2(\cos(dx+c))^3-2/15\,(\cos(dx+c))^3\right)+6\,a^2b^2\left(-1/5\,\sin(dx+c)(\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(1/5*b^4*sin(d*x+c)^5+4*a*b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(
d*x+c)^3)+6*a^2*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin
(d*x+c))-4/5*a^3*b*cos(d*x+c)^5+1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c))
```

Maxima [A] time = 1.15388, size = 166, normalized size = 1.01

$$-\frac{12\,a^3b\cos(dx+c)^5-3\,b^4\sin(dx+c)^5-\left(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c)\right)a^4+6\left(3\,\sin(dx+c)^5-\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(12*a^3*b*cos(d*x + c)^5 - 3*b^4*sin(d*x + c)^5 - (3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 6*(3*sin(d*x + c)^5 - 5*sin(d*x
 + c)^3)*a^2*b^2 - 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b^3)/d
```

Fricas [A] time = 0.517038, size = 277, normalized size = 1.68

$$-\frac{20\,ab^3\cos(dx+c)^3+12\left(a^3b-ab^3\right)\cos(dx+c)^5-\left(3\left(a^4-6\,a^2b^2+b^4\right)\cos(dx+c)^4+8\,a^4+12\,a^2b^2+3\,b^4+2\left(2\,a^4\right.\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(20*a*b^3*cos(d*x + c)^3 + 12*(a^3*b - a*b^3)*cos(d*x + c)^5 - (3*(a^
4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*a^4 + 12*a^2*b^2 + 3*b^4 + 2*(2*a^4
 + 3*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.45826, size = 206, normalized size = 1.25

$$\begin{cases} \frac{8a^4\sin^5(c+dx)}{15d} + \frac{4a^4\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^4\sin(c+dx)\cos^4(c+dx)}{d} - \frac{4a^3b\cos^5(c+dx)}{5d} + \frac{4a^2b^2\sin^5(c+dx)}{5d} + \frac{2a^2b^2\sin^3(c+dx)\cos^2(c+dx)}{d} - \\ x(a\cos(c) + b\sin(c))^4\cos(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((8*a**4*sin(c + d*x)**5/(15*d) + 4*a**4*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**4/d - 4*a**3*b*cos(c + d*x)*
*5/(5*d) + 4*a**2*b**2*sin(c + d*x)**5/(5*d) + 2*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)**2/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*a*b*
*3*cos(c + d*x)**5/(15*d) + b**4*sin(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*co
s(c) + b*sin(c))**4*cos(c), True))
```

Giac [A] time = 1.19437, size = 223, normalized size = 1.35

$$-\frac{\left(a^3b - ab^3\right)\cos(5\,dx + 5\,c)}{20\,d} - \frac{\left(3\,a^3b + ab^3\right)\cos(3\,dx + 3\,c)}{12\,d} - \frac{\left(a^3b + ab^3\right)\cos(dx + c)}{2\,d} + \frac{\left(a^4 - 6\,a^2b^2 + b^4\right)\sin(5\,dx +}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/20*(a^3*b - a*b^3)*cos(5*d*x + 5*c)/d - 1/12*(3*a^3*b + a*b^3)*cos(3*d*x
 + 3*c)/d - 1/2*(a^3*b + a*b^3)*cos(d*x + c)/d + 1/80*(a^4 - 6*a^2*b^2 + b^
4)*sin(5*d*x + 5*c)/d + 1/48*(5*a^4 - 6*a^2*b^2 - 3*b^4)*sin(3*d*x + 3*c)/d
 + 1/8*(5*a^4 + 6*a^2*b^2 + b^4)*sin(d*x + c)/d
```

3.79 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=108

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(c+dx}{}$$

```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```

Rubi [A] time = 0.0441804, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3073, 8}

$$-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```

Rule 3073

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Si
n[c + d*x])^(n - 1))/(d*n), x] + Dist[((n - 1)*(a^2 + b^2))/n, Int[(a*Cos[c
 + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}+\frac{1}{4}\left(3\left(a^2+b^2\right.\right.$$
$$= -\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{8d}-\frac{(b\,c}{}$$
$$= \frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin}{8d}$$

Mathematica [A] time = 0.402489, size = 107, normalized size = 0.99

$$\frac{12\left(a^2+b^2\right)^2(c+dx)+8\left(a^4-b^4\right)\sin(2(c+dx))+\left(-6a^2b^2+a^4+b^4\right)\sin(4(c+dx))-16ab\left(a^2+b^2\right)\cos(2(c+dx))-}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*(
a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a^2
*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.071, size = 153, normalized size = 1.4

$$\frac{1}{d}\left(b^4\left(-\frac{\cos(dx+c)}{4}\left((\sin(dx+c))^3+\frac{3\sin(dx+c)}{2}\right)+\frac{3dx}{8}+\frac{3c}{8}\right)+ab^3(\sin(dx+c))^4+6a^2b^2\left(-1/4\sin(dx+c)(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(b^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*b^
3*sin(d*x+c)^4+6*a^2*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d
*x+c)+1/8*d*x+1/8*c)-cos(d*x+c)^4*a^3*b+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+
c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.1589, size = 184, normalized size = 1.7

$$-\frac{a^3b\cos(dx+c)^4}{d}+\frac{ab^3\sin(dx+c)^4}{d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d}+\frac{3\,(4\,dx+4\,c-\sin(4\,c}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*x
+ 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x +
2*c))*b^4/d
```

Fricas [A] time = 0.50395, size = 273, normalized size = 2.53

$$-\frac{16\,ab^3\cos\left(dx+c\right)^2+8\left(a^3b-ab^3\right)\cos\left(dx+c\right)^4-3\left(a^4+2\,a^2b^2+b^4\right)dx-\left(2\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3+\left(3\,a^4+\right.\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4 +
 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^4
+ 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 1.5365, size = 381, normalized size = 3.53

$$\begin{cases}\frac{3a^4x\sin^4\left(c+dx\right)}{8}+\frac{3a^4x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{3a^4x\cos^4\left(c+dx\right)}{8}+\frac{3a^4\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a^4\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{a^3b\cos^4\left(c+dx\right)}{d}+\frac{3a}{} \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^4\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)**4/
d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a*b**
3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2
```

```
*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x
*(a*cos(c) + b*sin(c))**4, True))
```

Giac [A] time = 1.10525, size = 165, normalized size = 1.53

$$\frac{3}{8}\left(a^4+2\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(4\,dx+4\,c\right)}{8\,d}-\frac{\left(a^3b+ab^3\right)\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1/
2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4*d
*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

3.80 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=150

$$\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}-\frac{a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}+\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4ab^3\cos(c+dx)}{d}-\frac{b^4 \text{s}}{}$$

```
[Out] (b^4*ArcTanh[Sin[c + d*x]])/d - (4*a*b^3*Cos[c + d*x])/d - (4*a^3*b*Cos[c +
 d*x]^3)/(3*d) + (4*a*b^3*Cos[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x])/d - (b
^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d) + (2*a^2*b^2*Sin[c + d*x]^3
)/d - (b^4*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.15296, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3090, 2633, 2565, 30, 2564, 2592, 302, 206}

$$\frac{2a^2b^2\sin^3(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}-\frac{a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}+\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4ab^3\cos(c+dx)}{d}-\frac{b^4 \text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^4*ArcTanh[Sin[c + d*x]])/d - (4*a*b^3*Cos[c + d*x])/d - (4*a^3*b*Cos[c +
 d*x]^3)/(3*d) + (4*a*b^3*Cos[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x])/d - (b
^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d) + (2*a^2*b^2*Sin[c + d*x]^3
)/d - (b^4*Sin[c + d*x]^3)/(3*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^3(c+dx)+4a^3b\cos^2(c+dx)\sin(c+dx)+6a^2b^2\cos(c+dx\right.$$

$$= a^4\int \cos^3(c+dx)\,dx + \left(4a^3b\right)\int \cos^2(c+dx)\sin(c+dx)\,dx + \left(6a^2b^2\right.$$

$$= -\frac{a^4\,\mathrm{Subst}\left(\int\left(1-x^2\right)dx, x, -\sin(c+dx)\right)}{d} - \frac{\left(4a^3b\right)\mathrm{Subst}\left(\int x^2\,dx,\right.}{d}$$

$$= -\frac{4ab^3\cos(c+dx)}{d} - \frac{4a^3b\cos^3(c+dx)}{3d} + \frac{4ab^3\cos^3(c+dx)}{3d} + \frac{a^4\sin(}{}$$

$$= -\frac{4ab^3\cos(c+dx)}{d} - \frac{4a^3b\cos^3(c+dx)}{3d} + \frac{4ab^3\cos^3(c+dx)}{3d} + \frac{a^4\sin(}{}$$

$$= \frac{b^4\tanh^{-1}(\sin(c+dx))}{d} - \frac{4ab^3\cos(c+dx)}{d} - \frac{4a^3b\cos^3(c+dx)}{3d} + \frac{4al}{}$$

Mathematica [A] time = 0.994209, size = 181, normalized size = 1.21

$$\frac{18a^2b^2\sin(c+dx) - 6a^2b^2\sin(3(c+dx)) - 12ab\left(a^2+3b^2\right)\cos(c+dx) + \left(4ab^3-4a^3b\right)\cos(3(c+dx)) + 9a^4\sin(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-12*a*b*(a^2 + 3*b^2)*Cos[c + d*x] + (-4*a^3*b + 4*a*b^3)*Cos[3*(c + d*x)]
 - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^4*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 9*a^4*Sin[c + d*x] + 18*a^2*b^2*Sin[c + d*x] -
15*b^4*Sin[c + d*x] + a^4*Sin[3*(c + d*x)] - 6*a^2*b^2*Sin[3*(c + d*x)] + b
^4*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.12, size = 163, normalized size = 1.1

$$\frac{(\cos(dx+c))^2\sin(dx+c)\,a^4}{3\,d} + \frac{2\,a^4\sin(dx+c)}{3\,d} - \frac{4\,a^3b\,(\cos(dx+c))^3}{3\,d} + 2\,\frac{a^2b^2\,(\sin(dx+c))^3}{d} - \frac{4\,\cos(dx+c)\,(\sin(dx}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/3/d*cos(d*x+c)^2*sin(d*x+c)*a^4+2/3*a^4*sin(d*x+c)/d-4/3*a^3*b*cos(d*x+c)
^3/d+2*a^2*b^2*sin(d*x+c)^3/d-4/3/d*cos(d*x+c)*sin(d*x+c)^2*a*b^3-8/3*a*b^3
*cos(d*x+c)/d-1/3*b^4*sin(d*x+c)^3/d-b^4*sin(d*x+c)/d+1/d*b^4*ln(sec(d*x+c)
+tan(d*x+c))
```

Maxima [A] time = 1.17797, size = 170, normalized size = 1.13

$$-\frac{8\,a^3b\cos\left(dx+c\right)^3-12\,a^2b^2\sin\left(dx+c\right)^3+2\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^4-8\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)ab^3}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*a^3*b*cos(d*x + c)^3 - 12*a^2*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c)^
3 - 3*sin(d*x + c))*a^4 - 8*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^3 + (2*si
n(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*
x + c))*b^4)/d
```

Fricas [A] time = 0.523691, size = 289, normalized size = 1.93

$$-\frac{24\,ab^3\cos\left(dx+c\right)-3\,b^4\log\left(\sin\left(dx+c\right)+1\right)+3\,b^4\log\left(-\sin\left(dx+c\right)+1\right)+8\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3-2\left(2\,a^4+6\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(24*a*b^3*cos(d*x + c) - 3*b^4*log(sin(d*x + c) + 1) + 3*b^4*log(-sin(
d*x + c) + 1) + 8*(a^3*b - a*b^3)*cos(d*x + c)^3 - 2*(2*a^4 + 6*a^2*b^2 - 4
*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22915, size = 293, normalized size = 1.95

$$\frac{3\, b^4 \log\left(\left|\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) + 1\right|\right) - 3\, b^4 \log\left(\left|\tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right) - 1\right|\right) + \frac{2\left(3\, a^4 \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^5 - 3\, b^4 \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^5 - 12\, a^3 b \tan\left(\frac{1}{2}\, dx+\frac{1}{2}\, c\right)^4}{}}{3\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*b^4*tan(1/2*d*x + 1/2*c
)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 24*a
^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*b^4*tan(1/2*d*x + 1/2*c)^3 - 24*a*b^3*ta
n(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) - 3*b^4*tan(1/2*d*x + 1/2
*c) - 4*a^3*b - 8*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.81 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=119

$$\frac{\sin^2(c+dx)\left(\left(-6a^2b^2+a^4+b^4\right)\cot(c+dx)+4ab\left(a^2-b^2\right)\right)}{2d}+\frac{1}{2}x\left(6a^2b^2+a^4-3b^4\right)-\frac{4ab^3\log(\sin(c+dx))}{d}+\frac{4ab^3\ ...}{}$$

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/2 - (4*a*b^3*Log[Sin[c + d*x]])/d + (4*a*b^3*
Log[Tan[c + d*x]])/d + ((4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Cot[c
+ d*x])*Sin[c + d*x]^2)/(2*d) + (b^4*Tan[c + d*x])/d
```

Rubi [A] time = 0.183296, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3088, 1805, 1802, 635, 203, 260}

$$\frac{\sin^2(c+dx)\left(\left(-6a^2b^2+a^4+b^4\right)\cot(c+dx)+4ab\left(a^2-b^2\right)\right)}{2d}+\frac{1}{2}x\left(6a^2b^2+a^4-3b^4\right)-\frac{4ab^3\log(\sin(c+dx))}{d}+\frac{4ab^3\ ...}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/2 - (4*a*b^3*Log[Sin[c + d*x]])/d + (4*a*b^3*
Log[Tan[c + d*x]])/d + ((4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Cot[c
+ d*x])*Sin[c + d*x]^2)/(2*d) + (b^4*Tan[c + d*x])/d
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4}{x^2(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d} + \frac{\text{Sub}}{}$$

$$= \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+dx)\right)\sin^2(c+dx)}{2d} + \frac{\text{Sub}}{}$$

$$= \frac{4ab^3\log(\tan(c+dx))}{d} + \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+\right.}{2d}$$

$$= \frac{4ab^3\log(\tan(c+dx))}{d} + \frac{\left(4ab\left(a^2-b^2\right)+\left(a^4-6a^2b^2+b^4\right)\cot(c+\right.}{2d}$$

$$= \frac{1}{2}\left(a^4+6a^2b^2-3b^4\right)x - \frac{4ab^3\log(\sin(c+dx))}{d} + \frac{4ab^3\log(\tan(c+}{d}$$

Mathematica [B] time = 6.26261, size = 477, normalized size = 4.01

$$b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^5\left(ab\tan(c+dx)+b^2\right)}{2b^4\left(a^2+b^2\right)} - \frac{\left(3b^2-5a^2\right)\left(b\left(6a^2-b^2\right)\tan(c+dx)+\frac{1}{2}\left(\frac{-6a^2b^2+a^4+b^4}{\sqrt{-b^2}}+4a(a-b)(a+b)\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(4a(a-\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^5*(b^2 + a*b*Tan[c + d*x]))/(2*b
^4*(a^2 + b^2)) - ((-5*a^2 + 3*b^2)*(((4*a*(a - b)*(a + b) + (a^4 - 6*a^2*b
^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a*(a - b)*(
a + b) - (a^4 - 6*a^2*b^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x
]])/2 + b*(6*a^2 - b^2)*Tan[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c
+ d*x]^3)/3) + 4*a*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 + 5*a*b^
4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^2*b^2 +
 b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*Tan[c + d
*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4))/(2*b^2*(a^
2 + b^2))))/d
```

Maple [A] time = 0.133, size = 210, normalized size = 1.8

$$\frac{a^4\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^4x}{2}+\frac{a^4c}{2d}-2\frac{(\cos(dx+c))^2a^3b}{d}-3\frac{a^2b^2\cos(dx+c)\sin(dx+c)}{d}+3a^2b^2x+3\frac{a^2b^2c}{d}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*cos(d*x+c)*sin(d*x+c)/d+1/2*a^4*x+1/2/d*a^4*c-2/d*cos(d*x+c)^2*a^3*
b-3*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+3*a^2*b^2*x+3/d*a^2*b^2*c-2/d*a*b^3*sin
(d*x+c)^2-4/d*a*b^3*ln(cos(d*x+c))+1/d*b^4*sin(d*x+c)^5/cos(d*x+c)+1/d*b^4*
cos(d*x+c)*sin(d*x+c)^3+3/2*b^4*cos(d*x+c)*sin(d*x+c)/d-3/2*b^4*x-3/2/d*b^4
*c
```

Maxima [A] time = 1.78476, size = 182, normalized size = 1.53

$$\frac{8a^3b\sin(dx+c)^2+(2dx+2c+\sin(2dx+2c))a^4+6(2dx+2c-\sin(2dx+2c))a^2b^2-8\left(\sin(dx+c)^2+\log\left(\sin(d\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 6*(2*d
*x + 2*c - sin(2*d*x + 2*c))*a^2*b^2 - 8*(sin(d*x + c)^2 + log(sin(d*x + c)
^2 - 1))*a*b^3 - 2*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan
(d*x + c))*b^4)/d
```

Fricas [A] time = 0.52448, size = 312, normalized size = 2.62

$$-\frac{8ab^3\cos(dx+c)\log(-\cos(dx+c))+4\left(a^3b-ab^3\right)\cos(dx+c)^3-\left(2a^3b-2ab^3+\left(a^4+6a^2b^2-3b^4\right)dx\right)\cos(dx+c}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a*b^3*cos(d*x + c)*log(-cos(d*x + c)) + 4*(a^3*b - a*b^3)*cos(d*x +
 c)^3 - (2*a^3*b - 2*a*b^3 + (a^4 + 6*a^2*b^2 - 3*b^4)*d*x)*cos(d*x + c) -
(2*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
 c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21104, size = 173, normalized size = 1.45

$$\frac{4\,ab^3\log\left(\tan\left(dx+c\right)^2+1\right)+2\,b^4\tan\left(dx+c\right)+\left(a^4+6\,a^2b^2-3\,b^4\right)\left(dx+c\right)-\frac{4\,ab^3\tan(dx+c)^2-a^4\tan(dx+c)+6\,a^2b^2\tan(dx+c)-}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b^3*log(tan(d*x + c)^2 + 1) + 2*b^4*tan(d*x + c) + (a^4 + 6*a^2*b^
2 - 3*b^4)*(d*x + c) - (4*a*b^3*tan(d*x + c)^2 - a^4*tan(d*x + c) + 6*a^2*b
^2*tan(d*x + c) - b^4*tan(d*x + c) + 4*a^3*b)/(tan(d*x + c)^2 + 1))/d
```

3.82 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=151

$$-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c+}{d}$$

```
[Out] (6*a^2*b^2*ArcTanh[Sin[c + d*x]])/d - (3*b^4*ArcTanh[Sin[c + d*x]])/(2*d) -
 (4*a^3*b*Cos[c + d*x])/d + (4*a*b^3*Cos[c + d*x])/d + (4*a*b^3*Sec[c + d*x
])/d + (a^4*Sin[c + d*x])/d - (6*a^2*b^2*Sin[c + d*x])/d + (3*b^4*Sin[c + d
*x])/(2*d) + (b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.164174, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {3090, 2637, 2638, 2592, 321, 206, 2590, 14, 288}

$$-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```



```
[Out] (6*a^2*b^2*ArcTanh[Sin[c + d*x]])/d - (3*b^4*ArcTanh[Sin[c + d*x]])/(2*d) -
 (4*a^3*b*Cos[c + d*x])/d + (4*a*b^3*Cos[c + d*x])/d + (4*a*b^3*Sec[c + d*x
])/d + (a^4*Sin[c + d*x])/d - (6*a^2*b^2*Sin[c + d*x])/d + (3*b^4*Sin[c + d
*x])/(2*d) + (b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos(c+dx)+4a^3b\sin(c+dx)+6a^2b^2\sin(c+dx)\tan(c+dx\right.$$
$$= a^4\int \cos(c+dx)\,dx+\left(4a^3b\right)\int \sin(c+dx)\,dx+\left(6a^2b^2\right)\int \sin(c+$$
$$= -\frac{4a^3b\cos(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}+\frac{\left(6a^2b^2\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\text{s}\right.}{d}$$
$$= -\frac{4a^3b\cos(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{b^4\sin(c+}{}$$
$$= \frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{4ab^3\cos(c+dx)}{d}+$$
$$= \frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{3b^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3b\cos(c-}{d}$$

Mathematica [A] time = 2.29426, size = 268, normalized size = 1.77

$$-24a^2b^2\sin(c+dx)-16ab\left(a^2-b^2\right)\cos(c+dx)-24a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+24a^2b^2\log\left(\sin\left(\frac{1}{2}(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (16*a*b^3 - 16*a*b*(a^2 - b^2)*Cos[c + d*x] - 24*a^2*b^2*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 6*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2
4*a^2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 6*b^4*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + b^4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 32
*a*b^3*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^4/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 + 4*a^4*Sin[c + d*x] - 24*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*
x])/(4*d)
```

Maple [A] time = 0.136, size = 211, normalized size = 1.4

$$\frac{a^4\sin(dx+c)}{d}-4\,\frac{a^3b\cos(dx+c)}{d}-6\,\frac{a^2b^2\sin(dx+c)}{d}+6\,\frac{a^2b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\,\frac{ab^3(\sin(dx+c))^4}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] a^4*sin(d*x+c)/d-4*a^3*b*cos(d*x+c)/d-6*a^2*b^2*sin(d*x+c)/d+6/d*a^2*b^2*ln
(sec(d*x+c)+tan(d*x+c))+4/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)+4/d*cos(d*x+c)*si
n(d*x+c)^2*a*b^3+8*a*b^3*cos(d*x+c)/d+1/2/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2+1
/2*b^4*sin(d*x+c)^3/d+3/2*b^4*sin(d*x+c)/d-3/2/d*b^4*ln(sec(d*x+c)+tan(d*x+
c))
```

Maxima [A] time = 1.23948, size = 192, normalized size = 1.27

$$-\frac{b^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+3\log(\sin(dx+c)+1)-3\log(\sin(dx+c)-1)-4\sin(dx+c)\right)-16\,ab^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)-}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/4*(b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 16*a*b^3*(1/cos(d*x + c) + cos(
d*x + c)) - 12*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*s
in(d*x + c)) + 16*a^3*b*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

Fricas [A] time = 0.522919, size = 363, normalized size = 2.4

$$\frac{16\,ab^3\cos(dx+c)-16\left(a^3b-ab^3\right)\cos(dx+c)^3+3\left(4\,a^2b^2-b^4\right)\cos(dx+c)^2\log(\sin(dx+c)+1)-3\left(4\,a^2b^2-b^4\right)\text{c}}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a*b^3*cos(d*x + c) - 16*(a^3*b - a*b^3)*cos(d*x + c)^3 + 3*(4*a^2*b
^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(4*a^2*b^2 - b^4)*cos(d*
x + c)^2*log(-sin(d*x + c) + 1) + 2*(b^4 + 2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*
x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27284, size = 278, normalized size = 1.84

$$\frac{3\left(4a^2b^2-b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(4a^2b^2-b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{4\left(a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}{\tan\left(\frac{1}{2}dx+\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*a^2*b^2 - b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b^2
- b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(a^4*tan(1/2*d*x + 1/2*c) - 6
*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/2*c) - 4*a^3*b + 4*a*b^
3)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*t
an(1/2*d*x + 1/2*c)^2 + b^4*tan(1/2*d*x + 1/2*c) + 8*a*b^3)/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

3.83 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=103

$$\frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d}-\frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d}+x\left(-6a^2b^2+a^4+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\,\mathrm{t}}{}$$

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x - (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(3*a^2 - b^2)*Tan[c + d*x])/d + (a*b*(a + b*Tan[c + d*x])^2)/d + (b*(a + b*
Tan[c + d*x])^3)/(3*d)
```

Rubi [A] time = 0.156476, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3482, 3528, 3525, 3475}

$$\frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d}-\frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d}+x\left(-6a^2b^2+a^4+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\,\mathrm{t}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x - (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(3*a^2 - b^2)*Tan[c + d*x])/d + (a*b*(a + b*Tan[c + d*x])^2)/d + (b*(a + b*
Tan[c + d*x])^3)/(3*d)
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int (a+b\tan(c+dx))^4\,dx$$
$$= \frac{b(a+b\tan(c+dx))^3}{3d} + \int (a+b\tan(c+dx))^2\left(a^2-b^2+2ab\tan(c\right.$$
$$= \frac{ab(a+b\tan(c+dx))^2}{d} + \frac{b(a+b\tan(c+dx))^3}{3d} + \int (a+b\tan(c+d$$
$$= \left(a^4-6a^2b^2+b^4\right)x + \frac{b^2\left(3a^2-b^2\right)\tan(c+dx)}{d} + \frac{ab(a+b\tan(c+d}{d}$$
$$= \left(a^4-6a^2b^2+b^4\right)x - \frac{4ab\left(a^2-b^2\right)\log(\cos(c+dx))}{d} + \frac{b^2\left(3a^2-b^2\right)}{d}$$

Mathematica [C] time = 0.39153, size = 105, normalized size = 1.02

$$\frac{-6b^2\left(b^2-6a^2\right)\tan(c+dx)+12ab^3\tan^2(c+dx)+3i(a-ib)^4\log(\tan(c+dx)+i)-3i(a+ib)^4\log(-\tan(c+dx)+i)+}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((-3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] + (3*I)*(a - I*b)^4*Log[I + Tan[c
 + d*x]] - 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 12*a*b^3*Tan[c + d*x]^2 + 2*
```

```
b^4*Tan[c + d*x]^3)/(6*d)
```

Maple [A] time = 0.14, size = 145, normalized size = 1.4

$$a^4x+\frac{a^4c}{d}-4\,\frac{a^3b\ln(\cos(dx+c))}{d}-6\,a^2b^2x+6\,\frac{\tan(dx+c)\,a^2b^2}{d}-6\,\frac{a^2b^2c}{d}+2\,\frac{ab^3\,(\tan(dx+c))^2}{d}+4\,\frac{ab^3\ln(\cos(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] a^4*x+1/d*a^4*c-4/d*a^3*b*ln(cos(d*x+c))-6*a^2*b^2*x+6/d*tan(d*x+c)*a^2*b^2
-6/d*a^2*b^2*c+2/d*a*b^3*tan(d*x+c)^2+4/d*a*b^3*ln(cos(d*x+c))+1/3/d*b^4*ta
n(d*x+c)^3-b^4*tan(d*x+c)/d+b^4*x+1/d*b^4*c
```

Maxima [A] time = 1.83705, size = 157, normalized size = 1.52

$$\frac{3\,(dx+c)a^4-18\,(dx+c-\tan(dx+c))a^2b^2+\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)b^4-6\,ab^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 - 18*(d*x + c - tan(d*x + c))*a^2*b^2 + (tan(d*x + c)^
3 + 3*d*x + 3*c - 3*tan(d*x + c))*b^4 - 6*a*b^3*(1/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c)^2 - 1)) - 6*a^3*b*log(-sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.514935, size = 282, normalized size = 2.74

$$\frac{3\left(a^4-6\,a^2b^2+b^4\right)dx\cos(dx+c)^3+6\,ab^3\cos(dx+c)-12\left(a^3b-ab^3\right)\cos(dx+c)^3\log(-\cos(dx+c))+\left(b^4+2\left(9\,a\right.\right.}{3\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*d*x*cos(d*x + c)^3 + 6*a*b^3*cos(d*x + c) -
12*(a^3*b - a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) + (b^4 + 2*(9*a^2*b^2
- 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19885, size = 140, normalized size = 1.36

$$\frac{b^4\tan(dx+c)^3+6\,ab^3\tan(dx+c)^2+18\,a^2b^2\tan(dx+c)-3\,b^4\tan(dx+c)+3\left(a^4-6\,a^2b^2+b^4\right)(dx+c)+6\left(a^3b-a\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
- 3*b^4*tan(d*x + c) + 3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c) + 6*(a^3*b - a*b
^3)*log(tan(d*x + c)^2 + 1))/d
```

3.84 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=168

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{4a^3b\sec(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\sec^3}{3a}$$

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/d + (3*b^
4*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*b*Sec[c + d*x])/d - (4*a*b^3*Sec[c
+ d*x])/d + (4*a*b^3*Sec[c + d*x]^3)/(3*d) + (3*a^2*b^2*Sec[c + d*x]*Tan[c
+ d*x])/d - (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]*Tan
[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.194526, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3090, 3770, 2606, 8, 2611}

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{4a^3b\sec(c+dx)}{d}+\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\sec^3}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/d + (3*b^
4*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*b*Sec[c + d*x])/d - (4*a*b^3*Sec[c
+ d*x])/d + (4*a*b^3*Sec[c + d*x]^3)/(3*d) + (3*a^2*b^2*Sec[c + d*x]*Tan[c
+ d*x])/d - (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]*Tan
[c + d*x]^3)/(4*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec(c+dx)+4a^3b\sec(c+dx)\tan(c+dx)+6a^2b^2\sec(c+dx\right.$$

$$= a^4\int \sec(c+dx)\,dx + \left(4a^3b\right)\int \sec(c+dx)\tan(c+dx)\,dx + \left(6a^2b^2\right.$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} + \frac{3a^2b^2\sec(c+dx)\tan(c+dx)}{d} + \frac{b^4\sec(c+}{}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{4a^3b\sec(c+}{d}$$

$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{3b^4\tanh^{-1}(s}{8a}$$

Mathematica [B] time = 6.23842, size = 936, normalized size = 5.57

$$\frac{2ab\left(6a^2-5b^2\right)\cos^4(c+dx)(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4} + \frac{\left(-8a^4+24b^2a^2-3b^4\right)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+\right.\right.}{8d(a\cos(c+dx)+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (2*a*b*(6*a^2 - 5*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 24*a^2*b^2 - 3*b^4)*Cos[c + d*x]^
4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 24*a^2*b^2 + 3*b^4)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Tan[c +
d*x])^4)/(16*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + ((72*a^2*b^2 + 16*a*b^3 - 15*b^4)*Cos[c + d*x]^4*(a + b*
Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) + (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*
Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*
x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(16*d
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^
4) - (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
) + ((-72*a^2*b^2 + 16*a*b^3 + 15*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^
4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^4) + (2*Cos[c + d*x]^4*(6*a^3*b*Sin[(c + d*x)/2] - 5*a*b^3*Sin[(c +
 d*x)/2])*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*Cos[c + d*x]^4*(6*a^3*b*Sin[(c
+ d*x)/2] - 5*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.147, size = 297, normalized size = 1.8

$$\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{d}+4\,\frac{a^3b}{d\cos(dx+c)}+3\,\frac{a^2b^2(\sin(dx+c))^3}{d(\cos(dx+c))^2}+3\,\frac{a^2b^2\sin(dx+c)}{d}-3\,\frac{a^2b^2\ln(\sec(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/d*a^3*b/cos(d*x+c)+3/d*a^2*b^2*sin(d*x+
c)^3/cos(d*x+c)^2+3*a^2*b^2*sin(d*x+c)/d-3/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+
c))+4/3/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^3-4/3/d*a*b^3*sin(d*x+c)^4/cos(d*x+
c)-4/3/d*cos(d*x+c)*sin(d*x+c)^2*a*b^3-8/3*a*b^3*cos(d*x+c)/d+1/4/d*b^4*sin
(d*x+c)^5/cos(d*x+c)^4-1/8/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2-1/8*b^4*sin(d*x+
c)^3/d-3/8*b^4*sin(d*x+c)/d+3/8/d*b^4*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.20938, size = 259, normalized size = 1.54

$$\frac{3\,b^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-72\,a^2b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)\right.\right.}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) - 72*a
^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 24*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 192*a^3*b/cos(d*x + c) - 64*(3*cos(d*x + c)^2 - 1)*a*b^3/cos(d*x + c)^3)/
d
```

Fricas [A] time = 0.512719, size = 392, normalized size = 2.33

$$\frac{3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right.}{48\,d\cos\left(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 64*
a*b^3*cos(d*x + c) + 192*(a^3*b - a*b^3)*cos(d*x + c)^3 + 6*(2*b^4 + (24*a^
2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

[Out] Timed out

Giac [B] time = 1.25489, size = 439, normalized size = 2.61

$$\frac{3\left(8a^4-24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(8a^4-24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(72a^2b^2\tan\ldots\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(72*a^
2*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*b^4*tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*tan(
1/2*d*x + 1/2*c)^6 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 33*b^4*tan(1/2*d*x
 + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 192*a*b^3*tan(1/2*d*x + 1/
2*c)^4 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 33*b^4*tan(1/2*d*x + 1/2*c)^3
- 288*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 256*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 72*
a^2*b^2*tan(1/2*d*x + 1/2*c) - 9*b^4*tan(1/2*d*x + 1/2*c) + 96*a^3*b - 64*a
*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

3.85 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=30

$$\frac{\tan^5(c+dx)(a\cot(c+dx)+b)^5}{5bd}$$

```
[Out] ((b + a*Cot[c + d*x])^5*Tan[c + d*x]^5)/(5*b*d)
```

Rubi [A] time = 0.0476237, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$\frac{\tan^5(c+dx)(a\cot(c+dx)+b)^5}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((b + a*Cot[c + d*x])^5*Tan[c + d*x]^5)/(5*b*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\mathrm{Subst}\left(\int \frac{(b+ax)^4}{x^6}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{(b+a\cot(c+dx))^5\tan^5(c+dx)}{5bd}$$

Mathematica [B] time = 0.308616, size = 73, normalized size = 2.43

$$\frac{\tan(c+dx)\left(10a^2b^2\tan^2(c+dx)+10a^3b\tan(c+dx)+5a^4+5ab^3\tan^3(c+dx)+b^4\tan^4(c+dx)\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (Tan[c + d*x]*(5*a^4 + 10*a^3*b*Tan[c + d*x] + 10*a^2*b^2*Tan[c + d*x]^2 +
5*a*b^3*Tan[c + d*x]^3 + b^4*Tan[c + d*x]^4))/(5*d)
```

Maple [B] time = 0.146, size = 96, normalized size = 3.2

$$\frac{1}{d}\left(a^4\tan(dx+c)+2\,\frac{a^3b}{(\cos(dx+c))^2}+2\,\frac{a^2b^2(\sin(dx+c))^3}{(\cos(dx+c))^3}+\frac{ab^3(\sin(dx+c))^4}{(\cos(dx+c))^4}+\frac{b^4(\sin(dx+c))^5}{5\,(\cos(dx+c))^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(a^4*tan(d*x+c)+2*a^3*b/cos(d*x+c)^2+2*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^
3+a*b^3*sin(d*x+c)^4/cos(d*x+c)^4+1/5*b^4*sin(d*x+c)^5/cos(d*x+c)^5)
```

Maxima [B] time = 1.23157, size = 139, normalized size = 4.63

$$\frac{b^4\tan(dx+c)^5+10\,a^2b^2\tan(dx+c)^3+5\,a^4\tan(dx+c)+\frac{5\left(2\,\sin(dx+c)^2-1\right)ab^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\frac{10\,a^3b}{\sin(dx+c)^2-1}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*tan(d*x + c)^5 + 10*a^2*b^2*tan(d*x + c)^3 + 5*a^4*tan(d*x + c) +
5*(2*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 10
*a^3*b/(sin(d*x + c)^2 - 1))/d
```

Fricas [B] time = 0.484743, size = 250, normalized size = 8.33

$$\frac{5\,ab^3\cos\left(dx+c\right)+10\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+\left(\left(5\,a^4-10\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+b^4+2\left(5\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2}{5\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/5*(5*a*b^3*cos(d*x + c) + 10*(a^3*b - a*b^3)*cos(d*x + c)^3 + ((5*a^4 - 1
0*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 2*(5*a^2*b^2 - b^4)*cos(d*x + c)^2)
*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2078, size = 99, normalized size = 3.3

$$\frac{b^4\tan\left(dx+c\right)^5+5\,ab^3\tan\left(dx+c\right)^4+10\,a^2b^2\tan\left(dx+c\right)^3+10\,a^3b\tan\left(dx+c\right)^2+5\,a^4\tan\left(dx+c\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/5*(b^4*tan(d*x + c)^5 + 5*a*b^3*tan(d*x + c)^4 + 10*a^2*b^2*tan(d*x + c)^
3 + 10*a^3*b*tan(d*x + c)^2 + 5*a^4*tan(d*x + c))/d
```

3.86 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{3a^2b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{4d}+\frac{4a^3b\sec^3(c+dx)}{3d}+\frac{a^4}{}$$

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^3*b*Sec[c + d*x]^3)/(3*d) - (4
*a*b^3*Sec[c + d*x]^3)/(3*d) + (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (a^4*Sec[c
+ d*x]*Tan[c + d*x])/(2*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) +
(b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (3*a^2*b^2*Sec[c + d*x]^3*Tan[c +
d*x])/(2*d) - (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]^3
*Tan[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.294117, antiderivative size = 258, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{3a^2b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{4d}+\frac{4a^3b\sec^3(c+dx)}{3d}+\frac{a^4}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^3*b*Sec[c + d*x]^3)/(3*d) - (4
*a*b^3*Sec[c + d*x]^3)/(3*d) + (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (a^4*Sec[c
+ d*x]*Tan[c + d*x])/(2*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) +
(b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (3*a^2*b^2*Sec[c + d*x]^3*Tan[c +
d*x])/(2*d) - (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]^3
*Tan[c + d*x]^3)/(6*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^3(c+dx)+4a^3b\sec^3(c+dx)\tan(c+dx)+6a^2b^2\sec^3(c+\right.$$
$$= a^4\int \sec^3(c+dx)\,dx + \left(4a^3b\right)\int \sec^3(c+dx)\tan(c+dx)\,dx + \left(6a^2\right.$$
$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d} + \frac{3a^2b^2\sec^3(c+dx)\tan(c+dx)}{2d} + \frac{b^4\sec}{}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} + \frac{4a^3b\sec^3(c+dx)}{3d} + \frac{a^4\sec(c+dx)\tan(c+}{2d}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{4a^3b\sec^3(c+}{3d}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d} + \frac{b^4\tanh^{-1}(\sin}{16a}$$

Mathematica [B] time = 6.25837, size = 1342, normalized size = 5.2

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a*b*(20*a^2 - 11*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(30*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 12*a^2*b^2 - b^4)*Cos[c + d*x]^4
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(16*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 12*a^2*b^2 + b^4)*Cos[c + d*x
]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(16*d*
(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d
*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*S
in[c + d*x])^4) + ((30*a^2*b^2 + 8*a*b^3 - 5*b^4)*Cos[c + d*x]^4*(a + b*Tan
[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^4) + ((120*a^4 + 160*a^3*b - 180*a^2*b^2 - 88*a*b^3 + 15
*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(480*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a*b^3*Cos[c + d*x]
^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a
 + b*Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) - (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a +
 b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + ((-30*a^2*b^2 + 8*a*b^3 + 5*b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-120*a^4 + 160*a^3*b + 180*a^2*b^2 -
```

```
88*a*b^3 - 15*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(480*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c
 + d*x]^4*(20*a^3*b*Sin[(c + d*x)/2] - 11*a*b^3*Sin[(c + d*x)/2])*(a + b*Ta
n[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x
] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(20*a^3*b*Sin[(c + d*x)/2] - 11*a*
b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-20*
a^3*b*Sin[(c + d*x)/2] + 11*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)
/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4) + (Cos[c + d*x]^4*(-20*a^3*b*Sin[(c + d*x)/2] + 11*a*b^3*Sin[(c +
d*x)/2])*(a + b*Tan[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A] time = 0.146, size = 394, normalized size = 1.5

$$\frac{a^4\sec(dx+c)\tan(dx+c)}{2d}+\frac{a^4\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{4a^3b}{3d(\cos(dx+c))^3}+\frac{3a^2b^2(\sin(dx+c))^3}{2d(\cos(dx+c))^4}+\frac{3a^2b^2(\cdots}{4d(c\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/2*a^4*sec(d*x+c)*tan(d*x+c)/d+1/2/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+4/3/d*a
^3*b/cos(d*x+c)^3+3/2/d*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/4/d*a^2*b^2*sin
(d*x+c)^3/cos(d*x+c)^2+3/4*a^2*b^2*sin(d*x+c)/d-3/4/d*a^2*b^2*ln(sec(d*x+c)
+tan(d*x+c))+4/5/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^5+4/15/d*a*b^3*sin(d*x+c)^
4/cos(d*x+c)^3-4/15/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)-4/15/d*cos(d*x+c)*sin(d
*x+c)^2*a*b^3-8/15*a*b^3*cos(d*x+c)/d+1/6/d*b^4*sin(d*x+c)^5/cos(d*x+c)^6+1
/24/d*b^4*sin(d*x+c)^5/cos(d*x+c)^4-1/48/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2-1/
48*b^4*sin(d*x+c)^3/d-1/16*b^4*sin(d*x+c)/d+1/16/d*b^4*ln(sec(d*x+c)+tan(d*
x+c))
```

Maxima [A] time = 1.19383, size = 339, normalized size = 1.31

$$-\frac{5b^4\left(\frac{2\left(3\sin(dx+c)^5+8\sin(dx+c)^3-3\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-180a^2b^2\left(\frac{2\left(\sin(dx+c)^3+\sin\cdots\right.}{\sin(dx+c)^4-2\sin\cdots}\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(5*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 180*a^2*b^2*(2*(sin(d*x + c)^3 + sin(d*
x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(
sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*a^3*b/cos(d*x + c)^3 + 128
*(5*cos(d*x + c)^2 - 3)*a*b^3/cos(d*x + c)^5)/d
```

Fricas [A] time = 0.544456, size = 458, normalized size = 1.78

$$\frac{15\left(8a^4-12a^2b^2+b^4\right)\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\left(8a^4-12a^2b^2+b^4\right)\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1) -
 15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 384*
a*b^3*cos(d*x + c) + 640*(a^3*b - a*b^3)*cos(d*x + c)^3 + 10*(3*(8*a^4 - 12
*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*b^4 + 2*(36*a^2*b^2 - 7*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28739, size = 724, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 1
5*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(120*a^
4*tan(1/2*d*x + 1/2*c)^11 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 15*b^4*ta
n(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^10 - 360*a^4*tan(1/2
*d*x + 1/2*c)^9 + 900*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 85*b^4*tan(1/2*d*x +
 1/2*c)^9 + 2880*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 1920*a*b^3*tan(1/2*d*x + 1/
2*c)^8 + 240*a^4*tan(1/2*d*x + 1/2*c)^7 - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c)
^7 + 570*b^4*tan(1/2*d*x + 1/2*c)^7 - 3200*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 1
280*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 240*a^4*tan(1/2*d*x + 1/2*c)^5 - 1080*a^
2*b^2*tan(1/2*d*x + 1/2*c)^5 + 570*b^4*tan(1/2*d*x + 1/2*c)^5 + 1920*a^3*b*
tan(1/2*d*x + 1/2*c)^4 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 + 900*a^2*b^2*tan(1
/2*d*x + 1/2*c)^3 + 85*b^4*tan(1/2*d*x + 1/2*c)^3 - 960*a^3*b*tan(1/2*d*x +
 1/2*c)^2 + 768*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 120*a^4*tan(1/2*d*x + 1/2*c)
 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/2*d*x + 1/2*c) + 320*a^3
*b - 128*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

3.87 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=143

$$\frac{b^2\left(6a^2+b^2\right)\tan^5(c+dx)}{5d}+\frac{ab\left(a^2+b^2\right)\tan^4(c+dx)}{d}+\frac{a^2\left(a^2+6b^2\right)\tan^3(c+dx)}{3d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^4\tan(c+d}{d}$$

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 6*b^2)*Tan[
c + d*x]^3)/(3*d) + (a*b*(a^2 + b^2)*Tan[c + d*x]^4)/d + (b^2*(6*a^2 + b^2)
*Tan[c + d*x]^5)/(5*d) + (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan[c + d*x]
^7)/(7*d)
```

Rubi [A] time = 0.123162, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{b^2\left(6a^2+b^2\right)\tan^5(c+dx)}{5d}+\frac{ab\left(a^2+b^2\right)\tan^4(c+dx)}{d}+\frac{a^2\left(a^2+6b^2\right)\tan^3(c+dx)}{3d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^4\tan(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 6*b^2)*Tan[
c + d*x]^3)/(3*d) + (a*b*(a^2 + b^2)*Tan[c + d*x]^4)/d + (b^2*(6*a^2 + b^2)
*Tan[c + d*x]^5)/(5*d) + (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan[c + d*x]
^7)/(7*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4(1+x^2)}{x^8}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^8}+\frac{4ab^3}{x^7}+\frac{6a^2b^2+b^4}{x^6}+\frac{4ab(a^2+b^2)}{x^5}+\frac{a^4+6a^2b^2}{x^4}+\frac{4a^3b}{x^3}+\frac{a^4}{x^2}\right.\right.}{d}$$
$$= \frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2\left(a^2+6b^2\right)\tan^3(c+dx)}{3d}+$$

Mathematica [A] time = 0.557767, size = 54, normalized size = 0.38

$$\frac{(a+b\tan(c+dx))^5\left(a^2-5ab\tan(c+dx)+15b^2\tan^2(c+dx)+21b^2\right)}{105b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a + b*Tan[c + d*x])^5*(a^2 + 21*b^2 - 5*a*b*Tan[c + d*x] + 15*b^2*Tan[c +
 d*x]^2))/(105*b^3*d)
```

Maple [A] time = 0.149, size = 171, normalized size = 1.2

$$\frac{1}{d}\left(-a^4\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)+\frac{a^3b}{(\cos(dx+c))^4}+6\,a^2b^2\left(1/5\,\frac{(\sin(dx+c))^3}{(\cos(dx+c))^5}+2/15\,\frac{(\sin(dx+c))^3}{(\cos(dx+c))^3}\right)+4\,ab\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*b/cos(d*x+c)^4+6*a^2*b^2*(
1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+4*a*b^3*(1/6*
sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+b^4*(1/7*sin(d*x+
```

```
c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5))
```

Maxima [A] time = 1.12172, size = 204, normalized size = 1.43

$$\frac{35\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{4}+42\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)a^{2}b^{2}+3\left(5\,\tan\left(dx+c\right)^{7}+7\,\tan\left(dx+c\right)^{5}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/105*(35*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 42*(3*tan(d*x + c)^5 + 5*
tan(d*x + c)^3)*a^2*b^2 + 3*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*b^4 - 35*
(3*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x
 + c)^2 - 1) + 105*a^3*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A] time = 0.511883, size = 333, normalized size = 2.33

$$\frac{70\,ab^{3}\cos\left(dx+c\right)+105\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{3}+\left(2\left(35\,a^{4}-42\,a^{2}b^{2}+3\,b^{4}\right)\cos\left(dx+c\right)^{6}+\left(35\,a^{4}-42\,a^{2}b^{2}+3\,b^{4}\right)\mathrm{co}\right.}{105\,d\cos\left(dx+c\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(70*a*b^3*cos(d*x + c) + 105*(a^3*b - a*b^3)*cos(d*x + c)^3 + (2*(35*
a^4 - 42*a^2*b^2 + 3*b^4)*cos(d*x + c)^6 + (35*a^4 - 42*a^2*b^2 + 3*b^4)*co
s(d*x + c)^4 + 15*b^4 + 6*(21*a^2*b^2 - 4*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17608, size = 194, normalized size = 1.36

$$\frac{15\,b^4\tan\left(dx+c\right)^7+70\,ab^3\tan\left(dx+c\right)^6+126\,a^2b^2\tan\left(dx+c\right)^5+21\,b^4\tan\left(dx+c\right)^5+105\,a^3b\tan\left(dx+c\right)^4+105\,a}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 70*a*b^3*tan(d*x + c)^6 + 126*a^2*b^2*tan(d*
x + c)^5 + 21*b^4*tan(d*x + c)^5 + 105*a^3*b*tan(d*x + c)^4 + 105*a*b^3*tan
(d*x + c)^4 + 35*a^4*tan(d*x + c)^3 + 210*a^2*b^2*tan(d*x + c)^3 + 210*a^3*
b*tan(d*x + c)^2 + 105*a^4*tan(d*x + c))/d
```

3.88 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=330

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{d}-\frac{a^2b^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{3a^2b^2\tan(c+dx)\sec(c+}{8d}$$

```
[Out] (3*a^4*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (4*a^3*b*Sec[c + d*x]^5)/(5*d)
 - (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (3*a^4
*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(
8*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d) - (a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b^4*Sec[c
+ d*x]^3*Tan[c + d*x])/(64*d) + (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/d - (
b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (b^4*Sec[c + d*x]^5*Tan[c + d*x]^
3)/(8*d)
```

Rubi [A] time = 0.34335, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{d}-\frac{a^2b^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{3a^2b^2\tan(c+dx)\sec(c+}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (3*a^4*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (4*a^3*b*Sec[c + d*x]^5)/(5*d)
 - (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (3*a^4
*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(
8*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d) - (a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b^4*Sec[c
+ d*x]^3*Tan[c + d*x])/(64*d) + (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/d - (
b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (b^4*Sec[c + d*x]^5*Tan[c + d*x]^
3)/(8*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^5(c+dx)+4a^3b\sec^5(c+dx)\tan(c+dx)+6a^2b^2\sec^5(c+\right.$$

$$= a^4\int \sec^5(c+dx)\,dx+\left(4a^3b\right)\int \sec^5(c+dx)\tan(c+dx)\,dx+\left(6a^2\right.$$

$$= \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{a^2b^2\sec^5(c+dx)\tan(c+dx)}{d}+\frac{b^4\sec}{}$$

$$= \frac{4a^3b\sec^5(c+dx)}{5d}+\frac{3a^4\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^4\sec^3(c+dx)\tan}{4d}$$

$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3b\sec^5(c+dx)}{5d}-\frac{4ab^3\sec^5(c+dx)}{5d}+$$

$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3b\sec^5(c}{5d}$$

$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b^4\tanh^{-1}(}{12}$$

Mathematica [B] time = 6.38568, size = 1732, normalized size = 5.25

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a*b*(42*a^2 - 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(140*d*(a*Cos
[c + d*x] + b*Sin[c + d*x])^4) - (3*(16*a^4 - 16*a^2*b^2 + b^4)*Cos[c + d*x
]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(128*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(16*a^4 - 16*a^2*b^2 + b^4)*Cos[
c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)
/(128*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*T
an[c + d*x])^4)/(128*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^8*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) + ((56*a^2*b^2 + 16*a*b^3 - 7*b^4)*Cos[c + d*x]^4*
(a + b*Tan[c + d*x])^4)/(448*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6*(a*C
os[c + d*x] + b*Sin[c + d*x])^4) + ((560*a^4 + 896*a^3*b - 256*a*b^3 - 35*b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((1680*a^4 + 1344*a^
3*b - 1680*a^2*b^2 - 544*a*b^3 + 105*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x
])^4)/(8960*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^4) + (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x
])^4)/(14*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin
[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(128*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*b
```

```
^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(14*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-56*
a^2*b^2 + 16*a*b^3 + 7*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(448*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
 + ((-560*a^4 + 896*a^3*b - 256*a*b^3 + 35*b^4)*Cos[c + d*x]^4*(a + b*Tan[c
 + d*x])^4)/(8960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^4) + ((-1680*a^4 + 1344*a^3*b + 1680*a^2*b^2 - 544*a*b^3
 - 105*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x
]^4*(42*a^3*b*Sin[(c + d*x)/2] - 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c +
d*x])^4)/(140*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b
*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(42*a^3*b*Sin[(c + d*x)/2] - 17*a*b^3*S
in[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(140*d*(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(7*a^3*b*
Sin[(c + d*x)/2] - 2*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(35*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
) + (Cos[c + d*x]^4*(-7*a^3*b*Sin[(c + d*x)/2] + 2*a*b^3*Sin[(c + d*x)/2])*
(a + b*Tan[c + d*x])^4)/(35*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Co
s[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-42*a^3*b*Sin[(c + d*x)/
2] + 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(140*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c +
 d*x]^4*(-42*a^3*b*Sin[(c + d*x)/2] + 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan
[c + d*x])^4)/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^4)
```

Maple [A] time = 0.159, size = 491, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/4*a^4*sec(d*x+c)^3*tan(d*x+c)/d+3/8*a^4*sec(d*x+c)*tan(d*x+c)/d+3/8/d*a^4
*ln(sec(d*x+c)+tan(d*x+c))+4/5/d*a^3*b/cos(d*x+c)^5+1/d*a^2*b^2*sin(d*x+c)^
3/cos(d*x+c)^6+3/4/d*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a^2*b^2*sin(d*
x+c)^3/cos(d*x+c)^2+3/8*a^2*b^2*sin(d*x+c)/d-3/8/d*a^2*b^2*ln(sec(d*x+c)+ta
n(d*x+c))+4/7/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^7+12/35/d*a*b^3*sin(d*x+c)^4/
cos(d*x+c)^5+4/35/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)^3-4/35/d*a*b^3*sin(d*x+c)
^4/cos(d*x+c)-4/35/d*cos(d*x+c)*sin(d*x+c)^2*a*b^3-8/35*a*b^3*cos(d*x+c)/d+
1/8/d*b^4*sin(d*x+c)^5/cos(d*x+c)^8+1/16/d*b^4*sin(d*x+c)^5/cos(d*x+c)^6+1/
64/d*b^4*sin(d*x+c)^5/cos(d*x+c)^4-1/128/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2-1/
```

```
128*b^4*sin(d*x+c)^3/d-3/128*b^4*sin(d*x+c)/d+3/128/d*b^4*ln(sec(d*x+c)+tan
(d*x+c))
```

Maxima [A] time = 1.17099, size = 435, normalized size = 1.32

$$-\frac{35\,b^4\left(\frac{2\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-560\,a^2b^2\left(\frac{2\,\cdots}{\mathrm{s}\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/8960*(35*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x + c)^
3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 560*a^2*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) + 560*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d
*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) - 7168*a^3*b/cos(d*x + c)^5 + 1024*(7*cos(d*x +
c)^2 - 5)*a*b^3/cos(d*x + c)^7)/d
```

Fricas [A] time = 0.584715, size = 533, normalized size = 1.62

$$\frac{105\left(16\,a^4-16\,a^2b^2+b^4\right)\cos\left(dx+c\right)^8\log\left(\sin\left(dx+c\right)+1\right)-105\left(16\,a^4-16\,a^2b^2+b^4\right)\cos\left(dx+c\right)^8\log\left(-\sin\left(dx+c\right)\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/8960*(105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(sin(d*x + c) + 1
) - 105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(-sin(d*x + c) + 1) +
 5120*a*b^3*cos(d*x + c) + 7168*(a^3*b - a*b^3)*cos(d*x + c)^3 + 70*(3*(16*
a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(16*a^4 - 16*a^2*b^2 + b^4)*cos(
d*x + c)^4 + 16*b^4 + 8*(16*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/
(d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28803, size = 953, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/4480*(105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(2
800*a^4*tan(1/2*d*x + 1/2*c)^15 + 1680*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 - 10
5*b^4*tan(1/2*d*x + 1/2*c)^15 - 17920*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 9520*
a^4*tan(1/2*d*x + 1/2*c)^13 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 805*b
^4*tan(1/2*d*x + 1/2*c)^13 + 53760*a^3*b*tan(1/2*d*x + 1/2*c)^12 - 35840*a*
b^3*tan(1/2*d*x + 1/2*c)^12 + 11760*a^4*tan(1/2*d*x + 1/2*c)^11 - 7280*a^2*
b^2*tan(1/2*d*x + 1/2*c)^11 + 11655*b^4*tan(1/2*d*x + 1/2*c)^11 - 89600*a^3
*b*tan(1/2*d*x + 1/2*c)^10 - 5040*a^4*tan(1/2*d*x + 1/2*c)^9 - 17360*a^2*b^
2*tan(1/2*d*x + 1/2*c)^9 + 23485*b^4*tan(1/2*d*x + 1/2*c)^9 + 125440*a^3*b*
tan(1/2*d*x + 1/2*c)^8 - 35840*a*b^3*tan(1/2*d*x + 1/2*c)^8 - 5040*a^4*tan(
1/2*d*x + 1/2*c)^7 - 17360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 23485*b^4*tan(1
/2*d*x + 1/2*c)^7 - 111104*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 57344*a*b^3*tan(1
/2*d*x + 1/2*c)^6 + 11760*a^4*tan(1/2*d*x + 1/2*c)^5 - 7280*a^2*b^2*tan(1/2
*d*x + 1/2*c)^5 + 11655*b^4*tan(1/2*d*x + 1/2*c)^5 + 46592*a^3*b*tan(1/2*d*
x + 1/2*c)^4 + 7168*a*b^3*tan(1/2*d*x + 1/2*c)^4 - 9520*a^4*tan(1/2*d*x + 1
/2*c)^3 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 805*b^4*tan(1/2*d*x + 1/2*
c)^3 - 10752*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 8192*a*b^3*tan(1/2*d*x + 1/2*c)
^2 + 2800*a^4*tan(1/2*d*x + 1/2*c) + 1680*a^2*b^2*tan(1/2*d*x + 1/2*c) - 10
5*b^4*tan(1/2*d*x + 1/2*c) + 3584*a^3*b - 1024*a*b^3)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^8)/d
```

3.89 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=201

$$\frac{2b^2\left(3a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{2ab\left(a^2+2b^2\right)\tan^6(c+dx)}{3d}+\frac{\left(12a^2b^2+a^4+b^4\right)\tan^5(c+dx)}{5d}+\frac{ab\left(2a^2+b^2\right)\tan^4(c+d}{d}$$

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (2*a^2*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (a*b*(2*a^2 + b^2)*Tan[c + d*x]^4)/d + ((a^4 + 12*a^2
*b^2 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + 2*b^2)*Tan[c + d*x]^6)/(3
*d) + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(
2*d) + (b^4*Tan[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.171375, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{2b^2\left(3a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{2ab\left(a^2+2b^2\right)\tan^6(c+dx)}{3d}+\frac{\left(12a^2b^2+a^4+b^4\right)\tan^5(c+dx)}{5d}+\frac{ab\left(2a^2+b^2\right)\tan^4(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (2*a^2*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (a*b*(2*a^2 + b^2)*Tan[c + d*x]^4)/d + ((a^4 + 12*a^2
*b^2 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + 2*b^2)*Tan[c + d*x]^6)/(3
*d) + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(
2*d) + (b^4*Tan[c + d*x]^9)/(9*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4\left(1+x^2\right)^2}{x^{10}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^{10}} + \frac{4ab^3}{x^9} + \frac{2\left(3a^2b^2+b^4\right)}{x^8} + \frac{4ab\left(a^2+2b^2\right)}{x^7} + \frac{a^4+12a^2b^2+b^4}{x^6} + \cdots\right.\right.}{d}$$

$$= \frac{a^4\tan(c+dx)}{d} + \frac{2a^3b\tan^2(c+dx)}{d} + \frac{2a^2\left(a^2+3b^2\right)\tan^3(c+dx)}{3d} \cdots$$

Mathematica [A] time = 0.847717, size = 115, normalized size = 0.57

$$\frac{\frac{2}{7}\left(3a^2+b^2\right)(a+b\tan(c+dx))^7 - \frac{2}{3}a\left(a^2+b^2\right)(a+b\tan(c+dx))^6 + \frac{1}{5}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^5 + \frac{1}{9}(a+b\tan(c \cdots}{b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^5)/5 - (2*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^6)/3 + (2*(3*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(a + b*Tan[c
+ d*x])^8)/2 + (a + b*Tan[c + d*x])^9/9)/(b^5*d)
```

Maple [A] time = 0.147, size = 236, normalized size = 1.2

$$\frac{1}{d}\left(-a^4\left(-\frac{8}{15} - \frac{(\sec(dx+c))^4}{5} - \frac{4(\sec(dx+c))^2}{15}\right)\tan(dx+c) + \frac{2a^3b}{3(\cos(dx+c))^6} + 6a^2b^2\left(1/7\frac{(\sin(dx+c))^3}{(\cos(dx+c))^7} + \frac{4 \cdots}{35 \cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2/3*a^3*b/c
os(d*x+c)^6+6*a^2*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(
d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+4*a*b^3*(1/8*sin(d*x+c)^4/cos(d*x
+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+b^4*(1
/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c
)^5/cos(d*x+c)^5))
```

Maxima [A] time = 1.24377, size = 261, normalized size = 1.3

$$\frac{42\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{4}+36\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] 1/630*(42*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 36
*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^2*b^2 + 2*(3
5*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*b^4 + 105*(4*sin(
d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4
 - 4*sin(d*x + c)^2 + 1) - 420*a^3*b/(sin(d*x + c)^2 - 1)^3)/d
```

Fricas [A] time = 0.540231, size = 400, normalized size = 1.99

$$\frac{315\,ab^{3}\cos\left(dx+c\right)+420\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{3}+2\left(8\left(21\,a^{4}-18\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{8}+4\left(21\,a^{4}-18\,a^{2}b^{2}+b^{4}\right)\right.}{630\,d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/630*(315*a*b^3*cos(d*x + c) + 420*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(8*(
21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^8 + 4*(21*a^4 - 18*a^2*b^2 + b^4)*c
os(d*x + c)^6 + 3*(21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + 35*b^4 + 10*
(27*a^2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23587, size = 289, normalized size = 1.44

$$\frac{70\,b^4\tan\left(dx+c\right)^9+315\,ab^3\tan\left(dx+c\right)^8+540\,a^2b^2\tan\left(dx+c\right)^7+180\,b^4\tan\left(dx+c\right)^7+420\,a^3b\tan\left(dx+c\right)^6+84}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/630*(70*b^4*tan(d*x + c)^9 + 315*a*b^3*tan(d*x + c)^8 + 540*a^2*b^2*tan(d
*x + c)^7 + 180*b^4*tan(d*x + c)^7 + 420*a^3*b*tan(d*x + c)^6 + 840*a*b^3*t
an(d*x + c)^6 + 126*a^4*tan(d*x + c)^5 + 1512*a^2*b^2*tan(d*x + c)^5 + 126*
b^4*tan(d*x + c)^5 + 1260*a^3*b*tan(d*x + c)^4 + 630*a*b^3*tan(d*x + c)^4 +
 420*a^4*tan(d*x + c)^3 + 1260*a^2*b^2*tan(d*x + c)^3 + 1260*a^3*b*tan(d*x
+ c)^2 + 630*a^4*tan(d*x + c))/d
```

3.90 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=408

$$-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{3a^2b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{8d}-\frac{5a^2b^2\tan(c+dx)\sec^3}{32d}$$

```
[Out] (5*a^4*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a^2*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (4*a^3*b*Sec[c + d*x]^7)/(7
*d) - (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (4*a*b^3*Sec[c + d*x]^9)/(9*d) + (5*
a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a^2*b^2*Sec[c + d*x]*Tan[c + d*
x])/(64*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(256*d) + (5*a^4*Sec[c + d*x
]^3*Tan[c + d*x])/(24*d) - (5*a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(32*d) +
 (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*
x])/(6*d) - (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]
^5*Tan[c + d*x])/(160*d) + (3*a^2*b^2*Sec[c + d*x]^7*Tan[c + d*x])/(4*d) -
(3*b^4*Sec[c + d*x]^7*Tan[c + d*x])/(80*d) + (b^4*Sec[c + d*x]^7*Tan[c + d*
x]^3)/(10*d)
```

Rubi [A] time = 0.397941, antiderivative size = 408, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14}

$$-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{3a^2b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{8d}-\frac{5a^2b^2\tan(c+dx)\sec^3}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (5*a^4*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a^2*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (4*a^3*b*Sec[c + d*x]^7)/(7
*d) - (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (4*a*b^3*Sec[c + d*x]^9)/(9*d) + (5*
a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a^2*b^2*Sec[c + d*x]*Tan[c + d*
x])/(64*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(256*d) + (5*a^4*Sec[c + d*x
]^3*Tan[c + d*x])/(24*d) - (5*a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(32*d) +
 (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*
x])/(6*d) - (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]
^5*Tan[c + d*x])/(160*d) + (3*a^2*b^2*Sec[c + d*x]^7*Tan[c + d*x])/(4*d) -
(3*b^4*Sec[c + d*x]^7*Tan[c + d*x])/(80*d) + (b^4*Sec[c + d*x]^7*Tan[c + d*
x]^3)/(10*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^7(c+dx)+4a^3b\sec^7(c+dx)\tan(c+dx)+6a^2b^2\sec^7(c+\right.$$

$$= a^4\int \sec^7(c+dx)\,dx+\left(4a^3b\right)\int \sec^7(c+dx)\tan(c+dx)\,dx+\left(6a^2\right.$$

$$= \frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{3a^2b^2\sec^7(c+dx)\tan(c+dx)}{4d}+\frac{b^4 s}{}$$

$$= \frac{4a^3b\sec^7(c+dx)}{7d}+\frac{5a^4\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^4\sec^5(c+dx)}{6d}$$

$$= \frac{4a^3b\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^7(c+dx)}{7d}+\frac{4ab^3\sec^9(c+dx)}{9d}+\frac{5a^4 se}{}$$

$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4a^3b\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^7(c+dx)}{7d}+$$

$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{4a^3b\sec^7}{7d}$$

$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{3b^4\tanh}{}$$

Mathematica [A] time = 1.32976, size = 242, normalized size = 0.59

$$\frac{10\sec^9(c+dx)\left(189\left(1604a^2b^2+592a^4+739b^4\right)\tan(c+dx)+32768ab\left(27a^2+b^2\right)\right)-80640\left(-60a^2b^2+80a^4+3b^4\right)\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-80640*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c + d*x]^10*(98304
0*a*b*(a^2 - b^2)*Cos[3*(c + d*x)] + 420*(1552*a^4 + 1908*a^2*b^2 - 505*b^4
)*Sin[3*(c + d*x)] + 7*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(628*Sin[5*(c + d*x)]
+ 145*Sin[7*(c + d*x)] + 15*Sin[9*(c + d*x)])) + 10*Sec[c + d*x]^9*(32768*a
*b*(27*a^2 + b^2) + 189*(592*a^4 + 1604*a^2*b^2 + 739*b^4)*Tan[c + d*x]))/(
20643840*d)
```

Maple [A] time = 0.143, size = 590, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 3/256/d*b^4*ln(sec(d*x+c)+tan(d*x+c))+5/16*a^4*sec(d*x+c)*tan(d*x+c)/d+5/24
*a^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*x+c)/d+1/128/d*b^
4*sin(d*x+c)^5/cos(d*x+c)^4-1/256*b^4*sin(d*x+c)^3/d+20/63/d*a*b^3*sin(d*x+
c)^4/cos(d*x+c)^7+1/16/d*b^4*sin(d*x+c)^5/cos(d*x+c)^8+4/7/d*a^3*b/cos(d*x+
c)^7+1/10/d*b^4*sin(d*x+c)^5/cos(d*x+c)^10+15/64/d*a^2*b^2*sin(d*x+c)^3/cos
(d*x+c)^2-3/256*b^4*sin(d*x+c)/d-4/63/d*a*b^3*sin(d*x+c)^4/cos(d*x+c)+5/8/d
*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^6-15/64/d*a^2*b^2*ln(sec(d*x+c)+tan(d*x+c)
)-1/256/d*b^4*sin(d*x+c)^5/cos(d*x+c)^2-8/63*a*b^3*cos(d*x+c)/d+15/64*a^2*b
^2*sin(d*x+c)/d+5/16/d*a^4*ln(sec(d*x+c)+tan(d*x+c))+15/32/d*a^2*b^2*sin(d*
x+c)^3/cos(d*x+c)^4+1/32/d*b^4*sin(d*x+c)^5/cos(d*x+c)^6+4/63/d*a*b^3*sin(d
*x+c)^4/cos(d*x+c)^3-4/63/d*cos(d*x+c)*sin(d*x+c)^2*a*b^3+4/21/d*a*b^3*sin(
d*x+c)^4/cos(d*x+c)^5+3/4/d*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^8+4/9/d*a*b^3*s
in(d*x+c)^4/cos(d*x+c)^9
```

Maxima [A] time = 1.23752, size = 516, normalized size = 1.26

$$-\frac{63\, b^4\left(\frac{2\left(15\, \sin(dx+c)^9-70\, \sin(dx+c)^7+128\, \sin(dx+c)^5+70\, \sin(dx+c)^3-15\, \sin(dx+c)\right)}{\sin(dx+c)^{10}-5\, \sin(dx+c)^8+10\, \sin(dx+c)^6-10\, \sin(dx+c)^4+5\, \sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] -1/161280*(63*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d*x +
 c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(d*x +
 c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 1260*a^2*b^2*(2*(15*sin
(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin
(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) -
 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 1680*a^4*(2*(15*sin
(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(
d*x + c) - 1)) - 92160*a^3*b/cos(d*x + c)^7 + 10240*(9*cos(d*x + c)^2 - 7)*
a*b^3/cos(d*x + c)^9)/d
```

Fricas [A] time = 0.627859, size = 626, normalized size = 1.53

$$\frac{315\left(80\,a^4-60\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^{10}\log\left(\sin\left(dx+c\right)+1\right)-315\left(80\,a^4-60\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^{10}\log\left(-\sin\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/161280*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(sin(d*x + c
) + 1) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(-sin(d*x + c
) + 1) + 71680*a*b^3*cos(d*x + c) + 92160*(a^3*b - a*b^3)*cos(d*x + c)^3 +
42*(15*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^8 + 10*(80*a^4 - 60*a^2*b
^2 + 3*b^4)*cos(d*x + c)^6 + 8*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^4
 + 384*b^4 + 48*(60*a^2*b^2 - 11*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(
d*x + c)^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31296, size = 1188, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/80640*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
 2*(55440*a^4*tan(1/2*d*x + 1/2*c)^19 + 18900*a^2*b^2*tan(1/2*d*x + 1/2*c)^
```

19 - 945*b^4*tan(1/2*d*x + 1/2*c)^19 - 322560*a^3*b*tan(1/2*d*x + 1/2*c)^18 - 213360*a^4*tan(1/2*d*x + 1/2*c)^17 + 462420*a^2*b^2*tan(1/2*d*x + 1/2*c)^17 + 9135*b^4*tan(1/2*d*x + 1/2*c)^17 + 967680*a^3*b*tan(1/2*d*x + 1/2*c)^16 - 645120*a*b^3*tan(1/2*d*x + 1/2*c)^16 + 450240*a^4*tan(1/2*d*x + 1/2*c)^15 + 146160*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 + 218484*b^4*tan(1/2*d*x + 1/2*c)^15 - 2580480*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 430080*a*b^3*tan(1/2*d*x + 1/2*c)^14 - 624960*a^4*tan(1/2*d*x + 1/2*c)^13 + 468720*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 653940*b^4*tan(1/2*d*x + 1/2*c)^13 + 5160960*a^3*b*tan(1/2*d*x + 1/2*c)^12 - 2150400*a*b^3*tan(1/2*d*x + 1/2*c)^12 + 332640*a^4*tan(1/2*d*x + 1/2*c)^11 - 1096200*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 1183770*b^4*tan(1/2*d*x + 1/2*c)^11 - 5806080*a^3*b*tan(1/2*d*x + 1/2*c)^10 + 1290240*a*b^3*tan(1/2*d*x + 1/2*c)^10 + 332640*a^4*tan(1/2*d*x + 1/2*c)^9 - 1096200*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 1183770*b^4*tan(1/2*d*x + 1/2*c)^9 + 4515840*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 624960*a^4*tan(1/2*d*x + 1/2*c)^7 + 468720*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 653940*b^4*tan(1/2*d*x + 1/2*c)^7 - 2949120*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 1658880*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 450240*a^4*tan(1/2*d*x + 1/2*c)^5 + 146160*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 218484*b^4*tan(1/2*d*x + 1/2*c)^5 + 1105920*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 184320*a*b^3*tan(1/2*d*x + 1/2*c)^4 - 213360*a^4*tan(1/2*d*x + 1/2*c)^3 + 462420*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 9135*b^4*tan(1/2*d*x + 1/2*c)^3 - 138240*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 102400*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 55440*a^4*tan(1/2*d*x + 1/2*c) + 18900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 945*b^4*tan(1/2*d*x + 1/2*c) + 46080*a^3*b - 10240*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^10)/d

3.91 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=254

$$\frac{b^2\left(2a^2+b^2\right)\tan^9(c+dx)}{3d}+\frac{ab\left(a^2+3b^2\right)\tan^8(c+dx)}{2d}+\frac{\left(18a^2b^2+a^4+3b^4\right)\tan^7(c+dx)}{7d}+\frac{2ab\left(a^2+b^2\right)\tan^6(c+dx)}{d}$$

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 2*b^2)*Tan[
c + d*x]^3)/d + (a*b*(3*a^2 + b^2)*Tan[c + d*x]^4)/d + ((3*a^4 + 18*a^2*b^2
 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + b^2)*Tan[c + d*x]^6)/d + ((a^
4 + 18*a^2*b^2 + 3*b^4)*Tan[c + d*x]^7)/(7*d) + (a*b*(a^2 + 3*b^2)*Tan[c +
d*x]^8)/(2*d) + (b^2*(2*a^2 + b^2)*Tan[c + d*x]^9)/(3*d) + (2*a*b^3*Tan[c +
 d*x]^10)/(5*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.219142, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{b^2\left(2a^2+b^2\right)\tan^9(c+dx)}{3d}+\frac{ab\left(a^2+3b^2\right)\tan^8(c+dx)}{2d}+\frac{\left(18a^2b^2+a^4+3b^4\right)\tan^7(c+dx)}{7d}+\frac{2ab\left(a^2+b^2\right)\tan^6(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 2*b^2)*Tan[
c + d*x]^3)/d + (a*b*(3*a^2 + b^2)*Tan[c + d*x]^4)/d + ((3*a^4 + 18*a^2*b^2
 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + b^2)*Tan[c + d*x]^6)/d + ((a^
4 + 18*a^2*b^2 + 3*b^4)*Tan[c + d*x]^7)/(7*d) + (a*b*(a^2 + 3*b^2)*Tan[c +
d*x]^8)/(2*d) + (b^2*(2*a^2 + b^2)*Tan[c + d*x]^9)/(3*d) + (2*a*b^3*Tan[c +
 d*x]^10)/(5*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4\left(1+x^2\right)^3}{x^{12}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^{12}} + \frac{4ab^3}{x^{11}} + \frac{3\left(2a^2b^2+b^4\right)}{x^{10}} + \frac{4ab\left(a^2+3b^2\right)}{x^9} + \frac{a^4+18a^2b^2+3b^4}{x^8} + \right.\right.}{}$$

$$= \frac{a^4\tan(c+dx)}{d} + \frac{2a^3b\tan^2(c+dx)}{d} + \frac{a^2\left(a^2+2b^2\right)\tan^3(c+dx)}{d}$$

Mathematica [A] time = 1.74173, size = 175, normalized size = 0.69

$$\frac{\frac{1}{3}\left(5a^2+b^2\right)(a+b\tan(c+dx))^9 - \frac{1}{2}a\left(5a^2+3b^2\right)(a+b\tan(c+dx))^8 + \frac{3}{7}\left(a^2+b^2\right)\left(5a^2+b^2\right)(a+b\tan(c+dx))^7 - a}{b^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^5)/5 - a*(a^2 + b^2)^2*(a + b*Tan[c +
d*x])^6 + (3*(a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(5*a^
2 + 3*b^2)*(a + b*Tan[c + d*x])^8)/2 + ((5*a^2 + b^2)*(a + b*Tan[c + d*x])^
9)/3 - (3*a*(a + b*Tan[c + d*x])^10)/5 + (a + b*Tan[c + d*x])^11/11)/(b^7*d
)
```

Maple [A] time = 0.142, size = 300, normalized size = 1.2

$$\frac{1}{d}\left(-a^4\left(-\frac{16}{35} - \frac{(\sec(dx+c))^6}{7} - \frac{6(\sec(dx+c))^4}{35} - \frac{8(\sec(dx+c))^2}{35}\right)\tan(dx+c) + \frac{a^3b}{2(\cos(dx+c))^8} + 6a^2b^2\left(1/9\,\frac{(\mathrm{s}}{(\mathrm{c}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-a^4*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+1/2*a^3*b/cos(d*x+c)^8+6*a^2*b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/2
1*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+
c)^3/cos(d*x+c)^3)+4*a*b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)
^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)
^4)+b^4*(1/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/2
31*sin(d*x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5))
```

Maxima [A] time = 1.23292, size = 315, normalized size = 1.24

$$\frac{66\left(5\tan(dx+c)^7+21\tan(dx+c)^5+35\tan(dx+c)^3+35\tan(dx+c)\right)a^4+44\left(35\tan(dx+c)^9+135\tan(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] 1/2310*(66*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*a^4 + 44*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x
 + c)^5 + 105*tan(d*x + c)^3)*a^2*b^2 + 2*(105*tan(d*x + c)^11 + 385*tan(d*
x + c)^9 + 495*tan(d*x + c)^7 + 231*tan(d*x + c)^5)*b^4 - 231*(5*sin(d*x +
c)^2 - 1)*a*b^3/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 1
0*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) + 1155*a^3*b/(sin(d*x + c)^2 - 1)^
4)/d
```

Fricas [A] time = 0.579139, size = 471, normalized size = 1.85

$$\frac{924\,ab^3\cos(dx+c)+1155\left(a^3b-ab^3\right)\cos(dx+c)^3+2\left(16\left(33\,a^4-22\,a^2b^2+b^4\right)\cos(dx+c)^{10}+8\left(33\,a^4-22\,a^2b^2+b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/2310*(924*a*b^3*cos(d*x + c) + 1155*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(1
6*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^10 + 8*(33*a^4 - 22*a^2*b^2 + b^
4)*cos(d*x + c)^8 + 6*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^6 + 5*(33*a^
4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^4 + 105*b^4 + 70*(11*a^2*b^2 - 2*b^4)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21455, size = 383, normalized size = 1.51

$$\frac{210\,b^4\tan\left(dx+c\right)^{11}+924\,ab^3\tan\left(dx+c\right)^{10}+1540\,a^2b^2\tan\left(dx+c\right)^9+770\,b^4\tan\left(dx+c\right)^9+1155\,a^3b\tan\left(dx+c\right)^8}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2310*(210*b^4*tan(d*x + c)^11 + 924*a*b^3*tan(d*x + c)^10 + 1540*a^2*b^2*
tan(d*x + c)^9 + 770*b^4*tan(d*x + c)^9 + 1155*a^3*b*tan(d*x + c)^8 + 3465*
a*b^3*tan(d*x + c)^8 + 330*a^4*tan(d*x + c)^7 + 5940*a^2*b^2*tan(d*x + c)^7
 + 990*b^4*tan(d*x + c)^7 + 4620*a^3*b*tan(d*x + c)^6 + 4620*a*b^3*tan(d*x
+ c)^6 + 1386*a^4*tan(d*x + c)^5 + 8316*a^2*b^2*tan(d*x + c)^5 + 462*b^4*ta
n(d*x + c)^5 + 6930*a^3*b*tan(d*x + c)^4 + 2310*a*b^3*tan(d*x + c)^4 + 2310
*a^4*tan(d*x + c)^3 + 4620*a^2*b^2*tan(d*x + c)^3 + 4620*a^3*b*tan(d*x + c)
^2 + 2310*a^4*tan(d*x + c))/d
```

3.92 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=515

$$\frac{a^2b^3\cos^{10}(c+dx)}{d}-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^3b^2\sin(c+dx)\cos^9(c+dx)}{d}+\frac{a^3b^2\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3b^2\sin(c+$$

```
[Out] (63*a^5*x)/256 + (35*a^3*b^2*x)/128 + (15*a*b^4*x)/256 - (5*a^2*b^3*Cos[c +
 d*x]^8)/(4*d) - (a^4*b*Cos[c + d*x]^10)/(2*d) + (a^2*b^3*Cos[c + d*x]^10)/
d + (63*a^5*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (35*a^3*b^2*Cos[c + d*x]*S
in[c + d*x])/(128*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (21*a
^5*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (35*a^3*b^2*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (21*a^5*C
os[c + d*x]^5*Sin[c + d*x])/(160*d) + (7*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(48*d) + (a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) + (9*a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(80*d) + (a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3
*a*b^4*Cos[c + d*x]^7*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x]^9*Sin[c + d*
x])/(10*d) - (a^3*b^2*Cos[c + d*x]^9*Sin[c + d*x])/d - (a*b^4*Cos[c + d*x]^
7*Sin[c + d*x]^3)/(2*d) + (b^5*Sin[c + d*x]^6)/(6*d) - (b^5*Sin[c + d*x]^8)
/(4*d) + (b^5*Sin[c + d*x]^10)/(10*d)
```

Rubi [A] time = 0.48451, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14, 2564, 266, 43}

$$\frac{a^2b^3\cos^{10}(c+dx)}{d}-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^3b^2\sin(c+dx)\cos^9(c+dx)}{d}+\frac{a^3b^2\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3b^2\sin(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] (63*a^5*x)/256 + (35*a^3*b^2*x)/128 + (15*a*b^4*x)/256 - (5*a^2*b^3*Cos[c +
 d*x]^8)/(4*d) - (a^4*b*Cos[c + d*x]^10)/(2*d) + (a^2*b^3*Cos[c + d*x]^10)/
d + (63*a^5*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (35*a^3*b^2*Cos[c + d*x]*S
in[c + d*x])/(128*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (21*a
^5*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (35*a^3*b^2*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (21*a^5*C
os[c + d*x]^5*Sin[c + d*x])/(160*d) + (7*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(48*d) + (a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) + (9*a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(80*d) + (a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3
*a*b^4*Cos[c + d*x]^7*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x]^9*Sin[c + d*
```

```
x])/(10*d) - (a^3*b^2*Cos[c + d*x]^9*Sin[c + d*x])/d - (a*b^4*Cos[c + d*x]^
7*Sin[c + d*x]^3)/(2*d) + (b^5*Sin[c + d*x]^6)/(6*d) - (b^5*Sin[c + d*x]^8)
/(4*d) + (b^5*Sin[c + d*x]^10)/(10*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^{10}(c+dx)+5a^4b\cos^9(c+dx)\sin(c+dx)+10a^3b^2\cos^8(\right.$$
$$= a^5\int \cos^{10}(c+dx)\,dx+\left(5a^4b\right)\int \cos^9(c+dx)\sin(c+dx)\,dx+\left(1\right.$$
$$= \frac{a^5\cos^9(c+dx)\sin(c+dx)}{10d}-\frac{a^3b^2\cos^9(c+dx)\sin(c+dx)}{d}-\frac{ab^4\,\mathrm{c}}{}$$
$$= -\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{9a^5\cos^7(c+dx)\sin(c+dx)}{80d}+\frac{a^3b^2\cos^7(c+}{}$$
$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c+dx)}{d}+\frac{2}{}$$
$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c+dx)}{d}+\frac{2}{}$$
$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c+dx)}{d}+\frac{6}{}$$
$$= \frac{63a^5x}{256}+\frac{35}{128}a^3b^2x+\frac{15}{256}ab^4x-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c}{2d}$$

Mathematica [A] time = 1.26564, size = 307, normalized size = 0.6

$$\frac{120a\left(70a^2b^2+63a^4+15b^4\right)(c+dx)+300a\left(14a^2b^2+21a^4+b^4\right)\sin(2(c+dx))+600a\left(-2a^2b^2+3a^4-b^4\right)\sin(4(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a*(63*a^4 + 70*a^2*b^2 + 15*b^4)*(c + d*x) - 300*b*(21*a^4 + 14*a^2*b^
2 + b^4)*Cos[2*(c + d*x)] - 1200*a^2*b*(3*a^2 + b^2)*Cos[4*(c + d*x)] + 50*
b*(-27*a^4 + 6*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 300*a^2*b*(a^2 - b^2)*Cos[
8*(c + d*x)] - 6*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[10*(c + d*x)] + 300*a*(21
*a^4 + 14*a^2*b^2 + b^4)*Sin[2*(c + d*x)] + 600*a*(3*a^4 - 2*a^2*b^2 - b^4)
*Sin[4*(c + d*x)] + 50*a*(9*a^4 - 26*a^2*b^2 - 3*b^4)*Sin[6*(c + d*x)] + 75
*a*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c + d*x)] + 6*a*(a^4 - 10*a^2*b^2 + 5*b^4
)*Sin[10*(c + d*x)])/(30720*d)
```

Maple [A] time = 0.303, size = 335, normalized size = 0.7

$$\frac{1}{d}\left(b^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^6}{10}-\frac{(\sin(dx+c))^2(\cos(dx+c))^6}{20}-\frac{(\cos(dx+c))^6}{60}\right)+5\,ab^4\left(-1/10\,(\sin(dx+c))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/
60*cos(d*x+c)^6)+5*a*b^4*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*c
os(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+3/256*d*x+3/256*c)+10*a^2*b^3*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d
*x+c)^8)+10*a^3*b^2*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*c
os(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256
*c)-1/2*a^4*b*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*
cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*x+
63/256*c))
```

Maxima [A] time = 1.23067, size = 392, normalized size = 0.76

$$-\frac{15360\,a^4b\cos\left(dx+c\right)^{10}-3\left(32\,\sin\left(2\,dx+2\,c\right)^5-640\,\sin\left(2\,dx+2\,c\right)^3+2520\,dx+2520\,c+25\,\sin\left(8\,dx+8\,c\right)+600}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/30720*(15360*a^4*b*cos(d*x + c)^10 - 3*(32*sin(2*d*x + 2*c)^5 - 640*sin(
2*d*x + 2*c)^3 + 2520*d*x + 2520*c + 25*sin(8*d*x + 8*c) + 600*sin(4*d*x +
4*c) + 2560*sin(2*d*x + 2*c))*a^5 + 10*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d
*x + 2*c)^3 - 840*d*x - 840*c + 45*sin(8*d*x + 8*c) + 120*sin(4*d*x + 4*c))
*a^3*b^2 + 7680*(4*sin(d*x + c)^10 - 15*sin(d*x + c)^8 + 20*sin(d*x + c)^6
- 10*sin(d*x + c)^4)*a^2*b^3 - 15*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c
+ 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a*b^4 - 512*(6*sin(d*x + c)^10
- 15*sin(d*x + c)^8 + 10*sin(d*x + c)^6)*b^5)/d
```

Fricas [A] time = 0.605827, size = 595, normalized size = 1.16

$$-\frac{640\,b^5\cos\left(dx+c\right)^6+384\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^{10}+960\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^8-15\left(63\,a^5+70\,a^3b^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/3840*(640*b^5*cos(d*x + c)^6 + 384*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x
+ c)^10 + 960*(5*a^2*b^3 - b^5)*cos(d*x + c)^8 - 15*(63*a^5 + 70*a^3*b^2 +
15*a*b^4)*d*x - (384*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^9 + 48*(9*a^
5 + 10*a^3*b^2 - 55*a*b^4)*cos(d*x + c)^7 + 8*(63*a^5 + 70*a^3*b^2 + 15*a*b
^4)*cos(d*x + c)^5 + 10*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^3 + 1
5*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 48.4455, size = 979, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((63*a**5*x*sin(c + d*x)**10/256 + 315*a**5*x*sin(c + d*x)**8*cos(
c + d*x)**2/256 + 315*a**5*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 315*a**5
*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 315*a**5*x*sin(c + d*x)**2*cos(c +
 d*x)**8/256 + 63*a**5*x*cos(c + d*x)**10/256 + 63*a**5*sin(c + d*x)**9*cos
(c + d*x)/(256*d) + 147*a**5*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + 21*a
**5*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 237*a**5*sin(c + d*x)**3*cos(c
 + d*x)**7/(128*d) + 193*a**5*sin(c + d*x)*cos(c + d*x)**9/(256*d) - a**4*b
*cos(c + d*x)**10/(2*d) + 35*a**3*b**2*x*sin(c + d*x)**10/128 + 175*a**3*b*
*2*x*sin(c + d*x)**8*cos(c + d*x)**2/128 + 175*a**3*b**2*x*sin(c + d*x)**6*
cos(c + d*x)**4/64 + 175*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**6/64 + 1
75*a**3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**8/128 + 35*a**3*b**2*x*cos(c +
 d*x)**10/128 + 35*a**3*b**2*sin(c + d*x)**9*cos(c + d*x)/(128*d) + 245*a**
3*b**2*sin(c + d*x)**7*cos(c + d*x)**3/(192*d) + 7*a**3*b**2*sin(c + d*x)**
5*cos(c + d*x)**5/(3*d) + 395*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**7/(19
2*d) - 35*a**3*b**2*sin(c + d*x)*cos(c + d*x)**9/(128*d) - 5*a**2*b**3*sin(
c + d*x)**2*cos(c + d*x)**8/(4*d) - a**2*b**3*cos(c + d*x)**10/(4*d) + 15*a
*b**4*x*sin(c + d*x)**10/256 + 75*a*b**4*x*sin(c + d*x)**8*cos(c + d*x)**2/
256 + 75*a*b**4*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 75*a*b**4*x*sin(c +
 d*x)**4*cos(c + d*x)**6/128 + 75*a*b**4*x*sin(c + d*x)**2*cos(c + d*x)**8/
256 + 15*a*b**4*x*cos(c + d*x)**10/256 + 15*a*b**4*sin(c + d*x)**9*cos(c +
d*x)/(256*d) + 35*a*b**4*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + a*b**4*s
in(c + d*x)**5*cos(c + d*x)**5/(2*d) - 35*a*b**4*sin(c + d*x)**3*cos(c + d*
x)**7/(128*d) - 15*a*b**4*sin(c + d*x)*cos(c + d*x)**9/(256*d) - b**5*sin(c
 + d*x)**4*cos(c + d*x)**6/(6*d) - b**5*sin(c + d*x)**2*cos(c + d*x)**8/(12
*d) - b**5*cos(c + d*x)**10/(60*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*
cos(c)**5, True))
```

Giac [A] time = 1.37491, size = 462, normalized size = 0.9

$$\frac{1}{256}\left(63\,a^5+70\,a^3b^2+15\,ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(10\,dx+10\,c\right)}{5120\,d}-\frac{5\left(a^4b-a^2b^3\right)\cos\left(8\,dx+8\,c\right)}{512\,d}-\frac{5\left(27\,a^4b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/256*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*x - 1/5120*(5*a^4*b - 10*a^2*b^3 + b
^5)*cos(10*d*x + 10*c)/d - 5/512*(a^4*b - a^2*b^3)*cos(8*d*x + 8*c)/d - 5/3
072*(27*a^4*b - 6*a^2*b^3 - b^5)*cos(6*d*x + 6*c)/d - 5/128*(3*a^4*b + a^2*
b^3)*cos(4*d*x + 4*c)/d - 5/512*(21*a^4*b + 14*a^2*b^3 + b^5)*cos(2*d*x + 2
*c)/d + 1/5120*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(10*d*x + 10*c)/d + 5/2048*(
a^5 - 6*a^3*b^2 + a*b^4)*sin(8*d*x + 8*c)/d + 5/3072*(9*a^5 - 26*a^3*b^2 -
3*a*b^4)*sin(6*d*x + 6*c)/d + 5/256*(3*a^5 - 2*a^3*b^2 - a*b^4)*sin(4*d*x +
 4*c)/d + 5/512*(21*a^5 + 14*a^3*b^2 + a*b^4)*sin(2*d*x + 2*c)/d
```

3.93 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=337

$$-\frac{10a^3b^2\sin^9(c+dx)}{9d}+\frac{30a^3b^2\sin^7(c+dx)}{7d}-\frac{6a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^9(c+dx)}{9d}-\frac{10a^2b}{}$$

```
[Out] -(b^5*Cos[c + d*x]^5)/(5*d) - (10*a^2*b^3*Cos[c + d*x]^7)/(7*d) + (2*b^5*Co
s[c + d*x]^7)/(7*d) - (5*a^4*b*Cos[c + d*x]^9)/(9*d) + (10*a^2*b^3*Cos[c +
d*x]^9)/(9*d) - (b^5*Cos[c + d*x]^9)/(9*d) + (a^5*Sin[c + d*x])/d - (4*a^5*
Sin[c + d*x]^3)/(3*d) + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (6*a^5*Sin[c +
d*x]^5)/(5*d) - (6*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (
4*a^5*Sin[c + d*x]^7)/(7*d) + (30*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (10*a*b^4
*Sin[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x]^9)/(9*d) - (10*a^3*b^2*Sin[c + d
*x]^9)/(9*d) + (5*a*b^4*Sin[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.299991, antiderivative size = 337, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{10a^3b^2\sin^9(c+dx)}{9d}+\frac{30a^3b^2\sin^7(c+dx)}{7d}-\frac{6a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^9(c+dx)}{9d}-\frac{10a^2b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] -(b^5*Cos[c + d*x]^5)/(5*d) - (10*a^2*b^3*Cos[c + d*x]^7)/(7*d) + (2*b^5*Co
s[c + d*x]^7)/(7*d) - (5*a^4*b*Cos[c + d*x]^9)/(9*d) + (10*a^2*b^3*Cos[c +
d*x]^9)/(9*d) - (b^5*Cos[c + d*x]^9)/(9*d) + (a^5*Sin[c + d*x])/d - (4*a^5*
Sin[c + d*x]^3)/(3*d) + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (6*a^5*Sin[c +
d*x]^5)/(5*d) - (6*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (
4*a^5*Sin[c + d*x]^7)/(7*d) + (30*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (10*a*b^4
*Sin[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x]^9)/(9*d) - (10*a^3*b^2*Sin[c + d
*x]^9)/(9*d) + (5*a*b^4*Sin[c + d*x]^9)/(9*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^9(c+dx)+5a^4b\cos^8(c+dx)\sin(c+dx)+10a^3b^2\cos^7(c\right.$$
$$= a^5\int \cos^9(c+dx)\,dx+\left(5a^4b\right)\int \cos^8(c+dx)\sin(c+dx)\,dx+\left(10\right.$$
$$= -\frac{a^5\,\mathrm{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{(5}{}$$
$$= -\frac{5a^4b\cos^9(c+dx)}{9d}+\frac{a^5\sin(c+dx)}{d}-\frac{4a^5\sin^3(c+dx)}{3d}+\frac{6a^5\sin^5}{5}$$
$$= -\frac{b^5\cos^5(c+dx)}{5d}-\frac{10a^2b^3\cos^7(c+dx)}{7d}+\frac{2b^5\cos^7(c+dx)}{7d}-\frac{5a^4b}{}$$

Mathematica [A] time = 1.02615, size = 278, normalized size = 0.82

$$\frac{630a\left(70a^2b^2+63a^4+15b^4\right)\sin(c+dx)+420a\left(21a^4-5b^4\right)\sin(3(c+dx))+252a\left(-20a^2b^2+9a^4-5b^4\right)\sin(5(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-630*b*(35*a^4 + 30*a^2*b^2 + 3*b^4)*Cos[c + d*x] - 420*b*(35*a^4 + 20*a^2
*b^2 + b^4)*Cos[3*(c + d*x)] + 252*b*(-25*a^4 + b^4)*Cos[5*(c + d*x)] + 45*
b*(-35*a^4 + 30*a^2*b^2 + b^4)*Cos[7*(c + d*x)] - 35*b*(5*a^4 - 10*a^2*b^2
+ b^4)*Cos[9*(c + d*x)] + 630*a*(63*a^4 + 70*a^2*b^2 + 15*b^4)*Sin[c + d*x]
 + 420*a*(21*a^4 - 5*b^4)*Sin[3*(c + d*x)] + 252*a*(9*a^4 - 20*a^2*b^2 - 5*
b^4)*Sin[5*(c + d*x)] + 45*a*(9*a^4 - 50*a^2*b^2 + 5*b^4)*Sin[7*(c + d*x)]
+ 35*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[9*(c + d*x)])/(80640*d)
```

Maple [A] time = 0.213, size = 291, normalized size = 0.9

$$\frac{1}{d}\left(b^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^5}{9}-\frac{4\,(\sin(dx+c))^2(\cos(dx+c))^5}{63}-\frac{8\,(\cos(dx+c))^5}{315}\right)+5\,ab^4\left(-1/9\,(\sin(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/3
15*cos(d*x+c)^5)+5*a*b^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*co
s(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+10*a^2*b^3
*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+10*a^3*b^2*(-1/9*sin(d*
x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2
)*sin(d*x+c))-5/9*a^4*b*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d
*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.20285, size = 302, normalized size = 0.9

$$-\frac{175\,a^4b\cos(dx+c)^9-\left(35\,\sin(dx+c)^9-180\,\sin(dx+c)^7+378\,\sin(dx+c)^5-420\,\sin(dx+c)^3+315\,\sin(dx+c)\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/315*(175*a^4*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
+ 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5 + 10*(35*
sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)
^3)*a^3*b^2 - 50*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2*b^3 - 5*(35*sin(
d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a*b^4 + (35*cos(d*x + c
)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^5)/d
```

Fricas [A] time = 0.561943, size = 504, normalized size = 1.5

$$-\frac{63\,b^5\cos(dx+c)^5+35\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(dx+c)^9+90\left(5\,a^2b^3-b^5\right)\cos(dx+c)^7-\left(35\left(a^5-10\,a^3b^2+5\,ab^4\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(63*b^5*cos(d*x + c)^5 + 35*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^9 + 90*(5*a^2*b^3 - b^5)*cos(d*x + c)^7 - (35*(a^5 - 10*a^3*b^2 + 5*a*b^4
)*cos(d*x + c)^8 + 10*(4*a^5 + 5*a^3*b^2 - 25*a*b^4)*cos(d*x + c)^6 + 128*a
^5 + 160*a^3*b^2 + 40*a*b^4 + 3*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c
)^4 + 4*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 32.2598, size = 440, normalized size = 1.31

$$\begin{cases} \frac{128a^5\sin^9(c+dx)}{315d} + \frac{64a^5\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{16a^5\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{8a^5\sin^3(c+dx)\cos^6(c+dx)}{3d} + \frac{a^5\sin(c+dx)\cos^8(c+dx)}{d} - \frac{5a^4b\cos \\ x\,(a\cos(c) + b\sin(c))^5\cos^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((128*a**5*sin(c + d*x)**9/(315*d) + 64*a**5*sin(c + d*x)**7*cos(c
 + d*x)**2/(35*d) + 16*a**5*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**5*
sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**8/d
 - 5*a**4*b*cos(c + d*x)**9/(9*d) + 32*a**3*b**2*sin(c + d*x)**9/(63*d) + 1
6*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(7*d) + 4*a**3*b**2*sin(c + d*x
)**5*cos(c + d*x)**4/d + 10*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d)
 - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 20*a**2*b**3*cos(c
+ d*x)**9/(63*d) + 8*a*b**4*sin(c + d*x)**9/(63*d) + 4*a*b**4*sin(c + d*x)*
*7*cos(c + d*x)**2/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**4/d - b**5*
sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b**5*sin(c + d*x)**2*cos(c + d*x)
**7/(35*d) - 8*b**5*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*cos(c) + b*si
n(c))**5*cos(c)**4, True))
```

Giac [A] time = 1.43427, size = 423, normalized size = 1.26

$$-\frac{\left(5\,a^4b - 10\,a^2b^3 + b^5\right)\cos(9\,dx + 9\,c)}{2304\,d} - \frac{\left(35\,a^4b - 30\,a^2b^3 - b^5\right)\cos(7\,dx + 7\,c)}{1792\,d} - \frac{\left(25\,a^4b - b^5\right)\cos(5\,dx + 5\,c)}{320\,d} - \frac{(35$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/2304*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(9*d*x + 9*c)/d - 1/1792*(35*a^4*b
- 30*a^2*b^3 - b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b - b^5)*cos(5*d*x +
 5*c)/d - 1/192*(35*a^4*b + 20*a^2*b^3 + b^5)*cos(3*d*x + 3*c)/d - 1/128*(3
5*a^4*b + 30*a^2*b^3 + 3*b^5)*cos(d*x + c)/d + 1/2304*(a^5 - 10*a^3*b^2 + 5
*a*b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^5 - 50*a^3*b^2 + 5*a*b^4)*sin(7*d*
x + 7*c)/d + 1/320*(9*a^5 - 20*a^3*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d + 1/19
2*(21*a^5 - 5*a*b^4)*sin(3*d*x + 3*c)/d + 1/128*(63*a^5 + 70*a^3*b^2 + 15*a
*b^4)*sin(d*x + c)/d
```

3.94 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=426

$$\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^3b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{5a^3b^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{25a^3b^2\sin}{}$$

```
[Out] (35*a^5*x)/128 + (25*a^3*b^2*x)/64 + (15*a*b^4*x)/128 - (5*a^2*b^3*Cos[c +
d*x]^6)/(3*d) - (5*a^4*b*Cos[c + d*x]^8)/(8*d) + (5*a^2*b^3*Cos[c + d*x]^8)
/(4*d) + (35*a^5*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (25*a^3*b^2*Cos[c + d
*x]*Sin[c + d*x])/(64*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (
35*a^5*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*b^2*Cos[c + d*x]^3*Si
n[c + d*x])/(96*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^5*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (5*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(24*d) - (5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(8*d) - (5*a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (
5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d) + (b^5*Sin[c + d*x]^6)/(6*d) -
 (b^5*Sin[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.412553, antiderivative size = 426, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 2635, 8, 2565, 30, 2568, 14, 2564}

$$\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^3b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{5a^3b^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{25a^3b^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] (35*a^5*x)/128 + (25*a^3*b^2*x)/64 + (15*a*b^4*x)/128 - (5*a^2*b^3*Cos[c +
d*x]^6)/(3*d) - (5*a^4*b*Cos[c + d*x]^8)/(8*d) + (5*a^2*b^3*Cos[c + d*x]^8)
/(4*d) + (35*a^5*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (25*a^3*b^2*Cos[c + d
*x]*Sin[c + d*x])/(64*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (
35*a^5*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*b^2*Cos[c + d*x]^3*Si
n[c + d*x])/(96*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^5*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (5*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(24*d) - (5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(8*d) - (5*a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (
5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d) + (b^5*Sin[c + d*x]^6)/(6*d) -
 (b^5*Sin[c + d*x]^8)/(8*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^8(c+dx)+5a^4b\cos^7(c+dx)\sin(c+dx)+10a^3b^2\cos^6(c\right.$$

$$= a^5\int \cos^8(c+dx)\,dx+\left(5a^4b\right)\int \cos^7(c+dx)\sin(c+dx)\,dx+\left(10a\right.$$

$$= \frac{a^5\cos^7(c+dx)\sin(c+dx)}{8d}-\frac{5a^3b^2\cos^7(c+dx)\sin(c+dx)}{4d}-\frac{5ab^4}{}$$

$$= -\frac{5a^4b\cos^8(c+dx)}{8d}+\frac{7a^5\cos^5(c+dx)\sin(c+dx)}{48d}+\frac{5a^3b^2\cos^5(c+}{2}$$

$$= -\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^4b\cos^8(c+dx)}{8d}+\frac{5a^2b^3\cos^8(c+dx)}{4d}+\frac{35}{}$$

$$= -\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^4b\cos^8(c+dx)}{8d}+\frac{5a^2b^3\cos^8(c+dx)}{4d}+\frac{35}{}$$

$$= \frac{35a^5x}{128}+\frac{25}{64}a^3b^2x+\frac{15}{128}ab^4x-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^4b\cos^8(c+}{8d}$$

Mathematica [C] time = 0.877272, size = 259, normalized size = 0.61

$$\frac{120a(a-ib)(a+ib)\left(7a^2+3b^2\right)(c+dx)+96a^3\left(7a^2+5b^2\right)\sin(2(c+dx))+32a^3\left(a^2-5b^2\right)\sin(6(c+dx))+24a\left(-10a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a*(a - I*b)*(a + I*b)*(7*a^2 + 3*b^2)*(c + d*x) - 24*b*(35*a^4 + 30*a^
2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + 12*b*(-35*a^4 - 10*a^2*b^2 + b^4)*Cos[4*(
c + d*x)] + 8*b*(-15*a^4 + 10*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 3*b*(5*a^4
- 10*a^2*b^2 + b^4)*Cos[8*(c + d*x)] + 96*a^3*(7*a^2 + 5*b^2)*Sin[2*(c + d*
x)] + 24*a*(7*a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + 32*a^3*(a^2 - 5*
b^2)*Sin[6*(c + d*x)] + 3*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[8*(c + d*x)])/(3
072*d)
```

Maple [A] time = 0.197, size = 305, normalized size = 0.7

$$\frac{1}{d}\left(b^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^4}{8}-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{12}-\frac{(\cos(dx+c))^4}{24}\right)+5\,ab^4\left(-1/8\,(\sin(dx+c))^3\,(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/2
4*cos(d*x+c)^4)+5*a*b^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos
(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+
10*a^2*b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+10*a^3*b^2*(-
1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-5/8*a^4*b*cos(d*x+c)^8+a^5*(1/8*(cos(d*
x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/
128*d*x+35/128*c))
```

Maxima [A] time = 1.25331, size = 308, normalized size = 0.72

$$-\frac{1920\,a^4b\cos(dx+c)^8+\left(128\,\sin(2\,dx+2\,c)^3-840\,dx-840\,c-3\,\sin(8\,dx+8\,c)-168\,\sin(4\,dx+4\,c)-768\,\sin(2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/3072*(1920*a^4*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^5
 - 10*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*si
n(4*d*x + 4*c))*a^3*b^2 - 1280*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin
(d*x + c)^4)*a^2*b^3 - 15*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x +
 4*c))*a*b^4 + 128*(3*sin(d*x + c)^8 - 4*sin(d*x + c)^6)*b^5)/d
```

Fricas [A] time = 0.56612, size = 509, normalized size = 1.19

$$-\frac{96\,b^5\cos(dx+c)^4+48\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(dx+c)^8+128\left(5\,a^2b^3-b^5\right)\cos(dx+c)^6-15\left(7\,a^5+10\,a^3b^2+3\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/384*(96*b^5*cos(d*x + c)^4 + 48*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^8 + 128*(5*a^2*b^3 - b^5)*cos(d*x + c)^6 - 15*(7*a^5 + 10*a^3*b^2 + 3*a*b
^4)*d*x - (48*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 8*(7*a^5 + 10*a
^3*b^2 - 45*a*b^4)*cos(d*x + c)^5 + 10*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*cos(d
*x + c)^3 + 15*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A] time = 27.4128, size = 821, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((35*a**5*x*sin(c + d*x)**8/128 + 35*a**5*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**5*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**5*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**5*x*cos(c + d*x)**8/128 + 35*a**5*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**5*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**5*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**5*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - 5*a**4*b*cos(c + d*x)**8/(8*d) + 25*a
**3*b**2*x*sin(c + d*x)**8/64 + 25*a**3*b**2*x*sin(c + d*x)**6*cos(c + d*x)
**2/16 + 75*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 25*a**3*b**2*x
*sin(c + d*x)**2*cos(c + d*x)**6/16 + 25*a**3*b**2*x*cos(c + d*x)**8/64 + 2
5*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 275*a**3*b**2*sin(c + d*x
)**5*cos(c + d*x)**3/(192*d) + 365*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**
5/(192*d) - 25*a**3*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) - 5*a**2*b**3*
sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 5*a**2*b**3*cos(c + d*x)**8/(12*d)
+ 15*a*b**4*x*sin(c + d*x)**8/128 + 15*a*b**4*x*sin(c + d*x)**6*cos(c + d*x
)**2/32 + 45*a*b**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**4*x*sin(
c + d*x)**2*cos(c + d*x)**6/32 + 15*a*b**4*x*cos(c + d*x)**8/128 + 15*a*b**
4*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a*b**4*sin(c + d*x)**5*cos(c +
d*x)**3/(128*d) - 55*a*b**4*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 15*a*
b**4*sin(c + d*x)*cos(c + d*x)**7/(128*d) - b**5*sin(c + d*x)**4*cos(c + d*
x)**4/(4*d) - b**5*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - b**5*cos(c + d*x
)**8/(24*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos(c)**3, True))
```

Giac [A] time = 1.41208, size = 375, normalized size = 0.88

$$\frac{5}{128}\left(7\,a^5+10\,a^3b^2+3\,ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{\left(15\,a^4b-10\,a^2b^3-b^5\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{(35\,a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 5/128*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*x - 1/1024*(5*a^4*b - 10*a^2*b^3 + b^5
)*cos(8*d*x + 8*c)/d - 1/384*(15*a^4*b - 10*a^2*b^3 - b^5)*cos(6*d*x + 6*c)
/d - 1/256*(35*a^4*b + 10*a^2*b^3 - b^5)*cos(4*d*x + 4*c)/d - 1/128*(35*a^4
*b + 30*a^2*b^3 + 3*b^5)*cos(2*d*x + 2*c)/d + 1/1024*(a^5 - 10*a^3*b^2 + 5*
a*b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^5 - 5*a^3*b^2)*sin(6*d*x + 6*c)/d + 1/1
28*(7*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/d + 1/32*(7*a^5 + 5*a^3*
b^2)*sin(2*d*x + 2*c)/d
```

3.95 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=275

$$\frac{10a^3b^2\sin^7(c+dx)}{7d}-\frac{4a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^7(c+dx)}{7d}-\frac{2a^2b^3\cos^5(c+dx)}{d}-\frac{5a^4b\cos^7}{7d}$$

```
[Out] -(b^5*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^3*Cos[c + d*x]^5)/d + (2*b^5*Cos[c +
 d*x]^5)/(5*d) - (5*a^4*b*Cos[c + d*x]^7)/(7*d) + (10*a^2*b^3*Cos[c + d*x]^
7)/(7*d) - (b^5*Cos[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x])/d - (a^5*Sin[c +
 d*x]^3)/d + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (3*a^5*Sin[c + d*x]^5)/(5*
d) - (4*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (a^5*Sin[c +
 d*x]^7)/(7*d) + (10*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (5*a*b^4*Sin[c + d*x]^
7)/(7*d)
```

Rubi [A] time = 0.281352, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3090, 2633, 2565, 30, 2564, 270, 14}

$$\frac{10a^3b^2\sin^7(c+dx)}{7d}-\frac{4a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^7(c+dx)}{7d}-\frac{2a^2b^3\cos^5(c+dx)}{d}-\frac{5a^4b\cos^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] -(b^5*Cos[c + d*x]^3)/(3*d) - (2*a^2*b^3*Cos[c + d*x]^5)/d + (2*b^5*Cos[c +
 d*x]^5)/(5*d) - (5*a^4*b*Cos[c + d*x]^7)/(7*d) + (10*a^2*b^3*Cos[c + d*x]^
7)/(7*d) - (b^5*Cos[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x])/d - (a^5*Sin[c +
 d*x]^3)/d + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (3*a^5*Sin[c + d*x]^5)/(5*
d) - (4*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (a^5*Sin[c +
 d*x]^7)/(7*d) + (10*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (5*a*b^4*Sin[c + d*x]^
7)/(7*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^7(c+dx)+5a^4b\cos^6(c+dx)\sin(c+dx)+10a^3b^2\cos^5(c\right.$$

$$= a^5\int\cos^7(c+dx)\,dx+\left(5a^4b\right)\int\cos^6(c+dx)\sin(c+dx)\,dx+\left(10a^3\right.$$

$$= -\frac{a^5\,\mathrm{Subst}\left(\int\left(1-3x^2+3x^4-x^6\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(5a^4b\right)\mathrm{Su}}{}$$

$$= -\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{a^5\sin(c+dx)}{d}-\frac{a^5\sin^3(c+dx)}{d}+\frac{3a^5\sin^5(c}{5d}$$

$$= -\frac{b^5\cos^3(c+dx)}{3d}-\frac{2a^2b^3\cos^5(c+dx)}{d}+\frac{2b^5\cos^5(c+dx)}{5d}-\frac{5a^4b\,\mathrm{cc}}{}$$

Mathematica [A] time = 0.7679, size = 236, normalized size = 0.86

$$\frac{525a\left(10a^2b^2+7a^4+3b^4\right)\sin(c+dx)+35a\left(-10a^2b^2+21a^4-15b^4\right)\sin(3(c+dx))+21a\left(-30a^2b^2+7a^4-5b^4\right)\sin(5(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-525*b*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[c + d*x] - 35*b*(45*a^4 + 30*a^2*b^2
+ b^4)*Cos[3*(c + d*x)] + 21*b*(-25*a^4 + 10*a^2*b^2 + 3*b^4)*Cos[5*(c + d*
x)] - 15*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[7*(c + d*x)] + 525*a*(7*a^4 + 10*
a^2*b^2 + 3*b^4)*Sin[c + d*x] + 35*a*(21*a^4 - 10*a^2*b^2 - 15*b^4)*Sin[3*(
c + d*x)] + 21*a*(7*a^4 - 30*a^2*b^2 - 5*b^4)*Sin[5*(c + d*x)] + 15*a*(a^4
- 10*a^2*b^2 + 5*b^4)*Sin[7*(c + d*x)])/(6720*d)
```

Maple [A] time = 0.201, size = 261, normalized size = 1.

$$\frac{1}{d}\left(b^5\left(-\frac{(\sin(dx+c))^4(\cos(dx+c))^3}{7}-\frac{4\,(\sin(dx+c))^2(\cos(dx+c))^3}{35}-\frac{8\,(\cos(dx+c))^3}{105}\right)+5\,ab^4\left(-1/7\,(\sin(dx+c)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/1
05*cos(d*x+c)^3)+5*a*b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*co
```

```
s(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))+10*a^2*b^3*(-1/7*sin(d*x+c)^2*
cos(d*x+c)^5-2/35*cos(d*x+c)^5)+10*a^3*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/
35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-5/7*a^4*b*cos(d*x+c)^7+1
/7*a^5*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.08965, size = 262, normalized size = 0.95

$$-\frac{75\,a^4b\cos(dx+c)^7+3\left(5\,\sin(dx+c)^7-21\,\sin(dx+c)^5+35\,\sin(dx+c)^3-35\,\sin(dx+c)\right)a^5-10\left(15\,\sin(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/105*(75*a^4*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 +
 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5 - 10*(15*sin(d*x + c)^7 - 42*sin(
d*x + c)^5 + 35*sin(d*x + c)^3)*a^3*b^2 - 30*(5*cos(d*x + c)^7 - 7*cos(d*x
+ c)^5)*a^2*b^3 + 15*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a*b^4 + (15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*b^5)/d
```

Fricas [A] time = 0.537526, size = 429, normalized size = 1.56

$$-\frac{35\,b^5\cos(dx+c)^3+15\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos(dx+c)^7+42\left(5\,a^2b^3-b^5\right)\cos(dx+c)^5-\left(15\left(a^5-10\,a^3b^2+5\,ab^4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^5*cos(d*x + c)^3 + 15*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^7 + 42*(5*a^2*b^3 - b^5)*cos(d*x + c)^5 - (15*(a^5 - 10*a^3*b^2 + 5*a*b^4
)*cos(d*x + c)^6 + 48*a^5 + 80*a^3*b^2 + 30*a*b^4 + 6*(3*a^5 + 5*a^3*b^2 -
20*a*b^4)*cos(d*x + c)^4 + (24*a^5 + 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^2)
*sin(d*x + c))/d
```

Sympy [A] time = 10.632, size = 357, normalized size = 1.3

$$\begin{cases}\frac{16a^5\sin^7(c+dx)}{35d}+\frac{8a^5\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^5\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^5\sin(c+dx)\cos^6(c+dx)}{d}-\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{16a^3b^2\sin^7(c+dx)}{21d}+\\ x\left(a\cos(c)+b\sin(c)\right)^5\cos^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((16*a**5*sin(c + d*x)**7/(35*d) + 8*a**5*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**5*sin(c + d*x)**3*cos(c + d*x)**4/d + a**5*sin(c + d*x
)*cos(c + d*x)**6/d - 5*a**4*b*cos(c + d*x)**7/(7*d) + 16*a**3*b**2*sin(c +
 d*x)**7/(21*d) + 8*a**3*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(3*d) + 10*a*
*3*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - 2*a**2*b**3*sin(c + d*x)**2
*cos(c + d*x)**5/d - 4*a**2*b**3*cos(c + d*x)**7/(7*d) + 2*a*b**4*sin(c + d
*x)**7/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - b**5*sin(c + d*x)
**4*cos(c + d*x)**3/(3*d) - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) -
 8*b**5*cos(c + d*x)**7/(105*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos
(c)**2, True))
```

Giac [A] time = 1.3946, size = 350, normalized size = 1.27

$$-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(25\,a^4b-10\,a^2b^3-3\,b^5\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(45\,a^4b+30\,a^2b^3+b^5\right)\cos\left(3\,dx+\right.}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/448*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b -
10*a^2*b^3 - 3*b^5)*cos(5*d*x + 5*c)/d - 1/192*(45*a^4*b + 30*a^2*b^3 + b^5
)*cos(3*d*x + 3*c)/d - 5/64*(5*a^4*b + 6*a^2*b^3 + b^5)*cos(d*x + c)/d + 1/
448*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(7*d*x + 7*c)/d + 1/320*(7*a^5 - 30*a^3
*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d + 1/192*(21*a^5 - 10*a^3*b^2 - 15*a*b^4)
*sin(3*d*x + 3*c)/d + 5/64*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*sin(d*x + c)/d
```

3.96 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=126

$$\frac{5a\left(a^2+b^2\right)\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{16d}+\frac{5}{16}ax\left(a^2+b^2\right)^2+\frac{\sin^6(c+dx)(a\cot(c+dx)+b)^5}{6d}+$$

```
[Out] (5*a*(a^2 + b^2)^2*x)/16 + (5*a*(a^2 + b^2)*(b + a*Cot[c + d*x])*(a - b*Cot
[c + d*x])*Sin[c + d*x]^2)/(16*d) + (5*a*(b + a*Cot[c + d*x])^3*(a - b*Cot[
c + d*x])*Sin[c + d*x]^4)/(24*d) + ((b + a*Cot[c + d*x])^5*Sin[c + d*x]^6)/
(6*d)
```

Rubi [A] time = 0.0898664, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3088, 805, 723, 203}

$$\frac{5a\left(a^2+b^2\right)\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{16d}+\frac{5}{16}ax\left(a^2+b^2\right)^2+\frac{\sin^6(c+dx)(a\cot(c+dx)+b)^5}{6d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a*(a^2 + b^2)^2*x)/16 + (5*a*(a^2 + b^2)*(b + a*Cot[c + d*x])*(a - b*Cot
[c + d*x])*Sin[c + d*x]^2)/(16*d) + (5*a*(b + a*Cot[c + d*x])^3*(a - b*Cot[
c + d*x])*Sin[c + d*x]^4)/(24*d) + ((b + a*Cot[c + d*x])^5*Sin[c + d*x]^6)/
(6*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 805

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*
c*(p + 1)), x] - Dist[(m*(c*d*f + a*e*g))/(2*a*c*(p + 1)), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 723

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] +
 Dist[((2*p + 3)*(c*d^2 + a*e^2))/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax)^5}{(1+x^2)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(b+a\cot(c+dx))^5\sin^6(c+dx)}{6d} - \frac{(5a)\,\text{Subst}\left(\int \frac{(b+ax)^4}{(1+x^2)^3}\,dx, x, \cot(c+\right.}{6d}$$

$$= \frac{5a(b+a\cot(c+dx))^3(a-b\cot(c+dx))\sin^4(c+dx)}{24d} + \frac{(b+a\cot(c+}{}$$

$$= \frac{5a\left(a^2+b^2\right)(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+dx)}{16d} + \frac{5a(}{}$$

$$= \frac{5}{16}a\left(a^2+b^2\right)^2 x + \frac{5a\left(a^2+b^2\right)(b+a\cot(c+dx))(a-b\cot(c+dx))}{16d}$$

Mathematica [A] time = 0.617089, size = 188, normalized size = 1.49

$$\frac{60a\left(a^2+b^2\right)^2(c+dx)+15a\left(2a^2b^2+3a^4-b^4\right)\sin(2(c+dx))+3a\left(-10a^2b^2+3a^4-5b^4\right)\sin(4(c+dx))+a\left(-10a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (60*a*(a^2 + b^2)^2*(c + d*x) - 15*b*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[2*(c + d
*x)] + 6*b*(-5*a^4 + b^4)*Cos[4*(c + d*x)] - b*(5*a^4 - 10*a^2*b^2 + b^4)*C
os[6*(c + d*x)] + 15*a*(3*a^4 + 2*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*a*(3*
a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + a*(a^4 - 10*a^2*b^2 + 5*b^4)*S
in[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.18, size = 236, normalized size = 1.9

$$\frac{1}{d}\left(\frac{b^5(\sin(dx+c))^6}{6}+5\,ab^4\left(-1/6\,(\sin(dx+c))^3(\cos(dx+c))^3-1/8\,\sin(dx+c)(\cos(dx+c))^3+1/16\,\cos(dx+c)\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(1/6*b^5*sin(d*x+c)^6+5*a*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d
*x+c)*cos(d*x+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+10*a^2*b^3*(
-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+10*a^3*b^2*(-1/6*sin(d*x+
c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16
*c)-5/6*a^4*b*cos(d*x+c)^6+a^5*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos
(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A] time = 1.09425, size = 252, normalized size = 2.

$$-\frac{160\,a^4b\cos(dx+c)^6-32\,b^5\sin(dx+c)^6+\left(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/192*(160*a^4*b*cos(d*x + c)^6 - 32*b^5*sin(d*x + c)^6 + (4*sin(2*d*x + 2
*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^5 - 10*
(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3*b^2 + 160*(
2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^2*b^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12
*d*x - 12*c + 3*sin(4*d*x + 4*c))*a*b^4)/d
```

Fricas [A] time = 0.536616, size = 417, normalized size = 3.31

$$-\frac{24\,b^5\cos\left(dx+c\right)^2+8\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^6+24\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^4-15\left(a^5+2\,a^3b^2+ab^4\right)dx-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/48*(24*b^5*cos(d*x + c)^2 + 8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
6 + 24*(5*a^2*b^3 - b^5)*cos(d*x + c)^4 - 15*(a^5 + 2*a^3*b^2 + a*b^4)*d*x
- (8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(a^5 + 2*a^3*b^2 - 7*
a*b^4)*cos(d*x + c)^3 + 15*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [A] time = 9.86799, size = 663, normalized size = 5.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((5*a**5*x*sin(c + d*x)**6/16 + 15*a**5*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**5*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**5*x*cos(c
+ d*x)**6/16 + 5*a**5*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**5*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**5*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 + 5*a**4*b*sin(c + d*x)**6/(6*d) + 5*a**4*b*sin(c + d*x)**4*cos(c + d*x)**
2/(2*d) + 5*a**4*b*sin(c + d*x)**2*cos(c + d*x)**4/(2*d) + 5*a**3*b**2*x*si
n(c + d*x)**6/8 + 15*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a**
3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/8 + 5*a**3*b**2*x*cos(c + d*x)**6/
8 + 5*a**3*b**2*sin(c + d*x)**5*cos(c + d*x)/(8*d) + 5*a**3*b**2*sin(c + d*
x)**3*cos(c + d*x)**3/(3*d) - 5*a**3*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d
) + 5*a**2*b**3*sin(c + d*x)**6/(6*d) + 5*a**2*b**3*sin(c + d*x)**4*cos(c +
 d*x)**2/(2*d) + 5*a*b**4*x*sin(c + d*x)**6/16 + 15*a*b**4*x*sin(c + d*x)**
4*cos(c + d*x)**2/16 + 15*a*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a
*b**4*x*cos(c + d*x)**6/16 + 5*a*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) -
 5*a*b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*a*b**4*sin(c + d*x)*cos
(c + d*x)**5/(16*d) + b**5*sin(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a*cos(c) +
 b*sin(c))**5*cos(c), True))
```

Giac [A] time = 1.32448, size = 285, normalized size = 2.26

$$\frac{5}{16}\left(a^5+2\,a^3b^2+ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{\left(5\,a^4b-b^5\right)\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\left(5\,a^4b+6\,a^2b^3+b^5\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 5/16*(a^5 + 2*a^3*b^2 + a*b^4)*x - 1/192*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(6
*d*x + 6*c)/d - 1/32*(5*a^4*b - b^5)*cos(4*d*x + 4*c)/d - 5/64*(5*a^4*b + 6
*a^2*b^3 + b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin
(6*d*x + 6*c)/d + 1/64*(3*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/d +
5/64*(3*a^5 + 2*a^3*b^2 - a*b^4)*sin(2*d*x + 2*c)/d
```

3.97 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=94

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rubi [A] time = 0.0468233, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3072, 194}

$$\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{\left(a^2+b^2\right)^2(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rule 3072

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> -Dist[d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)^2 dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)-2a^2\left(1+\frac{b^2}{a^2}\right)x^2+x^4\right) dx, x, b\cos(c+dx)-a\sin\right.}{d}$$
$$= -\frac{\left(a^2+b^2\right)^2 (b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c}{3d}$$

Mathematica [A] time = 0.463477, size = 156, normalized size = 1.66

$$\frac{150a\left(a^2+b^2\right)^2\sin(c+dx)+25a\left(-2a^2b^2+a^4-3b^4\right)\sin(3(c+dx))+3a\left(-10a^2b^2+a^4+5b^4\right)\sin(5(c+dx))-150b\,(}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3*
(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2 +
 b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*
a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

Maple [A] time = 0.154, size = 175, normalized size = 1.9

$$\frac{1}{d}\left(-\frac{b^5\cos(dx+c)}{5}\left(\frac{8}{3}+(\sin(dx+c))^4+\frac{4\,(\sin(dx+c))^2}{3}\right)+ab^4(\sin(dx+c))^5+10\,a^2b^3\left(-1/5\,(\sin(dx+c))^2(\cos(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-1/5*b^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+
c)^5+10*a^2*b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+10*a^3*b
^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-a^4*b*co
s(d*x+c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A] time = 1.23263, size = 232, normalized size = 2.47

$$-\frac{a^4 b \cos(dx+c)^5}{d} + \frac{ab^4 \sin(dx+c)^5}{d} + \frac{\left(3 \sin(dx+c)^5 - 10 \sin(dx+c)^3 + 15 \sin(dx+c)\right)a^5}{15\,d} - \frac{2\left(3 \sin(dx+c)^5 - 5\right.}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*sin
(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b^3/
d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

Fricas [A] time = 0.513386, size = 354, normalized size = 3.77

$$-\frac{15\,b^5 \cos(dx+c) + 3\left(5\,a^4 b - 10\,a^2 b^3 + b^5\right)\cos(dx+c)^5 + 10\left(5\,a^2 b^3 - b^5\right)\cos(dx+c)^3 - \left(8\,a^5 + 20\,a^3 b^2 + 15\,ab^4 + 3\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
+ 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 3*
(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*a*b
^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A] time = 2.98121, size = 267, normalized size = 2.84

$$\begin{cases} \frac{8a^5 \sin^5(c+dx)}{15d} + \frac{4a^5 \sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^5 \sin(c+dx)\cos^4(c+dx)}{d} - \frac{a^4 b \cos^5(c+dx)}{d} + \frac{4a^3b^2 \sin^5(c+dx)}{3d} + \frac{10a^3b^2 \sin^3(c+dx)\cos^2(c+dx)}{3d} - 1 \\ x\left(a\cos(c) + b\sin(c)\right)^5 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)**5
```

```
/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 4*a
**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c + d*
x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**
5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, True))
```

Giac [B] time = 1.1573, size = 252, normalized size = 2.68

$$-\frac{\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\left(3\,a^{4}b+2\,a^{2}b^{3}-b^{5}\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{5\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)}{8\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*a
^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x +
c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5 - 2
*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*sin(
d*x + c)/d
```

3.98 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=170

$$-\frac{\sin^4(c+dx)\left(a\left(-10a^2b^2+a^4+5b^4\right)\cot(c+dx)+b\left(-10a^2b^2+5a^4+b^4\right)\right)}{4d}+\frac{\sin^2(c+dx)\left(5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(\right.}{8d}$$

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/8 - (b^5*Log[Sin[c + d*x]])/d + (b^5*Lo
g[Tan[c + d*x]])/d + ((4*b*(5*a^4 - b^4) + 5*a*(a^2 - 3*b^2)*(a^2 + b^2)*Co
t[c + d*x])*Sin[c + d*x]^2)/(8*d) - ((b*(5*a^4 - 10*a^2*b^2 + b^4) + a*(a^4
 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.221735, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3088, 1805, 801, 635, 203, 260}

$$-\frac{\sin^4(c+dx)\left(a\left(-10a^2b^2+a^4+5b^4\right)\cot(c+dx)+b\left(-10a^2b^2+5a^4+b^4\right)\right)}{4d}+\frac{\sin^2(c+dx)\left(5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/8 - (b^5*Log[Sin[c + d*x]])/d + (b^5*Lo
g[Tan[c + d*x]])/d + ((4*b*(5*a^4 - b^4) + 5*a*(a^2 - 3*b^2)*(a^2 + b^2)*Co
t[c + d*x])*Sin[c + d*x]^2)/(8*d) - ((b*(5*a^4 - 10*a^2*b^2 + b^4) + a*(a^4
 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^4)/(4*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x(1+x^2)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-10a^2b^2+5b^4\right)\cot(c+dx)\right)\sin^4(c+}{4d}$$

$$= \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{8d} - $$

$$= \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{8d} - $$

$$= \frac{b^5\log(\tan(c+dx))}{d} + \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(c+\right.}{8d}$$

$$= \frac{b^5\log(\tan(c+dx))}{d} + \frac{\left(4b\left(5a^4-b^4\right)+5a\left(a^2-3b^2\right)\left(a^2+b^2\right)\cot(c+\right.}{8d}$$

$$= \frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)x - \frac{b^5\log(\sin(c+dx))}{d} + \frac{b^5\log(\tan(c+dx}{d}$$

Mathematica [B] time = 6.44127, size = 711, normalized size = 4.18

$$b^5\left(\frac{\cos^4(c+dx)(a+b\tan(c+dx))^6\left(ab\tan(c+dx)+b^2\right)}{4b^6\left(a^2+b^2\right)} - \frac{\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6\left(b\left(a\left(2b^2-3a^2\right)+3ab^2\right)\tan(c+dx)-3a^2b^2+b^2\left(2b^2-3a^2\right)\right)}{2b^4\left(a^2+b^2\right)} - \frac{\left(-29a^2b^2+5a^2\left(3a^2-5b^2\right)+3a^4+8b^4\right.}{}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^5*((Cos[c + d*x]^4*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(4*b
^6*(a^2 + b^2)) - ((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(-3*a^2*b^2 + b^2
*(-3*a^2 + 2*b^2) + b*(3*a*b^2 + a*(-3*a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*
(a^2 + b^2)) - ((3*a^4 - 29*a^2*b^2 + 8*b^4 + 5*a^2*(3*a^2 - 5*b^2))*(((5*a
^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-
b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^3*b^2
+ 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(2*a^2 -
 b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c
```

```
 + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4) - 5*a*(3*a^2 - 5*b^2)*(((6*a^5 - 20*
a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[S
qrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5 - 20*a^3*b^2 + 6*a*b^4 - (a^6 - 15
*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/
2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c + d*x] + a*b^2*(10*a^2 - 3*b^2)*Tan
[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[c + d*x]^3)/3 + (3*a*b^4*Tan[c + d*x]
^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))
/d
```

Maple [A] time = 0.236, size = 272, normalized size = 1.6

$$\frac{a^5(\cos(dx+c))^3\sin(dx+c)}{4d}+\frac{3a^5\cos(dx+c)\sin(dx+c)}{8d}+\frac{3a^5x}{8}+\frac{3a^5c}{8d}-\frac{5a^4(\cos(dx+c))^4b}{4d}-\frac{5a^3b^2(\cos(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/4*a^5*cos(d*x+c)^3*sin(d*x+c)/d+3/8*a^5*cos(d*x+c)*sin(d*x+c)/d+3/8*a^5*x
+3/8/d*a^5*c-5/4/d*a^4*cos(d*x+c)^4*b-5/2*a^3*b^2*cos(d*x+c)^3*sin(d*x+c)/d
+5/4*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+5/4*a^3*b^2*x+5/4/d*a^3*b^2*c+5/2/d*a^
2*b^3*sin(d*x+c)^4-5/4/d*a*b^4*cos(d*x+c)*sin(d*x+c)^3-15/8*a*b^4*cos(d*x+c
)*sin(d*x+c)/d+15/8*a*b^4*x+15/8/d*a*b^4*c-1/4/d*b^5*sin(d*x+c)^4-1/2/d*sin
(d*x+c)^2*b^5-1/d*b^5*ln(cos(d*x+c))
```

Maxima [A] time = 1.09662, size = 230, normalized size = 1.35

$$\frac{80a^2b^3\sin(dx+c)^4-40\left(\sin(dx+c)^2-1\right)^2a^4b+(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c))a^5+10(4dx+4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/32*(80*a^2*b^3*sin(d*x + c)^4 - 40*(sin(d*x + c)^2 - 1)^2*a^4*b + (12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^5 + 10*(4*d*x + 4*c - si
n(4*d*x + 4*c))*a^3*b^2 + 5*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x
 + 2*c))*a*b^4 - 8*(sin(d*x + c)^4 + 2*sin(d*x + c)^2 + 2*log(sin(d*x + c)^
2 - 1))*b^5)/d
```

Fricas [A] time = 0.549745, size = 362, normalized size = 2.13

$$-\frac{8\,b^5\log\left(-\cos\left(dx+c\right)\right)+2\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)dx+8\left(5\,a^2b^3-b^5\right)\cos\left(dx+\cdots\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^5*log(-cos(d*x + c)) + 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^4 - (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*d*x + 8*(5*a^2*b^3 - b^5)*cos(d*x + c
)^2 - (2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2
- 25*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25981, size = 269, normalized size = 1.58

$$\frac{4\,b^5\log\left(\tan\left(dx+c\right)^2+1\right)+\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\left(dx+c\right)-\frac{6\,b^5\tan(dx+c)^4-3\,a^5\tan(dx+c)^3-10\,a^3b^2\tan(dx+c)^3+25\,ab^4\tan(dx+c)^3\cdots}{\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^5*log(tan(d*x + c)^2 + 1) + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x +
 c) - (6*b^5*tan(d*x + c)^4 - 3*a^5*tan(d*x + c)^3 - 10*a^3*b^2*tan(d*x + c
)^3 + 25*a*b^4*tan(d*x + c)^3 + 40*a^2*b^3*tan(d*x + c)^2 + 4*b^5*tan(d*x +
 c)^2 - 5*a^5*tan(d*x + c) + 10*a^3*b^2*tan(d*x + c) + 15*a*b^4*tan(d*x + c
) + 10*a^4*b + 20*a^2*b^3)/(tan(d*x + c)^2 + 1)^2)/d
```

3.99 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=205

$$\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}-\frac{10a^2b^3\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}-\frac{a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}$$

```
[Out] (5*a*b^4*ArcTanh[Sin[c + d*x]])/d - (10*a^2*b^3*Cos[c + d*x])/d + (2*b^5*Co
s[c + d*x])/d - (5*a^4*b*Cos[c + d*x]^3)/(3*d) + (10*a^2*b^3*Cos[c + d*x]^3
)/(3*d) - (b^5*Cos[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x])/d + (a^5*Sin[c +
d*x])/d - (5*a*b^4*Sin[c + d*x])/d - (a^5*Sin[c + d*x]^3)/(3*d) + (10*a^3*b
^2*Sin[c + d*x]^3)/(3*d) - (5*a*b^4*Sin[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.215913, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3090, 2633, 2565, 30, 2564, 2592, 302, 206, 2590, 270}

$$\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}-\frac{10a^2b^3\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}-\frac{a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] (5*a*b^4*ArcTanh[Sin[c + d*x]])/d - (10*a^2*b^3*Cos[c + d*x])/d + (2*b^5*Co
s[c + d*x])/d - (5*a^4*b*Cos[c + d*x]^3)/(3*d) + (10*a^2*b^3*Cos[c + d*x]^3
)/(3*d) - (b^5*Cos[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x])/d + (a^5*Sin[c +
d*x])/d - (5*a*b^4*Sin[c + d*x])/d - (a^5*Sin[c + d*x]^3)/(3*d) + (10*a^3*b
^2*Sin[c + d*x]^3)/(3*d) - (5*a*b^4*Sin[c + d*x]^3)/(3*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
```

```
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^3(c+dx)+5a^4b\cos^2(c+dx)\sin(c+dx)+10a^3b^2\cos(c$$

$$= a^5\int \cos^3(c+dx)\,dx+\left(5a^4b\right)\int \cos^2(c+dx)\sin(c+dx)\,dx+\left(10\right.$$

$$= -\frac{a^5\,\mathrm{Subst}\left(\int\left(1-x^2\right)dx,x,-\sin(c+dx)\right)}{d}-\frac{\left(5a^4b\right)\mathrm{Subst}\left(\int x^2\right.}{d}$$

$$= -\frac{10a^2b^3\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}+$$

$$= -\frac{10a^2b^3\cos(c+dx)}{d}+\frac{2b^5\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}+\frac{10a^2}{}$$

$$= \frac{5ab^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{10a^2b^3\cos(c+dx)}{d}+\frac{2b^5\cos(c+dx)}{d}$$

Mathematica [B] time = 6.30718, size = 632, normalized size = 3.08

$$-\frac{b\left(30a^2b^2+5a^4-7b^4\right)\cos^6(c+dx)(a+b\tan(c+dx))^5}{4d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{a\left(-10a^2b^2+a^4+5b^4\right)\sin(3(c+dx))\cos^5(c+dx)(a+b\,\mathrm{ta}}{12d(a\cos(c+dx)+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^5*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^5) - (b*(5*a^4 + 30*a^2*b^2 - 7*b^4)*Cos[c + d*x]^6*(a + b*Tan[c + d*
x])^5)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b*(5*a^4 - 10*a^2*b^2 +
 b^4)*Cos[c + d*x]^5*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^5) - (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c
```

```
+ d*x])^5) + (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*C
os[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*Cos[c + d*
x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (a*(3*a^4 + 10*a^2*b^2 -
 25*b^4)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x])^5)/(4*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^5) + (a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cos[c + d*x]^5*
Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^5)
```

Maple [A] time = 0.251, size = 251, normalized size = 1.2

$$\frac{(\cos(dx+c))^2\sin(dx+c)\,a^5}{3d}+\frac{2\,a^5\sin(dx+c)}{3d}-\frac{5\,a^4b\,(\cos(dx+c))^3}{3d}+\frac{10\,a^3b^2\,(\sin(dx+c))^3}{3d}-\frac{10\,\cos(dx+c)\,(\sin(\ldots}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/3/d*cos(d*x+c)^2*sin(d*x+c)*a^5+2/3*a^5*sin(d*x+c)/d-5/3*a^4*b*cos(d*x+c)
^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d-10/3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3-20/
3*a^2*b^3*cos(d*x+c)/d-5/3*a*b^4*sin(d*x+c)^3/d-5*a*b^4*sin(d*x+c)/d+5/d*a*
b^4*ln(sec(d*x+c)+tan(d*x+c))+1/d*b^5*sin(d*x+c)^6/cos(d*x+c)+8/3*b^5*cos(d
*x+c)/d+1/d*b^5*cos(d*x+c)*sin(d*x+c)^4+4/3/d*cos(d*x+c)*sin(d*x+c)^2*b^5
```

Maxima [A] time = 1.22015, size = 219, normalized size = 1.07

$$-\frac{10\,a^4b\cos(dx+c)^3-20\,a^3b^2\sin(dx+c)^3+2\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)a^5-20\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)a^2\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(10*a^4*b*cos(d*x + c)^3 - 20*a^3*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c)
^3 - 3*sin(d*x + c))*a^5 - 20*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b^3 + 5
*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*
sin(d*x + c))*a*b^4 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*
```

```
b^5)/d
```

Fricas [A] time = 0.536934, size = 429, normalized size = 2.09

$$\frac{15\,ab^4\cos(dx+c)\log(\sin(dx+c)+1)-15\,ab^4\cos(dx+c)\log(-\sin(dx+c)+1)+6\,b^5-2\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a*b^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 15*a*b^4*cos(d*x + c)*lo
g(-sin(d*x + c) + 1) + 6*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
4 - 12*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 2*((a^5 - 10*a^3*b^2 + 5*a*b^4)*c
os(d*x + c)^3 + 2*(a^5 + 5*a^3*b^2 - 10*a*b^4)*cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2754, size = 382, normalized size = 1.86

$$\frac{15\,ab^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,ab^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\,b^5}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(3\,a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-15\,ab^4\tan\left(\frac{1}{2}\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(15*a*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*b^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 6*b^5/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(3*a^5*tan(1/2
*d*x + 1/2*c)^5 - 15*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*b*tan(1/2*d*x +
1/2*c)^4 + 3*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x + 1/2*c)^3 + 40
*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 50*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^2*
b^3*tan(1/2*d*x + 1/2*c)^2 + 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*
d*x + 1/2*c) - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 5*a^4*b - 20*a^2*b^3 + 5*b^5
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

3.100 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=169

$$-\frac{2b^3\left(5a^2-b^2\right)\log(\sin(c+dx))}{d}+\frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d}+\frac{\sin^2(c+dx)\left(a\left(-10a^2b^2+a^4+5b^4\right)\cot(c+dx)\right.}{2d}$$

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/2 - (2*b^3*(5*a^2 - b^2)*Log[Sin[c + d*x]
])/d + (2*b^3*(5*a^2 - b^2)*Log[Tan[c + d*x]])/d + ((b*(5*a^4 - 10*a^2*b^2
+ b^4) + a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d) +
 (5*a*b^4*Tan[c + d*x])/d + (b^5*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.231263, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3088, 1805, 1802, 635, 203, 260}

$$-\frac{2b^3\left(5a^2-b^2\right)\log(\sin(c+dx))}{d}+\frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d}+\frac{\sin^2(c+dx)\left(a\left(-10a^2b^2+a^4+5b^4\right)\cot(c+dx)\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/2 - (2*b^3*(5*a^2 - b^2)*Log[Sin[c + d*x]
])/d + (2*b^3*(5*a^2 - b^2)*Log[Tan[c + d*x]])/d + ((b*(5*a^4 - 10*a^2*b^2
+ b^4) + a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d) +
 (5*a*b^4*Tan[c + d*x])/d + (b^5*Tan[c + d*x]^2)/(2*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
```

```
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x^3(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-10a^2b^2+5b^4\right)\cot(c+dx)\right)\sin^2(c}{2d}$$

$$= \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-10a^2b^2+5b^4\right)\cot(c+dx)\right)\sin^2(c}{2d}$$

$$= \frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d} + \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-\right.\right.}{}$$

$$= \frac{2b^3\left(5a^2-b^2\right)\log(\tan(c+dx))}{d} + \frac{\left(b\left(5a^4-10a^2b^2+b^4\right)+a\left(a^4-\right.\right.}{}$$

$$= \frac{1}{2}a\left(a^4+10a^2b^2-15b^4\right)x - \frac{2b^3\left(5a^2-b^2\right)\log(\sin(c+dx))}{d} + \frac{2b^3}{}$$

<u>Mathematica</u> [B] time = 6.37759, size = 571, normalized size = 3.38

$$b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6\left(ab\tan(c+dx)+b^2\right)}{2b^4\left(a^2+b^2\right)} - \frac{\left(4b^2-6a^2\right)\left(\frac{1}{2}b^2\left(10a^2-b^2\right)\tan^2(c+dx)+5ab\left(2a^2-b^2\right)\tan(c+dx)+\frac{1}{2}\left(-10a^2b^2+\frac{-10a^3b^2+a^5+5ab^4}{\sqrt{-b^2}}+5a^4+b\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(2*b
^4*(a^2 + b^2)) - ((-6*a^2 + 4*b^2)*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10
*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a
^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-
b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2
 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^
4)/4) + 5*a*(((6*a^5 - 20*a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^
4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5 - 20*a^3
*b^2 + 6*a*b^4 - (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt
[-b^2] + b*Tan[c + d*x]])/2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c + d*x] +
a*b^2*(10*a^2 - 3*b^2)*Tan[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[c + d*x]^3)
/3 + (3*a*b^4*Tan[c + d*x]^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b^2*(a^2 + b^
```

```
2))))/d
```

Maple [A] time = 0.253, size = 291, normalized size = 1.7

$$\frac{a^5\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^5x}{2}+\frac{a^5c}{2d}-\frac{5a^4b(\cos(dx+c))^2}{2d}-5\frac{a^3b^2\cos(dx+c)\sin(dx+c)}{d}+5a^3b^2x+5\frac{a^3b^2c}{d}-5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/2*a^5*cos(d*x+c)*sin(d*x+c)/d+1/2*a^5*x+1/2/d*a^5*c-5/2/d*a^4*b*cos(d*x+c
)^2-5*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+5*a^3*b^2*x+5/d*a^3*b^2*c-5/d*a^2*b^3
*sin(d*x+c)^2-10/d*a^2*b^3*ln(cos(d*x+c))+5/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)
+5/d*a*b^4*cos(d*x+c)*sin(d*x+c)^3+15/2*a*b^4*cos(d*x+c)*sin(d*x+c)/d-15/2*
a*b^4*x-15/2/d*a*b^4*c+1/2/d*b^5*sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^5*sin(d*
x+c)^4+1/d*sin(d*x+c)^2*b^5+2/d*b^5*ln(cos(d*x+c))
```

Maxima [A] time = 1.78528, size = 242, normalized size = 1.43

$$\frac{10a^4b\sin(dx+c)^2+(2dx+2c+\sin(2dx+2c))a^5+10(2dx+2c-\sin(2dx+2c))a^3b^2-20\left(\sin(dx+c)^2+\log\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/4*(10*a^4*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^5 + 10*(2
*d*x + 2*c - sin(2*d*x + 2*c))*a^3*b^2 - 20*(sin(d*x + c)^2 + log(sin(d*x +
 c)^2 - 1))*a^2*b^3 - 10*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) -
 2*tan(d*x + c))*a*b^4 + 2*(sin(d*x + c)^2 - 1/(sin(d*x + c)^2 - 1) + 2*log
(sin(d*x + c)^2 - 1))*b^5)/d
```

Fricas [A] time = 0.536343, size = 414, normalized size = 2.45

$$\frac{2b^5-2\left(5a^4b-10a^2b^3+b^5\right)\cos(dx+c)^4-8\left(5a^2b^3-b^5\right)\cos(dx+c)^2\log(-\cos(dx+c))+\left(5a^4b-10a^2b^3+b^5+2\right.}{4d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 - 8*(5*a^2*b^3 -
 b^5)*cos(d*x + c)^2*log(-cos(d*x + c)) + (5*a^4*b - 10*a^2*b^3 + b^5 + 2*(
a^5 + 10*a^3*b^2 - 15*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(10*a*b^4*cos(d*x + c)
 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27347, size = 234, normalized size = 1.38

$$\frac{b^5 \tan(dx+c)^2 + 10\,ab^4 \tan(dx+c) + \left(a^5 + 10\,a^3b^2 - 15\,ab^4\right)(dx+c) + 2\left(5\,a^2b^3 - b^5\right)\log\left(\tan(dx+c)^2 + 1\right) - \frac{10\,a^2b^3}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*tan(d*x + c)^2 + 10*a*b^4*tan(d*x + c) + (a^5 + 10*a^3*b^2 - 15*a*
b^4)*(d*x + c) + 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1) - (10*a^2*b^3*
tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 - a^5*tan(d*x + c) + 10*a^3*b^2*tan(d
*x + c) - 5*a*b^4*tan(d*x + c) + 5*a^4*b - b^5)/(tan(d*x + c)^2 + 1))/d
```

3.101 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=204

$$-\frac{10a^3b^2\sin(c+dx)}{d}+\frac{10a^2b^3\cos(c+dx)}{d}+\frac{10a^2b^3\sec(c+dx)}{d}+\frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5 s}{}$$

```
[Out] (10*a^3*b^2*ArcTanh[Sin[c + d*x]])/d - (15*a*b^4*ArcTanh[Sin[c + d*x]])/(2*
d) - (5*a^4*b*Cos[c + d*x])/d + (10*a^2*b^3*Cos[c + d*x])/d - (b^5*Cos[c +
d*x])/d + (10*a^2*b^3*Sec[c + d*x])/d - (2*b^5*Sec[c + d*x])/d + (b^5*Sec[c
 + d*x]^3)/(3*d) + (a^5*Sin[c + d*x])/d - (10*a^3*b^2*Sin[c + d*x])/d + (15
*a*b^4*Sin[c + d*x])/(2*d) + (5*a*b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.208001, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3090, 2637, 2638, 2592, 321, 206, 2590, 14, 288, 270}

$$-\frac{10a^3b^2\sin(c+dx)}{d}+\frac{10a^2b^3\cos(c+dx)}{d}+\frac{10a^2b^3\sec(c+dx)}{d}+\frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5 s}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (10*a^3*b^2*ArcTanh[Sin[c + d*x]])/d - (15*a*b^4*ArcTanh[Sin[c + d*x]])/(2*
d) - (5*a^4*b*Cos[c + d*x])/d + (10*a^2*b^3*Cos[c + d*x])/d - (b^5*Cos[c +
d*x])/d + (10*a^2*b^3*Sec[c + d*x])/d - (2*b^5*Sec[c + d*x])/d + (b^5*Sec[c
 + d*x]^3)/(3*d) + (a^5*Sin[c + d*x])/d - (10*a^3*b^2*Sin[c + d*x])/d + (15
*a*b^4*Sin[c + d*x])/(2*d) + (5*a*b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos(c+dx)+5a^4b\sin(c+dx)+10a^3b^2\sin(c+dx)\tan(c+dx\right.$$

$$= a^5\int\cos(c+dx)\,dx+\left(5a^4b\right)\int\sin(c+dx)\,dx+\left(10a^3b^2\right)\int\sin(c+$$

$$= -\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}+\frac{\left(10a^3b^2\right)\text{Subst}\left(\int\frac{x^2}{1-x^2}\,dx,x,\right.}{d}$$

$$= -\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}-\frac{10a^3b^2\sin(c+dx)}{d}+\frac{5ab^4\sin}{}$$

$$= \frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^4b\cos(c+dx)}{d}+\frac{10a^2b^3\cos(c+dx}{d}$$

$$= \frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^4b\cos}{}$$

Mathematica [A] time = 5.95975, size = 397, normalized size = 1.95

$$12a\left(-10a^2b^2+a^4+5b^4\right)\sin(c+dx)-12b\left(-10a^2b^2+5a^4+b^4\right)\cos(c+dx)+\frac{2b^3\left(60a^2-11b^2\right)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{2b^3\left(11b^2-60a^2\right)\sin}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a^2*b^3 - 22*b^5 - 12*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x] - 30*a
*b^2*(4*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 30*a*b^2*(4
*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^4*(15*a + b))/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^5*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b^3*(60*a^2 - 11*b^2)*Sin[(c + d*x)/2])
```

```
/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^5*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^3 + (b^4*(-15*a + b))/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (2*b^3*(-60*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]) + 12*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[c + d*x])/(1
2*d)
```

Maple [A] time = 0.259, size = 327, normalized size = 1.6

$$\frac{a^5\sin(dx+c)}{d} - 5\,\frac{a^4b\cos(dx+c)}{d} - 10\,\frac{a^3b^2\sin(dx+c)}{d} + 10\,\frac{a^3b^2\ln(\sec(dx+c)+\tan(dx+c))}{d} + 10\,\frac{a^2b^3(\sin(dx+}{d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] a^5*sin(d*x+c)/d-5*a^4*b*cos(d*x+c)/d-10*a^3*b^2*sin(d*x+c)/d+10/d*a^3*b^2*
ln(sec(d*x+c)+tan(d*x+c))+10/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)+10/d*cos(d*x
+c)*sin(d*x+c)^2*a^2*b^3+20*a^2*b^3*cos(d*x+c)/d+5/2/d*a*b^4*sin(d*x+c)^5/c
os(d*x+c)^2+5/2*a*b^4*sin(d*x+c)^3/d+15/2*a*b^4*sin(d*x+c)/d-15/2/d*a*b^4*l
n(sec(d*x+c)+tan(d*x+c))+1/3/d*b^5*sin(d*x+c)^6/cos(d*x+c)^3-1/d*b^5*sin(d*
x+c)^6/cos(d*x+c)-8/3*b^5*cos(d*x+c)/d-1/d*b^5*cos(d*x+c)*sin(d*x+c)^4-4/3/
d*cos(d*x+c)*sin(d*x+c)^2*b^5
```

Maxima [A] time = 1.08271, size = 244, normalized size = 1.2

$$-\frac{15\,ab^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + 3\,\log(\sin(dx+c)+1) - 3\,\log(\sin(dx+c)-1) - 4\,\sin(dx+c)\right) - 120\,a^2b^3\left(\frac{1}{\cos(dx+c)} + \cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/12*(15*a*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) +
 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 120*a^2*b^3*(1/cos(d*x +
c) + cos(d*x + c)) + 4*b^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d
*x + c)) - 60*a^3*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*si
n(d*x + c)) + 60*a^4*b*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

Fricas [A] time = 0.5368, size = 455, normalized size = 2.23

$$\frac{4\,b^5 - 12\left(5\,a^4b - 10\,a^2b^3 + b^5\right)\cos\left(dx+c\right)^4 + 15\left(4\,a^3b^2 - 3\,ab^4\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right) - 15\left(4\,a^3b^2 - 3\,ab^4\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 - 12*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 15*(4*a^3*b^
2 - 3*a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*(4*a^3*b^2 - 3*a*b^4
)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 24*(5*a^2*b^3 - b^5)*cos(d*x + c)
^2 + 6*(5*a*b^4*cos(d*x + c) + 2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^
3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29411, size = 379, normalized size = 1.86

$$15\left(4\,a^3b^2 - 3\,ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 15\left(4\,a^3b^2 - 3\,ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{12\left(a^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 10\,a^3b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^
3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^5*tan(1/2*d*x +
```

```
 1/2*c) - 10*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*a*b^4*tan(1/2*d*x + 1/2*c) -
5*a^4*b + 10*a^2*b^3 - b^5)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(15*a*b^4*tan(
1/2*d*x + 1/2*c)^5 - 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 + 6*b^5*tan(1/2*d*x
+ 1/2*c)^4 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*b^5*tan(1/2*d*x + 1/2*
c)^2 - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 60*a^2*b^3 + 10*b^5)/(tan(1/2*d*x +
1/2*c)^2 - 1)^3)/d
```

3.102 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=147

$$\frac{b(3a^2-b^2)(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2(a^2-b^2)\tan(c+dx)}{d}-\frac{b(-10a^2b^2+5a^4+b^4)\log(\cos(c+dx))}{d}+ax(-10a^2b^2\cdot$$

```
[Out] a*(a^4 - 10*a^2*b^2 + 5*b^4)*x - (b*(5*a^4 - 10*a^2*b^2 + b^4)*Log[Cos[c +
d*x]])/d + (4*a*b^2*(a^2 - b^2)*Tan[c + d*x])/d + (b*(3*a^2 - b^2)*(a + b*T
an[c + d*x])^2)/(2*d) + (2*a*b*(a + b*Tan[c + d*x])^3)/(3*d) + (b*(a + b*Ta
n[c + d*x])^4)/(4*d)
```

Rubi [A] time = 0.230165, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3482, 3528, 3525, 3475}

$$\frac{b(3a^2-b^2)(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2(a^2-b^2)\tan(c+dx)}{d}-\frac{b(-10a^2b^2+5a^4+b^4)\log(\cos(c+dx))}{d}+ax(-10a^2b^2\cdot$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] a*(a^4 - 10*a^2*b^2 + 5*b^4)*x - (b*(5*a^4 - 10*a^2*b^2 + b^4)*Log[Cos[c +
d*x]])/d + (4*a*b^2*(a^2 - b^2)*Tan[c + d*x])/d + (b*(3*a^2 - b^2)*(a + b*T
an[c + d*x])^2)/(2*d) + (2*a*b*(a + b*Tan[c + d*x])^3)/(3*d) + (b*(a + b*Ta
n[c + d*x])^4)/(4*d)
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3482

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n - 1))/(d*(n - 1)), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3528

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(d*(a + b*Tan[e + f*x])^m)/(f*m), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rule 3525

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[(b*d*Tan[e + f*x])/f, x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= \int (a+b\tan(c+dx))^5\,dx\\
&= \frac{b(a+b\tan(c+dx))^4}{4d}+\int (a+b\tan(c+dx))^3\left(a^2-b^2+2ab\tan(\right.\\
&= \frac{2ab(a+b\tan(c+dx))^3}{3d}+\frac{b(a+b\tan(c+dx))^4}{4d}+\int (a+b\tan(c\\
&= \frac{b\left(3a^2-b^2\right)(a+b\tan(c+dx))^2}{2d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}+\frac{b(a}{}\\
&= a\left(a^4-10a^2b^2+5b^4\right)x+\frac{4ab^2\left(a^2-b^2\right)\tan(c+dx)}{d}+\frac{b\left(3a^2-b^2\right)}{}\\
&= a\left(a^4-10a^2b^2+5b^4\right)x-\frac{b\left(5a^4-10a^2b^2+b^4\right)\log(\cos(c+dx))}{d}+\end{aligned}$$

Mathematica [C] time = 0.754068, size = 126, normalized size = 0.86

$$\frac{-6b^3\left(b^2-10a^2\right)\tan^2(c+dx)+60ab^2\left(2a^2-b^2\right)\tan(c+dx)+20ab^4\tan^3(c+dx)+6(b-ia)^5\log(-\tan(c+dx)+i)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (6*((-I)*a + b)^5*Log[I - Tan[c + d*x]] + 6*(I*a + b)^5*Log[I + Tan[c + d*x
]] + 60*a*b^2*(2*a^2 - b^2)*Tan[c + d*x] - 6*b^3*(-10*a^2 + b^2)*Tan[c + d*
x]^2 + 20*a*b^4*Tan[c + d*x]^3 + 3*b^5*Tan[c + d*x]^4)/(12*d)
```

Maple [A] time = 0.264, size = 202, normalized size = 1.4

$$a^5x+\frac{a^5c}{d}-5\,\frac{a^4b\ln(\cos(dx+c))}{d}-10\,a^3b^2x+10\,\frac{\tan(dx+c)\,a^3b^2}{d}-10\,\frac{a^3b^2c}{d}+5\,\frac{a^2b^3(\tan(dx+c))^2}{d}+10\,\frac{a^2b^3\ln(\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] a^5*x+1/d*a^5*c-5/d*a^4*b*ln(cos(d*x+c))-10*a^3*b^2*x+10/d*tan(d*x+c)*a^3*b
^2-10/d*a^3*b^2*c+5/d*a^2*b^3*tan(d*x+c)^2+10/d*a^2*b^3*ln(cos(d*x+c))+5/3/
d*a*b^4*tan(d*x+c)^3-5*a*b^4*tan(d*x+c)/d+5*a*b^4*x+5/d*a*b^4*c+1/4/d*b^5*t
an(d*x+c)^4-1/2*b^5*tan(d*x+c)^2/d-1/d*b^5*ln(cos(d*x+c))
```

Maxima [A] time = 1.74333, size = 235, normalized size = 1.6

$$\frac{12\,(dx+c)a^5-120\,(dx+c-\tan(dx+c))a^3b^2+20\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)ab^4+3\,b^5\left(\frac{4\,\sin(dx+}{\sin(dx+c)^4-2\,\sin}\right.}{12\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*a^5 - 120*(d*x + c - tan(d*x + c))*a^3*b^2 + 20*(tan(d*x
 + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b^4 + 3*b^5*((4*sin(d*x + c)^2 -
3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1)) - 6
0*a^2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 30*a^4*b*log
(-sin(d*x + c)^2 + 1))/d
```

Fricas [A] time = 0.533899, size = 367, normalized size = 2.5

$$\frac{12\left(a^5-10\,a^3b^2+5\,ab^4\right)dx\cos\left(dx+c\right)^4-12\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+3\,b^5+12\left(5\,a^2b^3\right.}{12\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*d*x*cos(d*x + c)^4 - 12*(5*a^4*b - 10
*a^2*b^3 + b^5)*cos(d*x + c)^4*log(-cos(d*x + c)) + 3*b^5 + 12*(5*a^2*b^3 -
 b^5)*cos(d*x + c)^2 + 20*(a*b^4*cos(d*x + c) + 2*(3*a^3*b^2 - 2*a*b^4)*cos
(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27717, size = 194, normalized size = 1.32

$$\frac{3\,b^5\tan\left(dx+c\right)^4+20\,ab^4\tan\left(dx+c\right)^3+60\,a^2b^3\tan\left(dx+c\right)^2-6\,b^5\tan\left(dx+c\right)^2+120\,a^3b^2\tan\left(dx+c\right)-60\,ab^4\tan}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^5*tan(d*x + c)^4 + 20*a*b^4*tan(d*x + c)^3 + 60*a^2*b^3*tan(d*x +
 c)^2 - 6*b^5*tan(d*x + c)^2 + 120*a^3*b^2*tan(d*x + c) - 60*a*b^4*tan(d*x
+ c) + 12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(d*x + c) + 6*(5*a^4*b - 10*a^2*b^3
+ b^5)*log(tan(d*x + c)^2 + 1))/d
```

3.103 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=224

$$\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{10a^2b^3\sec(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{5a^4b\sec(c+d}{d}$$

```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/d - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/d + (15*a
*b^4*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*b*Sec[c + d*x])/d - (10*a^2*b^3*
Sec[c + d*x])/d + (b^5*Sec[c + d*x])/d + (10*a^2*b^3*Sec[c + d*x]^3)/(3*d)
- (2*b^5*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^5)/(5*d) + (5*a^3*b^2*Se
c[c + d*x]*Tan[c + d*x])/d - (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (
5*a*b^4*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.233113, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3090, 3770, 2606, 8, 2611, 194}

$$\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{10a^2b^3\sec(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{5a^4b\sec(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```



```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/d - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/d + (15*a
*b^4*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*b*Sec[c + d*x])/d - (10*a^2*b^3*
Sec[c + d*x])/d + (b^5*Sec[c + d*x])/d + (10*a^2*b^3*Sec[c + d*x]^3)/(3*d)
- (2*b^5*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^5)/(5*d) + (5*a^3*b^2*Se
c[c + d*x]*Tan[c + d*x])/d - (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (
5*a*b^4*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= \int \left(a^5\sec(c+dx)+5a^4b\sec(c+dx)\tan(c+dx)+10a^3b^2\sec(c+\right.\\ &= a^5\int \sec(c+dx)\,dx+\left(5a^4b\right)\int \sec(c+dx)\tan(c+dx)\,dx+\left(10a^3\right.\\ &= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3b^2\sec(c+dx)\tan(c+dx)}{d}+\frac{5ab^4\sec}{}\\ &= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4b\sec(c}{d}\\ &= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{15ab^4\tanh}{}\end{aligned}$$

Mathematica [B] time = 6.32179, size = 1219, normalized size = 5.44

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b*(600*a^4 - 1000*a^2*b^2 + 89*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)
/(120*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 40*a^3*b^2 - 15*a
*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c
+ d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 40*a^3*b^2
 + 15*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b
*Tan[c + d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((25*a*b^4 +
2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(80*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((600*a^3*b^2 + 200
*a^2*b^3 - 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(240*
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])
^5) + (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(20*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
- (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(20*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((
-25*a*b^4 + 2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(80*d*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-600*
a^3*b^2 + 200*a^2*b^3 + 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d
*x])^5)/(240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-600*a^4*b*Sin[(c + d*x)/2] + 1000*a^2*
b^3*Sin[(c + d*x)/2] - 89*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(12
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])
^5) + (Cos[c + d*x]^5*(200*a^2*b^3*Sin[(c + d*x)/2] - 31*b^5*Sin[(c + d*x)/
2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*
(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-200*a^2*b^3*Sin[(c
 + d*x)/2] + 31*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (C
os[c + d*x]^5*(600*a^4*b*Sin[(c + d*x)/2] - 1000*a^2*b^3*Sin[(c + d*x)/2] +
 89*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
```

Maple [B] time = 0.26, size = 440, normalized size = 2.

$$\frac{a^5\ln(\sec(dx+c)+\tan(dx+c))}{d}+5\,\frac{a^4b}{d\cos(dx+c)}+5\,\frac{a^3b^2(\sin(dx+c))^3}{d(\cos(dx+c))^2}+5\,\frac{a^3b^2\sin(dx+c)}{d}-5\,\frac{a^3b^2\ln(\sec(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*a^5*ln(sec(d*x+c)+tan(d*x+c))+5/d*a^4*b/cos(d*x+c)+5/d*a^3*b^2*sin(d*x+
c)^3/cos(d*x+c)^2+5*a^3*b^2*sin(d*x+c)/d-5/d*a^3*b^2*ln(sec(d*x+c)+tan(d*x+
c))+10/3/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3-10/3/d*a^2*b^3*sin(d*x+c)^4/co
s(d*x+c)-10/3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3-20/3*a^2*b^3*cos(d*x+c)/d+5
/4/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^4-5/8/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^2-
5/8*a*b^4*sin(d*x+c)^3/d-15/8*a*b^4*sin(d*x+c)/d+15/8/d*a*b^4*ln(sec(d*x+c)
+tan(d*x+c))+1/5/d*b^5*sin(d*x+c)^6/cos(d*x+c)^5-1/15/d*b^5*sin(d*x+c)^6/co
s(d*x+c)^3+1/5/d*b^5*sin(d*x+c)^6/cos(d*x+c)+8/15*b^5*cos(d*x+c)/d+1/5/d*b^
5*cos(d*x+c)*sin(d*x+c)^4+4/15/d*cos(d*x+c)*sin(d*x+c)^2*b^5
```

Maxima [A] time = 1.13435, size = 311, normalized size = 1.39

$$75\,ab^4\left(\frac{2\,\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-600\,a^3b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(d\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/240*(75*a*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*
sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) -
600*a^3*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 120*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 1200*a^4*b/cos(d*x + c) - 800*(3*cos(d*x + c)^2 - 1)*a^2*b^3/cos(d*
x + c)^3 + 16*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*b^5/cos(d*x + c)^
5)/d
```

Fricas [A] time = 0.571347, size = 478, normalized size = 2.13

$$15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c) +
 1) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) +
 1) + 48*b^5 + 240*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 160*(5*a^2
*b^3 - b^5)*cos(d*x + c)^2 + 150*(2*a*b^4*cos(d*x + c) + (8*a^3*b^2 - 5*a*b
^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34456, size = 554, normalized size = 2.47

$$15\left(8a^5 - 40a^3b^2 + 15ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 15\left(8a^5 - 40a^3b^2 + 15ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right) + \frac{2(600}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
 2*(600*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*a*b^4*tan(1/2*d*x + 1/2*c)^9 -
 600*a^4*b*tan(1/2*d*x + 1/2*c)^8 - 1200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 + 1
050*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 2400
*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 3600*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 5600*
a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 640*b^5*tan(1/2*d*x + 1/2*c)^4 + 1200*a^3*
b^2*tan(1/2*d*x + 1/2*c)^3 - 1050*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2400*a^4*b
*tan(1/2*d*x + 1/2*c)^2 - 4000*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 320*b^5*tan
(1/2*d*x + 1/2*c)^2 - 600*a^3*b^2*tan(1/2*d*x + 1/2*c) + 225*a*b^4*tan(1/2*
d*x + 1/2*c) - 600*a^4*b + 800*a^2*b^3 - 64*b^5)/(tan(1/2*d*x + 1/2*c)^2 -
1)^5)/d
```

3.104 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=30

$$\frac{\tan^6(c+dx)(a\cot(c+dx)+b)^6}{6bd}$$

```
[Out] ((b + a*Cot[c + d*x])^6*Tan[c + d*x]^6)/(6*b*d)
```

Rubi [A] time = 0.0477584, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$\frac{\tan^6(c+dx)(a\cot(c+dx)+b)^6}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] ((b + a*Cot[c + d*x])^6*Tan[c + d*x]^6)/(6*b*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x^7}\,dx, x, \cot(c+dx)\right)}{d} = \frac{(b+a\cot(c+dx))^6\tan^6(c+dx)}{6bd}$$

Mathematica [B] time = 0.479837, size = 89, normalized size = 2.97

$$\frac{\tan(c+dx)\left(20a^3b^2\tan^2(c+dx)+15a^2b^3\tan^3(c+dx)+15a^4b\tan(c+dx)+6a^5+6ab^4\tan^4(c+dx)+b^5\tan^5(c+dx\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (Tan[c + d*x]*(6*a^5 + 15*a^4*b*Tan[c + d*x] + 20*a^3*b^2*Tan[c + d*x]^2 +
15*a^2*b^3*Tan[c + d*x]^3 + 6*a*b^4*Tan[c + d*x]^4 + b^5*Tan[c + d*x]^5))/(
6*d)
```

Maple [B] time = 0.257, size = 120, normalized size = 4.

$$\frac{1}{d}\left(a^5\tan(dx+c)+\frac{5\,a^4b}{2\,(\cos(dx+c))^2}+\frac{10\,a^3b^2(\sin(dx+c))^3}{3\,(\cos(dx+c))^3}+\frac{5\,a^2b^3(\sin(dx+c))^4}{2\,(\cos(dx+c))^4}+\frac{ab^4(\sin(dx+c))^5}{(\cos(dx+c))^5}+\frac{b^5(\sin(d}{6\,(\cos(d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(a^5*tan(d*x+c)+5/2*a^4*b/cos(d*x+c)^2+10/3*a^3*b^2*sin(d*x+c)^3/cos(d*
x+c)^3+5/2*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^4+a*b^4*sin(d*x+c)^5/cos(d*x+c)^
5+1/6*b^5*sin(d*x+c)^6/cos(d*x+c)^6)
```

Maxima [B] time = 1.20596, size = 224, normalized size = 7.47

$$\frac{6\,ab^4\tan(dx+c)^5+20\,a^3b^2\tan(dx+c)^3+6\,a^5\tan(dx+c)+\frac{15\left(2\sin(dx+c)^2-1\right)a^2b^3}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-\frac{\left(3\sin(dx+c)^4-3\sin(dx+c)^2+1\right)b^5}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*b^4*tan(d*x + c)^5 + 20*a^3*b^2*tan(d*x + c)^3 + 6*a^5*tan(d*x + c
) + 15*(2*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1) - (3*sin(d*x + c)^4 - 3*sin(d*x + c)^2 + 1)*b^5/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*a^4*b/(sin(d*x + c)^2 - 1))/d
```

Fricas [B] time = 0.507109, size = 329, normalized size = 10.97

$$\frac{b^5 + 3\left(5\,a^4b - 10\,a^2b^3 + b^5\right)\cos\left(dx+c\right)^4 + 3\left(5\,a^2b^3 - b^5\right)\cos\left(dx+c\right)^2 + 2\left(3\,ab^4\cos\left(dx+c\right) + \left(3\,a^5 - 10\,a^3b^2 + 3\,ab\right.\right.}{6\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5 + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 3*(5*a^2*b^3 - b
^5)*cos(d*x + c)^2 + 2*(3*a*b^4*cos(d*x + c) + (3*a^5 - 10*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^5 + 2*(5*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(
d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31267, size = 120, normalized size = 4.

$$\frac{b^5\tan\left(dx+c\right)^6 + 6\,ab^4\tan\left(dx+c\right)^5 + 15\,a^2b^3\tan\left(dx+c\right)^4 + 20\,a^3b^2\tan\left(dx+c\right)^3 + 15\,a^4b\tan\left(dx+c\right)^2 + 6\,a^5\tan\left(\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*tan(d*x + c)^6 + 6*a*b^4*tan(d*x + c)^5 + 15*a^2*b^3*tan(d*x + c)^
4 + 20*a^3*b^2*tan(d*x + c)^3 + 15*a^4*b*tan(d*x + c)^2 + 6*a^5*tan(d*x + c
))/d
```

3.105 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=318

$$\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5a^3b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{5a^3b^2\tan(c$$

```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (5*a*b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (5*a^4*b*Sec[c + d*x]^3)/(3*d)
- (10*a^2*b^3*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^3)/(3*d) + (2*a^2*b
^3*Sec[c + d*x]^5)/d - (2*b^5*Sec[c + d*x]^5)/(5*d) + (b^5*Sec[c + d*x]^7)/
(7*d) + (a^5*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(4*d) + (5*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (5*a^3*b^2*
Sec[c + d*x]^3*Tan[c + d*x])/(2*d) - (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/
(8*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.336903, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14, 270}

$$\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5a^3b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{5a^3b^2\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (5*a*b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (5*a^4*b*Sec[c + d*x]^3)/(3*d)
- (10*a^2*b^3*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^3)/(3*d) + (2*a^2*b
^3*Sec[c + d*x]^5)/d - (2*b^5*Sec[c + d*x]^5)/(5*d) + (b^5*Sec[c + d*x]^7)/
(7*d) + (a^5*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(4*d) + (5*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (5*a^3*b^2*
Sec[c + d*x]^3*Tan[c + d*x])/(2*d) - (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/
(8*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
```

```
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^3(c+dx)+5a^4b\sec^3(c+dx)\tan(c+dx)+10a^3b^2\sec^3(c$$

$$= a^5\int \sec^3(c+dx)\,dx+\left(5a^4b\right)\int \sec^3(c+dx)\tan(c+dx)\,dx+\left(10\right.$$

$$= \frac{a^5\sec(c+dx)\tan(c+dx)}{2d}+\frac{5a^3b^2\sec^3(c+dx)\tan(c+dx)}{2d}+\frac{5ab}{}$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^4b\sec^3(c+dx)}{3d}+\frac{a^5\sec(c+dx)\tan(c}{2d}$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5a^4b\sec^3(c}{3d}$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5ab^4\tanh^{-}}{1}$$

Mathematica [B] time = 6.3304, size = 1677, normalized size = 5.27

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b*(1400*a^4 - 1540*a^2*b^2 + 103*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^
5)/(1680*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 20*a^3*b^2 - 5
*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[
c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 20*a^3*
b^2 + 5*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a +
 b*Tan[c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((35*a*b^4
 + 3*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((350*a^3*b^2 +
 140*a^2*b^3 - 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(
560*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d
*x])^5) + ((840*a^5 + 1400*a^4*b - 2100*a^3*b^2 - 1540*a^2*b^3 + 525*a*b^4
+ 103*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(3360*d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*Cos[c +
d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*Cos[c + d*x]
^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c + d*x)/2] + Sin[(
```

```
c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-35*a*b^4 + 3*b^5)*
Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d*x)/2] + Sin[(c +
d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-350*a^3*b^2 + 140*a^2*
b^3 + 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(560*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) +
 ((-840*a^5 + 1400*a^4*b + 2100*a^3*b^2 - 1540*a^2*b^3 - 525*a*b^4 + 103*b^
5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(3360*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-140
0*a^4*b*Sin[(c + d*x)/2] + 1540*a^2*b^3*Sin[(c + d*x)/2] - 103*b^5*Sin[(c +
 d*x)/2])*(a + b*Tan[c + d*x])^5)/(1680*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-1400*a^4*b*
Sin[(c + d*x)/2] + 1540*a^2*b^3*Sin[(c + d*x)/2] - 103*b^5*Sin[(c + d*x)/2]
)*(a + b*Tan[c + d*x])^5)/(1680*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*
Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(70*a^2*b^3*Sin[(c + d*
x)/2] - 9*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(140*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c +
 d*x]^5*(-70*a^2*b^3*Sin[(c + d*x)/2] + 9*b^5*Sin[(c + d*x)/2])*(a + b*Tan[
c + d*x])^5)/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(1400*a^4*b*Sin[(c + d*x)/2] - 1540
*a^2*b^3*Sin[(c + d*x)/2] + 103*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^
5)/(1680*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[
c + d*x])^5) + (Cos[c + d*x]^5*(1400*a^4*b*Sin[(c + d*x)/2] - 1540*a^2*b^3*
Sin[(c + d*x)/2] + 103*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(1680*
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5
)
```

Maple [A] time = 0.266, size = 564, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] -4/3*a^2*b^3*cos(d*x+c)/d-5/16*a*b^4*sin(d*x+c)/d-5/48*a*b^4*sin(d*x+c)^3/d
+8/105*b^5*cos(d*x+c)/d+2/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^5+5/4*a^3*b^2*s
in(d*x+c)/d+1/2*a^5*sec(d*x+c)*tan(d*x+c)/d-5/4/d*a^3*b^2*ln(sec(d*x+c)+tan
(d*x+c))-1/105/d*b^5*sin(d*x+c)^6/cos(d*x+c)^3+5/3/d*a^4*b/cos(d*x+c)^3+1/7
/d*b^5*sin(d*x+c)^6/cos(d*x+c)^7-2/3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b^3+5/6/
d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^6+5/2/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^4+1
/2/d*a^5*ln(sec(d*x+c)+tan(d*x+c))+5/16/d*a*b^4*ln(sec(d*x+c)+tan(d*x+c))+1
/35/d*b^5*sin(d*x+c)^6/cos(d*x+c)+1/35/d*b^5*cos(d*x+c)*sin(d*x+c)^4+4/105/
```

```
d*cos(d*x+c)*sin(d*x+c)^2*b^5+1/35/d*b^5*sin(d*x+c)^6/cos(d*x+c)^5-2/3/d*a^
2*b^3*sin(d*x+c)^4/cos(d*x+c)+2/3/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3+5/24/
d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^4+5/4/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^2-5
/48/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^2
```

Maxima [A] time = 1.18801, size = 390, normalized size = 1.23

$$-\frac{175\,ab^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1} - 3\,\log\left(\sin\left(dx+c\right)+1\right) + 3\,\log\left(\sin\left(dx+c\right)-1\right)\right) - 2100\,a^3b^2\left(\frac{2\left(\sin(dx+\ldots\right.}{\sin(dx+c)^4-\ldots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/3360*(175*a*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 2100*a^3*b^2*(2*(sin(d*x + c)^3 +
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1)) + 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 5600*a^4*b/cos(d*x + c)^
3 + 2240*(5*cos(d*x + c)^2 - 3)*a^2*b^3/cos(d*x + c)^5 - 32*(35*cos(d*x + c
)^4 - 42*cos(d*x + c)^2 + 15)*b^5/cos(d*x + c)^7)/d
```

Fricas [A] time = 0.572712, size = 556, normalized size = 1.75

$$\frac{105\left(8\,a^5-20\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right) - 105\left(8\,a^5-20\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^7\log\left(-\sin\left(dx\ldots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3360*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(sin(d*x + c)
+ 1) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(-sin(d*x + c)
+ 1) + 480*b^5 + 1120*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 1344*(5
*a^2*b^3 - b^5)*cos(d*x + c)^2 + 70*(40*a*b^4*cos(d*x + c) + 3*(8*a^5 - 20*
a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(12*a^3*b^2 - 7*a*b^4)*cos(d*x + c)^
3)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31792, size = 918, normalized size = 2.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1680*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
+ 2*(840*a^5*tan(1/2*d*x + 1/2*c)^13 + 2100*a^3*b^2*tan(1/2*d*x + 1/2*c)^13
 - 525*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 8400*a^4*b*tan(1/2*d*x + 1/2*c)^12 -
 3360*a^5*tan(1/2*d*x + 1/2*c)^11 + 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 +
3500*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b*tan(1/2*d*x + 1/2*c)^10 -
33600*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 + 4200*a^5*tan(1/2*d*x + 1/2*c)^9 - 2
3100*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 + 16975*a*b^4*tan(1/2*d*x + 1/2*c)^9 -
53200*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 56000*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 -
 8960*b^5*tan(1/2*d*x + 1/2*c)^8 + 44800*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 224
00*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 4480*b^5*tan(1/2*d*x + 1/2*c)^6 - 4200*
a^5*tan(1/2*d*x + 1/2*c)^5 + 23100*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 16975*a
*b^4*tan(1/2*d*x + 1/2*c)^5 - 25200*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 13440*a^
2*b^3*tan(1/2*d*x + 1/2*c)^4 - 2688*b^5*tan(1/2*d*x + 1/2*c)^4 + 3360*a^5*t
an(1/2*d*x + 1/2*c)^3 - 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 11200*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 15680*a^2*b^3*t
an(1/2*d*x + 1/2*c)^2 + 896*b^5*tan(1/2*d*x + 1/2*c)^2 - 840*a^5*tan(1/2*d*
x + 1/2*c) - 2100*a^3*b^2*tan(1/2*d*x + 1/2*c) + 525*a*b^4*tan(1/2*d*x + 1/
2*c) - 2800*a^4*b + 2240*a^2*b^3 - 128*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)
/d
```

3.106 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=177

$$\frac{b^3\left(10a^2+b^2\right)\tan^6(c+dx)}{6d}+\frac{ab^2\left(2a^2+b^2\right)\tan^5(c+dx)}{d}+\frac{5a^2b\left(a^2+2b^2\right)\tan^4(c+dx)}{4d}+\frac{a^3\left(a^2+10b^2\right)\tan^3(c+dx)}{3d}$$

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (a^3*(a^2 + 10*b^2)
*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + 2*b^2)*Tan[c + d*x]^4)/(4*d) + (a*
b^2*(2*a^2 + b^2)*Tan[c + d*x]^5)/d + (b^3*(10*a^2 + b^2)*Tan[c + d*x]^6)/(
6*d) + (5*a*b^4*Tan[c + d*x]^7)/(7*d) + (b^5*Tan[c + d*x]^8)/(8*d)
```

Rubi [A] time = 0.151652, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{b^3\left(10a^2+b^2\right)\tan^6(c+dx)}{6d}+\frac{ab^2\left(2a^2+b^2\right)\tan^5(c+dx)}{d}+\frac{5a^2b\left(a^2+2b^2\right)\tan^4(c+dx)}{4d}+\frac{a^3\left(a^2+10b^2\right)\tan^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (a^3*(a^2 + 10*b^2)
*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + 2*b^2)*Tan[c + d*x]^4)/(4*d) + (a*
b^2*(2*a^2 + b^2)*Tan[c + d*x]^5)/d + (b^3*(10*a^2 + b^2)*Tan[c + d*x]^6)/(
6*d) + (5*a*b^4*Tan[c + d*x]^7)/(7*d) + (b^5*Tan[c + d*x]^8)/(8*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5(1+x^2)}{x^9}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^5}{x^9}+\frac{5ab^4}{x^8}+\frac{10a^2b^3+b^5}{x^7}+\frac{5ab^2(2a^2+b^2)}{x^6}+\frac{5a^2b(a^2+2b^2)}{x^5}+\frac{a^5+10}{x^4}\right.\right.}{d}$$
$$= \frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{a^3\left(a^2+10b^2\right)\tan^3(c+dx)}{3d}+$$

Mathematica [A] time = 0.443175, size = 54, normalized size = 0.31

$$\frac{(a+b\tan(c+dx))^6\left(a^2-6ab\tan(c+dx)+21b^2\tan^2(c+dx)+28b^2\right)}{168b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] ((a + b*Tan[c + d*x])^6*(a^2 + 28*b^2 - 6*a*b*Tan[c + d*x] + 21*b^2*Tan[c +
 d*x]^2))/(168*b^3*d)
```

Maple [A] time = 0.267, size = 217, normalized size = 1.2

$$\frac{1}{d}\left(-a^5\left(-\frac{2}{3}-\frac{(\sec(dx+c))^2}{3}\right)\tan(dx+c)+\frac{5a^4b}{4(\cos(dx+c))^4}+10a^3b^2\left(1/5\frac{(\sin(dx+c))^3}{(\cos(dx+c))^5}+2/15\frac{(\sin(dx+c))^3}{(\cos(dx+c))^3}\right)+10\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-a^5*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+5/4*a^4*b/cos(d*x+c)^4+10*a^3*
b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+10*a^2*b
^3*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+5*a*b^4*(
1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)+b^5*(1/8*sin(
```

```
d*x+c)^6/cos(d*x+c)^8+1/24*sin(d*x+c)^6/cos(d*x+c)^6))
```

Maxima [A] time = 1.28562, size = 301, normalized size = 1.7

$$\frac{56\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)a^{5}+112\left(3\,\tan\left(dx+c\right)^{5}+5\,\tan\left(dx+c\right)^{3}\right)a^{3}b^{2}+24\left(5\,\tan\left(dx+c\right)^{7}+7\,\tan\left(dx+\right.\right.}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/168*(56*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 + 112*(3*tan(d*x + c)^5 + 5
*tan(d*x + c)^3)*a^3*b^2 + 24*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*a*b^4 -
 140*(3*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 7*(6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1)*b^5/(sin(
d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) +
210*a^4*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A] time = 0.546022, size = 408, normalized size = 2.31

$$\frac{21\,b^{5}+42\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{4}+56\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{2}+8\left(2\left(7\,a^{5}-14\,a^{3}b^{2}+3\,ab^{4}\right)\cos\left(dx+c\right.\right.}{168\,d\cos\left(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^5 + 42*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 56*(5*a^2*
b^3 - b^5)*cos(d*x + c)^2 + 8*(2*(7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x + c
)^7 + 15*a*b^4*cos(d*x + c) + (7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^5
 + 6*(7*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33172, size = 238, normalized size = 1.34

$$\frac{21\,b^5\tan\left(dx+c\right)^8+120\,ab^4\tan\left(dx+c\right)^7+280\,a^2b^3\tan\left(dx+c\right)^6+28\,b^5\tan\left(dx+c\right)^6+336\,a^3b^2\tan\left(dx+c\right)^5+168\,a\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^5*tan(d*x + c)^8 + 120*a*b^4*tan(d*x + c)^7 + 280*a^2*b^3*tan(d
*x + c)^6 + 28*b^5*tan(d*x + c)^6 + 336*a^3*b^2*tan(d*x + c)^5 + 168*a*b^4*
tan(d*x + c)^5 + 210*a^4*b*tan(d*x + c)^4 + 420*a^2*b^3*tan(d*x + c)^4 + 56
*a^5*tan(d*x + c)^3 + 560*a^3*b^2*tan(d*x + c)^3 + 420*a^4*b*tan(d*x + c)^2
 + 168*a^5*tan(d*x + c))/d
```

3.107 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=391

$$\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^3b^2\tan(c+dx)\sec^5(c+dx)}{3d}-\frac{5a^3b^2\tan(c$$

```
[Out] (3*a^5*ArcTanh[Sin[c + d*x]])/(8*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (a^4*b*Sec[c + d*x]^5)/d -
(2*a^2*b^3*Sec[c + d*x]^5)/d + (b^5*Sec[c + d*x]^5)/(5*d) + (10*a^2*b^3*Sec
[c + d*x]^7)/(7*d) - (2*b^5*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^9)/(9
*d) + (3*a^5*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(8*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^5*Sec[
c + d*x]^3*Tan[c + d*x])/(4*d) - (5*a^3*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(1
2*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (5*a^3*b^2*Sec[c + d*
x]^5*Tan[c + d*x])/(3*d) - (5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (
5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*d)
```

Rubi [A] time = 0.389182, antiderivative size = 391, normalized size of antiderivative = 1., number of steps used = 22, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14, 270}

$$\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^3b^2\tan(c+dx)\sec^5(c+dx)}{3d}-\frac{5a^3b^2\tan(c$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```


```
[Out] (3*a^5*ArcTanh[Sin[c + d*x]])/(8*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (a^4*b*Sec[c + d*x]^5)/d -
(2*a^2*b^3*Sec[c + d*x]^5)/d + (b^5*Sec[c + d*x]^5)/(5*d) + (10*a^2*b^3*Sec
[c + d*x]^7)/(7*d) - (2*b^5*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^9)/(9
*d) + (3*a^5*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(8*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^5*Sec[
c + d*x]^3*Tan[c + d*x])/(4*d) - (5*a^3*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(1
2*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (5*a^3*b^2*Sec[c + d*
x]^5*Tan[c + d*x])/(3*d) - (5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (
5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*d)
```


Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^5(c+dx)+5a^4b\sec^5(c+dx)\tan(c+dx)+10a^3b^2\sec^5(\right.$$
$$= a^5\int \sec^5(c+dx)\,dx+\left(5a^4b\right)\int \sec^5(c+dx)\tan(c+dx)\,dx+\left(10\right.$$
$$= \frac{a^5\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{5a^3b^2\sec^5(c+dx)\tan(c+dx)}{3d}+\frac{5a}{}$$
$$= \frac{a^4b\sec^5(c+dx)}{d}+\frac{3a^5\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^5\sec^3(c+dx)\tan}{4d}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4b\sec^5(c+dx)}{d}-\frac{2a^2b^3\sec^5(c+dx)}{d}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4b\sec^5(}{d}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{15ab^4\tan}{}$$

Mathematica [A] time = 2.21531, size = 331, normalized size = 0.85

$$\frac{1260a\left(2320a^2b^2+656a^4+845b^4\right)\tan(c+dx)\sec^7(c+dx)-40320a\left(-80a^2b^2+48a^4+15b^4\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-40320*a*(48*a^4 - 80*a^2*b^2 + 15*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^9*(19353
60*a^4*b - 184320*a^2*b^3 + 223232*b^5 + 73728*(35*a^4*b - 20*a^2*b^3 - 3*b
^5)*Cos[2*(c + d*x)] + 129024*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c + d*x)]
 + 372960*a^5*Sin[4*(c + d*x)] + 453600*a^3*b^2*Sin[4*(c + d*x)] - 488250*a
*b^4*Sin[4*(c + d*x)] + 131040*a^5*Sin[6*(c + d*x)] - 218400*a^3*b^2*Sin[6*
(c + d*x)] + 40950*a*b^4*Sin[6*(c + d*x)] + 15120*a^5*Sin[8*(c + d*x)] - 25
200*a^3*b^2*Sin[8*(c + d*x)] + 4725*a*b^4*Sin[8*(c + d*x)]) + 1260*a*(656*a
^4 + 2320*a^2*b^2 + 845*b^4)*Sec[c + d*x]^7*Tan[c + d*x])/(5160960*d)
```

Maple [A] time = 0.266, size = 688, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] -4/7*a^2*b^3*cos(d*x+c)/d-15/128*a*b^4*sin(d*x+c)/d-5/128*a*b^4*sin(d*x+c)^
3/d+8/315*b^5*cos(d*x+c)/d+5/8/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^8+10/7/d*a^2
*b^3*sin(d*x+c)^4/cos(d*x+c)^7+6/7/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^5+5/8*
a^3*b^2*sin(d*x+c)/d+3/8*a^5*sec(d*x+c)*tan(d*x+c)/d+1/4*a^5*sec(d*x+c)^3*t
an(d*x+c)/d-5/8/d*a^3*b^2*ln(sec(d*x+c)+tan(d*x+c))-1/315/d*b^5*sin(d*x+c)^
6/cos(d*x+c)^3+1/21/d*b^5*sin(d*x+c)^6/cos(d*x+c)^7-2/7/d*cos(d*x+c)*sin(d*
x+c)^2*a^2*b^3+5/3/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^6+5/16/d*a*b^4*sin(d*x
+c)^5/cos(d*x+c)^6+5/4/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^4+3/8/d*a^5*ln(sec
(d*x+c)+tan(d*x+c))+1/9/d*b^5*sin(d*x+c)^6/cos(d*x+c)^9+1/d*a^4*b/cos(d*x+c
)^5+15/128/d*a*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/105/d*b^5*sin(d*x+c)^6/cos(d
*x+c)+1/105/d*b^5*cos(d*x+c)*sin(d*x+c)^4+4/315/d*cos(d*x+c)*sin(d*x+c)^2*b
^5+1/105/d*b^5*sin(d*x+c)^6/cos(d*x+c)^5-2/7/d*a^2*b^3*sin(d*x+c)^4/cos(d*x
+c)+2/7/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3+5/64/d*a*b^4*sin(d*x+c)^5/cos(d
*x+c)^4+5/8/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^2-5/128/d*a*b^4*sin(d*x+c)^5/
cos(d*x+c)^2
```

Maxima [A] time = 1.22289, size = 486, normalized size = 1.24

$$-\frac{1575\,ab^4\left(\frac{2\,\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-8400\,a^3\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] -1/80640*(1575*a*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x
+ c)^3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c
)^4 - 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1)) - 8400*a^3*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
```

```
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 5040*a^5*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 80640*a^4*b/cos(d*x + c)^5 + 23040*(7*c
os(d*x + c)^2 - 5)*a^2*b^3/cos(d*x + c)^7 - 256*(63*cos(d*x + c)^4 - 90*cos
(d*x + c)^2 + 35)*b^5/cos(d*x + c)^9)/d
```

Fricas [A] time = 0.628745, size = 643, normalized size = 1.64

$$\frac{315\left(48\,a^5-80\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^9\log\left(\sin\left(dx+c\right)+1\right)-315\left(48\,a^5-80\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^9\log\left(-\mathrm{si}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/80640*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(sin(d*x +
c) + 1) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(-sin(d*x
+ c) + 1) + 8960*b^5 + 16128*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 +
23040*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 210*(3*(48*a^5 - 80*a^3*b^2 + 15*a
*b^4)*cos(d*x + c)^7 + 240*a*b^4*cos(d*x + c) + 2*(48*a^5 - 80*a^3*b^2 + 15
*a*b^4)*cos(d*x + c)^5 + 40*(16*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^3)*sin(d*x
+ c))/(d*cos(d*x + c)^9)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33802, size = 1199, normalized size = 3.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/40320*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
+ 1)) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) + 2*(25200*a^5*tan(1/2*d*x + 1/2*c)^17 + 25200*a^3*b^2*tan(1/2*d*x + 1
/2*c)^17 - 4725*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 201600*a^4*b*tan(1/2*d*x +
1/2*c)^16 - 110880*a^5*tan(1/2*d*x + 1/2*c)^15 + 319200*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^15 + 40950*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 806400*a^4*b*tan(1/2*d
*x + 1/2*c)^14 - 806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 + 191520*a^5*tan(1/
2*d*x + 1/2*c)^13 - 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^13 + 488250*a*b^4*t
an(1/2*d*x + 1/2*c)^13 - 1612800*a^4*b*tan(1/2*d*x + 1/2*c)^12 + 806400*a^2
*b^3*tan(1/2*d*x + 1/2*c)^12 - 215040*b^5*tan(1/2*d*x + 1/2*c)^12 - 151200*
a^5*tan(1/2*d*x + 1/2*c)^11 - 151200*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 + 5323
50*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2419200*a^4*b*tan(1/2*d*x + 1/2*c)^10 -
806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 322560*b^5*tan(1/2*d*x + 1/2*c)^10
 - 2661120*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 2096640*a^2*b^3*tan(1/2*d*x + 1/2
*c)^8 - 451584*b^5*tan(1/2*d*x + 1/2*c)^8 + 151200*a^5*tan(1/2*d*x + 1/2*c)
^7 + 151200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 532350*a*b^4*tan(1/2*d*x + 1/2
*c)^7 + 1774080*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 1128960*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^6 - 129024*b^5*tan(1/2*d*x + 1/2*c)^6 - 191520*a^5*tan(1/2*d*x + 1
/2*c)^5 + 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 488250*a*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 645120*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 23040*a^2*b^3*tan(1/2*d*
x + 1/2*c)^4 - 36864*b^5*tan(1/2*d*x + 1/2*c)^4 + 110880*a^5*tan(1/2*d*x +
1/2*c)^3 - 319200*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 40950*a*b^4*tan(1/2*d*x
+ 1/2*c)^3 + 161280*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 207360*a^2*b^3*tan(1/2*d
*x + 1/2*c)^2 + 9216*b^5*tan(1/2*d*x + 1/2*c)^2 - 25200*a^5*tan(1/2*d*x + 1
/2*c) - 25200*a^3*b^2*tan(1/2*d*x + 1/2*c) + 4725*a*b^4*tan(1/2*d*x + 1/2*c
) - 40320*a^4*b + 23040*a^2*b^3 - 1024*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^9)
/d
```

3.108 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=242

$$\frac{b^3\left(5a^2+b^2\right)\tan^8(c+dx)}{4d}+\frac{10ab^2\left(a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{b\left(20a^2b^2+5a^4+b^4\right)\tan^6(c+dx)}{6d}+\frac{a\left(20a^2b^2+a^4+5b^4\right)}{5d}$$

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (2*a^3*(a^2 + 5*b^2
)*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + b^2)*Tan[c + d*x]^4)/(2*d) + (a*(
a^4 + 20*a^2*b^2 + 5*b^4)*Tan[c + d*x]^5)/(5*d) + (b*(5*a^4 + 20*a^2*b^2 +
b^4)*Tan[c + d*x]^6)/(6*d) + (10*a*b^2*(a^2 + b^2)*Tan[c + d*x]^7)/(7*d) +
(b^3*(5*a^2 + b^2)*Tan[c + d*x]^8)/(4*d) + (5*a*b^4*Tan[c + d*x]^9)/(9*d) +
 (b^5*Tan[c + d*x]^10)/(10*d)
```

Rubi [A] time = 0.221595, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 948}

$$\frac{b^3\left(5a^2+b^2\right)\tan^8(c+dx)}{4d}+\frac{10ab^2\left(a^2+b^2\right)\tan^7(c+dx)}{7d}+\frac{b\left(20a^2b^2+5a^4+b^4\right)\tan^6(c+dx)}{6d}+\frac{a\left(20a^2b^2+a^4+5b^4\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (2*a^3*(a^2 + 5*b^2
)*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + b^2)*Tan[c + d*x]^4)/(2*d) + (a*(
a^4 + 20*a^2*b^2 + 5*b^4)*Tan[c + d*x]^5)/(5*d) + (b*(5*a^4 + 20*a^2*b^2 +
b^4)*Tan[c + d*x]^6)/(6*d) + (10*a*b^2*(a^2 + b^2)*Tan[c + d*x]^7)/(7*d) +
(b^3*(5*a^2 + b^2)*Tan[c + d*x]^8)/(4*d) + (5*a*b^4*Tan[c + d*x]^9)/(9*d) +
 (b^5*Tan[c + d*x]^10)/(10*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5\left(1+x^2\right)^2}{x^{11}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^5}{x^{11}}+\frac{5ab^4}{x^{10}}+\frac{2\left(5a^2b^3+b^5\right)}{x^9}+\frac{10ab^2\left(a^2+b^2\right)}{x^8}+\frac{5a^4b+20a^2b^3+b^5}{x^7}+\right.\right.}{}$$

$$= \frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{2a^3\left(a^2+5b^2\right)\tan^3(c+dx)}{3d}$$

Mathematica [A] time = 1.20733, size = 115, normalized size = 0.48

$$\frac{\frac{1}{4}\left(3a^2+b^2\right)(a+b\tan(c+dx))^8-\frac{4}{7}a\left(a^2+b^2\right)(a+b\tan(c+dx))^7+\frac{1}{6}\left(a^2+b^2\right)^2(a+b\tan(c+dx))^6+\frac{1}{10}(a+b\tan(c}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^6)/6 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^7)/7 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^8)/4 - (4*a*(a + b*Tan[c
+ d*x])^9)/9 + (a + b*Tan[c + d*x])^10/10)/(b^5*d)
```

Maple [A] time = 0.256, size = 299, normalized size = 1.2

$$\frac{1}{d}\left(-a^5\left(-\frac{8}{15}-\frac{(\sec(dx+c))^4}{5}-\frac{4(\sec(dx+c))^2}{15}\right)\tan(dx+c)+\frac{5a^4b}{6(\cos(dx+c))^6}+10a^3b^2\left(1/7\frac{(\sin(dx+c))^3}{(\cos(dx+c))^7}+\frac{4(s}{35(}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] 1/d*(-a^5*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+5/6*a^4*b/c
os(d*x+c)^6+10*a^3*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos
(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+10*a^2*b^3*(1/8*sin(d*x+c)^4/cos
(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+5*
a*b^4*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*s
in(d*x+c)^5/cos(d*x+c)^5)+b^5*(1/10*sin(d*x+c)^6/cos(d*x+c)^10+1/20*sin(d*x
+c)^6/cos(d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6))
```

Maxima [A] time = 1.24709, size = 371, normalized size = 1.53

$$\frac{84\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{5}+120\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] 1/1260*(84*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^5 + 1
20*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^3*b^2 + 20
*(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*a*b^4 + 525*(4
*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x
 + c)^4 - 4*sin(d*x + c)^2 + 1) - 21*(10*sin(d*x + c)^4 - 5*sin(d*x + c)^2
+ 1)*b^5/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d
*x + c)^4 + 5*sin(d*x + c)^2 - 1) - 1050*a^4*b/(sin(d*x + c)^2 - 1)^3)/d
```

Fricas [A] time = 0.598797, size = 491, normalized size = 2.03

$$\frac{126\,b^{5}+210\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{4}+315\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{2}+4\left(8\left(21\,a^{5}-30\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/1260*(126*b^5 + 210*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 315*(5*
a^2*b^3 - b^5)*cos(d*x + c)^2 + 4*(8*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*
```

```
x + c)^9 + 4*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 175*a*b^4*cos
(d*x + c) + 3*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 50*(9*a^3*b^
2 - 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^10)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)

[Out] Timed out
```

Giac [A] time = 1.32152, size = 354, normalized size = 1.46

$$\frac{126\,b^5\tan(dx+c)^{10}+700\,ab^4\tan(dx+c)^9+1575\,a^2b^3\tan(dx+c)^8+315\,b^5\tan(dx+c)^8+1800\,a^3b^2\tan(dx+c)^7+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")

[Out] 1/1260*(126*b^5*tan(d*x + c)^10 + 700*a*b^4*tan(d*x + c)^9 + 1575*a^2*b^3*t
an(d*x + c)^8 + 315*b^5*tan(d*x + c)^8 + 1800*a^3*b^2*tan(d*x + c)^7 + 1800
*a*b^4*tan(d*x + c)^7 + 1050*a^4*b*tan(d*x + c)^6 + 4200*a^2*b^3*tan(d*x +
c)^6 + 210*b^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 5040*a^3*b^2*tan(d
*x + c)^5 + 1260*a*b^4*tan(d*x + c)^5 + 3150*a^4*b*tan(d*x + c)^4 + 3150*a^
2*b^3*tan(d*x + c)^4 + 840*a^5*tan(d*x + c)^3 + 4200*a^3*b^2*tan(d*x + c)^3
 + 3150*a^4*b*tan(d*x + c)^2 + 1260*a^5*tan(d*x + c))/d
```

3.109 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=472

$$\frac{10a^2b^3\sec^9(c+dx)}{9d}-\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{5a^3b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{5a^3b^2\tan}{}$$

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/(16*d) - (25*a^3*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (5*a^4*b*Sec[c + d*x]^7)
/(7*d) - (10*a^2*b^3*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^7)/(7*d) + (
10*a^2*b^3*Sec[c + d*x]^9)/(9*d) - (2*b^5*Sec[c + d*x]^9)/(9*d) + (b^5*Sec[
c + d*x]^11)/(11*d) + (5*a^5*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (25*a^3*b^
2*Sec[c + d*x]*Tan[c + d*x])/(64*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/
(256*d) + (5*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) - (25*a^3*b^2*Sec[c +
d*x]^3*Tan[c + d*x])/(96*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d)
 + (a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (5*a^3*b^2*Sec[c + d*x]^5*Tan[
c + d*x])/(24*d) + (a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(32*d) + (5*a^3*b^2*
Sec[c + d*x]^7*Tan[c + d*x])/(4*d) - (3*a*b^4*Sec[c + d*x]^7*Tan[c + d*x])/
(16*d) + (a*b^4*Sec[c + d*x]^7*Tan[c + d*x]^3)/(2*d)
```

Rubi [A] time = 0.466743, antiderivative size = 472, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3090, 3768, 3770, 2606, 30, 2611, 14, 270}

$$\frac{10a^2b^3\sec^9(c+dx)}{9d}-\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{5a^3b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{5a^3b^2\tan}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/(16*d) - (25*a^3*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (5*a^4*b*Sec[c + d*x]^7)
/(7*d) - (10*a^2*b^3*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^7)/(7*d) + (
10*a^2*b^3*Sec[c + d*x]^9)/(9*d) - (2*b^5*Sec[c + d*x]^9)/(9*d) + (b^5*Sec[
c + d*x]^11)/(11*d) + (5*a^5*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (25*a^3*b^
2*Sec[c + d*x]*Tan[c + d*x])/(64*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/
(256*d) + (5*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) - (25*a^3*b^2*Sec[c +
d*x]^3*Tan[c + d*x])/(96*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d)
 + (a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (5*a^3*b^2*Sec[c + d*x]^5*Tan[
c + d*x])/(24*d) + (a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(32*d) + (5*a^3*b^2*
Sec[c + d*x]^7*Tan[c + d*x])/(4*d) - (3*a*b^4*Sec[c + d*x]^7*Tan[c + d*x])/
```

```
(16*d) + (a*b^4*Sec[c + d*x]^7*Tan[c + d*x]^3)/(2*d)
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^7(c+dx)+5a^4b\sec^7(c+dx)\tan(c+dx)+10a^3b^2\sec^7(\right.$$

$$= a^5\int \sec^7(c+dx)\,dx+\left(5a^4b\right)\int \sec^7(c+dx)\tan(c+dx)\,dx+\left(1\right.$$

$$= \frac{a^5\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{5a^3b^2\sec^7(c+dx)\tan(c+dx)}{4d}+\frac{a}{}$$

$$= \frac{5a^4b\sec^7(c+dx)}{7d}+\frac{5a^5\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^5\sec^5(c+d}{6}$$

$$= \frac{5a^4b\sec^7(c+dx)}{7d}-\frac{10a^2b^3\sec^7(c+dx)}{7d}+\frac{b^5\sec^7(c+dx)}{7d}+\frac{10a^2}{}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^4b\sec^7(c+dx)}{7d}-\frac{10a^2b^3\sec^7(c+d}{7d}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{5a^4b\sec}{7}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{15ab^4\tan}{}$$

Mathematica [A] time = 1.85669, size = 374, normalized size = 0.79

$$\frac{13860a\left(2876a^2b^2+976a^4+1207b^4\right)\tan(c+dx)\sec^9(c+dx)-1774080a\left(-20a^2b^2+16a^4+3b^4\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-1774080*a*(16*a^4 - 20*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^11*(243
30240*a^4*b + 1802240*a^2*b^3 + 3031040*b^5 + 3604480*(9*a^4*b - 4*a^2*b^3
- b^5)*Cos[2*(c + d*x)] + 1622016*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c + d
```

```
*x)] + 6623232*a^5*Sin[4*(c + d*x)] + 5913600*a^3*b^2*Sin[4*(c + d*x)] - 65
64096*a*b^4*Sin[4*(c + d*x)] + 2857008*a^5*Sin[6*(c + d*x)] - 3571260*a^3*b
^2*Sin[6*(c + d*x)] + 535689*a*b^4*Sin[6*(c + d*x)] + 591360*a^5*Sin[8*(c +
 d*x)] - 739200*a^3*b^2*Sin[8*(c + d*x)] + 110880*a*b^4*Sin[8*(c + d*x)] +
55440*a^5*Sin[10*(c + d*x)] - 69300*a^3*b^2*Sin[10*(c + d*x)] + 10395*a*b^4
*Sin[10*(c + d*x)]) + 13860*a*(976*a^4 + 2876*a^2*b^2 + 1207*b^4)*Sec[c + d
*x]^9*Tan[c + d*x])/(90832896*d)
```

Maple [A] time = 0.259, size = 814, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x)
```

```
[Out] -20/63*a^2*b^3*cos(d*x+c)/d-15/256*a*b^4*sin(d*x+c)/d-5/256*a*b^4*sin(d*x+c
)^3/d+8/693*b^5*cos(d*x+c)/d+5/16/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^8+50/63/d
*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^7+10/9/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^9
+1/2/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)^10+5/4/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+
c)^8+10/21/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^5+25/64*a^3*b^2*sin(d*x+c)/d+5
/16*a^5*sec(d*x+c)*tan(d*x+c)/d+5/24*a^5*sec(d*x+c)^3*tan(d*x+c)/d+5/7/d*a^
4*b/cos(d*x+c)^7+1/11/d*b^5*sin(d*x+c)^6/cos(d*x+c)^11-25/64/d*a^3*b^2*ln(s
ec(d*x+c)+tan(d*x+c))-1/693/d*b^5*sin(d*x+c)^6/cos(d*x+c)^3+1/6*a^5*sec(d*x
+c)^5*tan(d*x+c)/d+5/231/d*b^5*sin(d*x+c)^6/cos(d*x+c)^7-10/63/d*cos(d*x+c)
*sin(d*x+c)^2*a^2*b^3+25/24/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^6+5/32/d*a*b^
4*sin(d*x+c)^5/cos(d*x+c)^6+25/32/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^4+5/16/
d*a^5*ln(sec(d*x+c)+tan(d*x+c))+5/99/d*b^5*sin(d*x+c)^6/cos(d*x+c)^9+15/256
/d*a*b^4*ln(sec(d*x+c)+tan(d*x+c))+1/231/d*b^5*sin(d*x+c)^6/cos(d*x+c)+1/23
1/d*b^5*cos(d*x+c)*sin(d*x+c)^4+4/693/d*cos(d*x+c)*sin(d*x+c)^2*b^5+1/231/d
*b^5*sin(d*x+c)^6/cos(d*x+c)^5-10/63/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)+10/6
3/d*a^2*b^3*sin(d*x+c)^4/cos(d*x+c)^3+5/128/d*a*b^4*sin(d*x+c)^5/cos(d*x+c)
^4+25/64/d*a^3*b^2*sin(d*x+c)^3/cos(d*x+c)^2-5/256/d*a*b^4*sin(d*x+c)^5/cos
(d*x+c)^2
```

Maxima [A] time = 1.23221, size = 567, normalized size = 1.2

$$-\frac{693\,ab^4\left(\frac{2\left(15\,\sin(dx+c)^9-70\,\sin(dx+c)^7+128\,\sin(dx+c)^5+70\,\sin(dx+c)^3-15\,\sin(dx+c)\right)}{\sin(dx+c)^{10}-5\,\sin(dx+c)^8+10\,\sin(dx+c)^6-10\,\sin(dx+c)^4+5\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] -1/354816*(693*a*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d*
x + c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(d*
x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) -
15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 4620*a^3*b^2*(2*(15*
sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x + c))/(
sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1
) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3696*a^5*(2*(15*
sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*s
in(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(s
in(d*x + c) - 1)) - 253440*a^4*b/cos(d*x + c)^7 + 56320*(9*cos(d*x + c)^2 -
 7)*a^2*b^3/cos(d*x + c)^9 - 512*(99*cos(d*x + c)^4 - 154*cos(d*x + c)^2 +
63)*b^5/cos(d*x + c)^11)/d
```

Fricas [A] time = 0.668564, size = 720, normalized size = 1.53

$$\frac{3465\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^{11}\log\left(\sin\left(dx+c\right)+1\right)-3465\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^{11}\log\left(-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/354816*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(sin(d*x
+ c) + 1) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(-sin(d
*x + c) + 1) + 32256*b^5 + 50688*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
4 + 78848*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 462*(15*(16*a^5 - 20*a^3*b^2 +
 3*a*b^4)*cos(d*x + c)^9 + 10*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^
7 + 384*a*b^4*cos(d*x + c) + 8*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)
^5 + 48*(20*a^3*b^2 - 11*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^11)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32433, size = 1480, normalized size = 3.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/177408*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(121968*a^5*tan(1/2*d*x + 1/2*c)^21 + 69300*a^3*b^2*tan(1/2*d*x +
 1/2*c)^21 - 10395*a*b^4*tan(1/2*d*x + 1/2*c)^21 - 887040*a^4*b*tan(1/2*d*x
 + 1/2*c)^20 - 591360*a^5*tan(1/2*d*x + 1/2*c)^19 + 1626240*a^3*b^2*tan(1/2
*d*x + 1/2*c)^19 + 110880*a*b^4*tan(1/2*d*x + 1/2*c)^19 + 3548160*a^4*b*tan
(1/2*d*x + 1/2*c)^18 - 3548160*a^2*b^3*tan(1/2*d*x + 1/2*c)^18 + 1459920*a^
5*tan(1/2*d*x + 1/2*c)^17 - 1159620*a^3*b^2*tan(1/2*d*x + 1/2*c)^17 + 23028
39*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 9757440*a^4*b*tan(1/2*d*x + 1/2*c)^16 +
1182720*a^2*b^3*tan(1/2*d*x + 1/2*c)^16 - 946176*b^5*tan(1/2*d*x + 1/2*c)^1
6 - 2365440*a^5*tan(1/2*d*x + 1/2*c)^15 + 1182720*a^3*b^2*tan(1/2*d*x + 1/2
*c)^15 + 4790016*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 21288960*a^4*b*tan(1/2*d*x
 + 1/2*c)^14 - 9461760*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 - 2365440*b^5*tan(1/
2*d*x + 1/2*c)^14 + 2106720*a^5*tan(1/2*d*x + 1/2*c)^13 - 5738040*a^3*b^2*t
an(1/2*d*x + 1/2*c)^13 + 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 30159360*a
^4*b*tan(1/2*d*x + 1/2*c)^12 + 18923520*a^2*b^3*tan(1/2*d*x + 1/2*c)^12 - 5
203968*b^5*tan(1/2*d*x + 1/2*c)^12 + 28385280*a^4*b*tan(1/2*d*x + 1/2*c)^10
 - 7096320*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 4257792*b^5*tan(1/2*d*x + 1/2*
c)^10 - 2106720*a^5*tan(1/2*d*x + 1/2*c)^9 + 5738040*a^3*b^2*tan(1/2*d*x +
1/2*c)^9 - 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^9 - 20528640*a^4*b*tan(1/2*d*
x + 1/2*c)^8 + 9123840*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 - 3041280*b^5*tan(1/2
*d*x + 1/2*c)^8 + 2365440*a^5*tan(1/2*d*x + 1/2*c)^7 - 1182720*a^3*b^2*tan(
1/2*d*x + 1/2*c)^7 - 4790016*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 11151360*a^4*b*
tan(1/2*d*x + 1/2*c)^6 - 8110080*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 608256*b^
5*tan(1/2*d*x + 1/2*c)^6 - 1459920*a^5*tan(1/2*d*x + 1/2*c)^5 + 1159620*a^3
*b^2*tan(1/2*d*x + 1/2*c)^5 - 2302839*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 342144
0*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 450560*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 11
2640*b^5*tan(1/2*d*x + 1/2*c)^4 + 591360*a^5*tan(1/2*d*x + 1/2*c)^3 - 16262
```

```
40*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 110880*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 5
06880*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 619520*a^2*b^3*tan(1/2*d*x + 1/2*c)^2
+ 22528*b^5*tan(1/2*d*x + 1/2*c)^2 - 121968*a^5*tan(1/2*d*x + 1/2*c) - 6930
0*a^3*b^2*tan(1/2*d*x + 1/2*c) + 10395*a*b^4*tan(1/2*d*x + 1/2*c) - 126720*
a^4*b + 56320*a^2*b^3 - 2048*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^11)/d
```

3.110 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=227

$$\frac{b^3\cos^2(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d\left(a^2+b^2\right)}$$

```
[Out] (a*b^4*x)/(a^2 + b^2)^3 + (a*b^2*x)/(2*(a^2 + b^2)^2) + (3*a*x)/(8*(a^2 + b
^2)) + (b^3*Cos[c + d*x]^2)/(2*(a^2 + b^2)^2*d) + (b*Cos[c + d*x]^4)/(4*(a^
2 + b^2)*d) + (b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d)
+ (a*b^2*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)^2*d) + (3*a*Cos[c + d*x]
*Sin[c + d*x])/(8*(a^2 + b^2)*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*(a^2
+ b^2)*d)
```

Rubi [A] time = 0.214251, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3100, 2635, 8, 3098, 3133}

$$\frac{b^3\cos^2(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*b^4*x)/(a^2 + b^2)^3 + (a*b^2*x)/(2*(a^2 + b^2)^2) + (3*a*x)/(8*(a^2 + b
^2)) + (b^3*Cos[c + d*x]^2)/(2*(a^2 + b^2)^2*d) + (b*Cos[c + d*x]^4)/(4*(a^
2 + b^2)*d) + (b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d)
+ (a*b^2*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)^2*d) + (3*a*Cos[c + d*x]
*Sin[c + d*x])/(8*(a^2 + b^2)*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*(a^2
+ b^2)*d)
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d} + \frac{a\int \cos^4(c+dx)\,dx}{a^2+b^2} + \frac{b^2\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4\left(a^2+b^2\right)d} + \frac{\left(ab^2\right)\int \cos^2(c+\cdots}{\left(a^2+b^2\right)^2\cdots}$$

$$= \frac{ab^4x}{\left(a^2+b^2\right)^3} + \frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d} + \frac{ab^2\cos(c+dx)\sin(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{3a\cdots}{\cdots}$$

$$= \frac{ab^4x}{\left(a^2+b^2\right)^3} + \frac{ab^2x}{2\left(a^2+b^2\right)^2} + \frac{3ax}{8\left(a^2+b^2\right)} + \frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d} + \frac{b^5\cdots}{\cdots}$$

Mathematica [A] time = 0.409526, size = 218, normalized size = 0.96

$$\frac{24a^3b^2\sin(2(c+dx)) + 2a^3b^2\sin(4(c+dx)) + 4b\left(4a^2b^2+a^4+3b^4\right)\cos(2(c+dx)) + b\left(a^2+b^2\right)^2\cos(4(c+dx)) + 40a^3b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (12*a^5*c + 40*a^3*b^2*c + 60*a*b^4*c + 12*a^5*d*x + 40*a^3*b^2*d*x + 60*a*
b^4*d*x + 4*b*(a^4 + 4*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^2*
Cos[4*(c + d*x)] + 32*b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 8*a^5*Sin[
2*(c + d*x)] + 24*a^3*b^2*Sin[2*(c + d*x)] + 16*a*b^4*Sin[2*(c + d*x)] + a^
5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c + d*x)] + a*b^4*Sin[4*(c + d*x)])/(
32*(a^2 + b^2)^3*d)
```

Maple [B] time = 0.123, size = 524, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^5/(a^2+b^2)^3*ln(a+b*tan(d*x+c))+3/8/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2
*tan(d*x+c)^3*a^5+5/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a^3*b^2
+7/8/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a*b^4+1/2/d/(a^2+b^2)^3/
(tan(d*x+c)^2+1)^2*tan(d*x+c)^2*a^2*b^3+1/2/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^
2*tan(d*x+c)^2*b^5+7/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a^3*b^2+
9/8/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a*b^4+5/8/d/(a^2+b^2)^3/(ta
n(d*x+c)^2+1)^2*tan(d*x+c)*a^5+1/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*a^4*b+1
/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)^2*a^2*b^3+3/4/d/(a^2+b^2)^3/(tan(d*x+c)^2+1
)^2*b^5-1/2/d/(a^2+b^2)^3*b^5*ln(tan(d*x+c)^2+1)+15/8/d/(a^2+b^2)^3*arctan(
tan(d*x+c))*a*b^4+3/8/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^5+5/4/d/(a^2+b^2)^
3*arctan(tan(d*x+c))*a^3*b^2
```

Maxima [B] time = 1.81493, size = 761, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^5*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*b^5*log(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (
3*a^5 + 10*a^3*b^2 + 15*a*b^4)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (16*b^3*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - (5*a^3 + 9*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 8*(a^2*b + 2*b^3)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^3 - a*b^2)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - (3*a^3 - a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 8*(a
^2*b + 2*b^3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^3 + 9*a*b^2)*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2
 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8))/d
```

Fricas [A] time = 0.562692, size = 471, normalized size = 2.07

$$\frac{4\,b^{5}\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)+2\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{4}+\left(3\,a^{5}+10\,a\right.}{8\left(a^{6}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^2
+ 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 +
a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19883, size = 435, normalized size = 1.92

$$\frac{\frac{8\,b^6\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,b^5\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{6\,b^5\tan(dx+c)^4+3\,a^5\tan(dx+c)^3+10\,a^3b^2\tan(dx+c)^3+7\,ab^4\tan(dx-}{}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3
*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x + c)^3
 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x + c)^
2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x + c) + 2
*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(d*x
+ c)^2 + 1)^2))/d
```

3.111 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=166

$$-\frac{a\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{b^3\cos(c+dx)}{d\left(a^2+b^2\right)^2}-\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] -((b^4*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3)/
(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d*x
])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rubi [A] time = 0.17495, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3100, 2633, 2637, 3074, 206}

$$-\frac{a\sin^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos^3(c+dx)}{3d\left(a^2+b^2\right)}+\frac{b^3\cos(c+dx)}{d\left(a^2+b^2\right)^2}-\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```


```
[Out] -((b^4*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3)/
(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d*x
])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```


Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{a\int \cos^3(c+dx)\,dx}{a^2+b^2} + \frac{b^2\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{\left(ab^2\right)\int \cos(c+dx)\,dx}{\left(a^2+b^2\right)^2} + \frac{b^4\int \frac{1}{a\cos(c+dx)+b\sin(c+d}}{\left(a^2+b^2\right)^2}$$

$$= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{ab^2\sin(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{a\sin(c+dx)}{\left(a^2+b^2\right)d} - \frac{a\sin^3(c+dx)}{3\left(a^2+b^2\right)d}$$

$$= -\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2} d} + \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d} + \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d} + \frac{ab^2\sin(c+}{\left(a^2+b^2\right)^2}$$

Mathematica [A] time = 1.01504, size = 137, normalized size = 0.83

$$\frac{\sqrt{a^2+b^2}\left(3b\left(a^2+5b^2\right)\cos(c+dx)+b\left(a^2+b^2\right)\cos(3(c+dx))+2a\sin(c+dx)\left(\left(a^2+b^2\right)\cos(2(c+dx))+5a^2+11b^2\right)\right.}{12d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b^2
)^(5/2)*d)
```

Maple [A] time = 0.135, size = 221, normalized size = 1.3

$$\frac{1}{d}\left(2\,\frac{b^4}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan\left(1/2\,dx+c/2\right)-2\,b}{\sqrt{a^2+b^2}}\right)-2\,\frac{\left(-a^3-2\,ab^2\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^5+\left(-a\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x
+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1/2
*d*x+1/2*c)^5+(-a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*tan(
1/2*d*x+1/2*c)^3-2*b^3*tan(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+1/2*
c)-1/3*a^2*b-4/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.558174, size = 598, normalized size = 3.6

$$\frac{3\sqrt{a^2+b^2}b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos(d}{6\left(a^6+3\,a^4b^2+3\,a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b
^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5)*co
s(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31919, size = 386, normalized size = 2.33

$$-\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3\right.}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,d\right.\right.}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2*d*
x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^
4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x +
1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 +
1)^3))/d
```

3.112 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{b\cos^2(c+dx)}{2d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)\cos(c+dx)}{2d\left(a^2+b^2\right)}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{2\left(a^2+b^2\right)}$$

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (a*x)/(2*(a^2 + b^2)) + (b*Cos[c + d*x]^2)/(2*(a^
2 + b^2)*d) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d)
+ (a*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)*d)
```

Rubi [A] time = 0.129146, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3100, 2635, 8, 3098, 3133}

$$\frac{b\cos^2(c+dx)}{2d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)\cos(c+dx)}{2d\left(a^2+b^2\right)}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (a*x)/(2*(a^2 + b^2)) + (b*Cos[c + d*x]^2)/(2*(a^
2 + b^2)*d) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d)
+ (a*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)*d)
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d} + \frac{a\int \cos^2(c+dx)\,dx}{a^2+b^2} + \frac{b^2\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d} + \frac{a\cos(c+dx)\sin(c+dx)}{2\left(a^2+b^2\right)d} + \frac{b^3\int \frac{b\cos(c+dx)-a\sin(c+d}{a\cos(c+dx)+b\sin(c+d}}{\left(a^2+b^2\right)^2}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{ax}{2\left(a^2+b^2\right)} + \frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d} + \frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^2 d}$$

Mathematica [C] time = 0.220539, size = 143, normalized size = 1.2

$$\frac{b\left(a^2+b^2\right)\cos(2(c+dx))+a^3\sin(2(c+dx))+2a^3c+2a^3dx+ab^2\sin(2(c+dx))+2b^3\log\left((a\cos(c+dx)+b\sin(c+\right.}{4d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d*
x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b^3
*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

Maple [B] time = 0.122, size = 236, normalized size = 2.

$$\frac{b^3\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^2+b^2\right)^2}+\frac{\tan\left(dx+c\right)a^3}{2\,d\left(a^2+b^2\right)^2\left(\left(\tan\left(dx+c\right)\right)^2+1\right)}+\frac{\tan\left(dx+c\right)ab^2}{2\,d\left(a^2+b^2\right)^2\left(\left(\tan\left(dx+c\right)\right)^2+1\right)}+\frac{a^2}{2\,d\left(a^2+b^2\right)^2\left((\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/2/d/(a^2+b^2)^2/(tan(d*x+c)^2+1)*t
an(d*x+c)*a^3+1/2/d/(a^2+b^2)^2/(tan(d*x+c)^2+1)*tan(d*x+c)*a*b^2+1/2/d/(a^
2+b^2)^2/(tan(d*x+c)^2+1)*a^2*b+1/2/d/(a^2+b^2)^2/(tan(d*x+c)^2+1)*b^3-1/2/
d/(a^2+b^2)^2*b^3*ln(tan(d*x+c)^2+1)+3/2/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a
*b^2+1/2/d/(a^2+b^2)^2*arctan(tan(d*x+c))*a^3
```

Maxima [B] time = 1.7359, size = 383, normalized size = 3.22

$$\frac{\frac{b^3\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2+b^2+\frac{2\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^2+b^2\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) + (a*sin(d*x + c)/(cos(d*x +
 c) + 1) - 2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - a*sin(d*x + c)^3/(cos(
```

```
d*x + c) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + (a^2 + b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Fricas [A] time = 0.52543, size = 278, normalized size = 2.34

$$\frac{b^3 \log\left(2\,ab\cos(dx+c)\sin(dx+c) + \left(a^2-b^2\right)\cos(dx+c)^2 + b^2\right) + \left(a^3+3\,ab^2\right)dx + \left(a^2b+b^3\right)\cos(dx+c)^2 + \left(a^3+\right.}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.14924, size = 246, normalized size = 2.07

$$\frac{\frac{2\,b^4\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5} - \frac{b^3\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4} + \frac{\left(a^3+3\,ab^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4} + \frac{b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+a^2b+2\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan(dx+c)^2+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*log
(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a
^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*tan(
d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

3.113 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}-\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -((b^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^2
+ b^2)*d)
```

Rubi [A] time = 0.0817828, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3100, 2637, 3074, 206}

$$\frac{a\sin(c+dx)}{d\left(a^2+b^2\right)}+\frac{b\cos(c+dx)}{d\left(a^2+b^2\right)}-\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((b^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^2
+ b^2)*d)
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{a\int \cos(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}-\frac{b^2\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c-\right.}{\left(a^2+b^2\right)d}$$

$$= -\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}d}+\frac{b\cos(c+dx)}{\left(a^2+b^2\right)d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}$$

Mathematica [A] time = 0.180227, size = 79, normalized size = 0.87

$$\frac{\sqrt{a^2+b^2}(a\sin(c+dx)+b\cos(c+dx))+2b^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]
*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [A] time = 0.132, size = 90, normalized size = 1.

$$\frac{1}{d}\left(2\,\frac{b^2}{\left(a^2+b^2\right)^{3/2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan\left(1/2\,dx+c/2\right)-2\,b}{\sqrt{a^2+b^2}}\right)-2\,\frac{-a\tan\left(1/2\,dx+c/2\right)-b}{\left(a^2+b^2\right)\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^
2)^(1/2))-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.520005, size = 436, normalized size = 4.79

$$\frac{\sqrt{a^2+b^2}b^2\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)+2\left(a^2b+b^3\right)\cos\left(dx+c\right)+}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28761, size = 159, normalized size = 1.75

$$-\frac{\frac{b^2 \log\left(\frac{\left|2 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-2 b-2 \sqrt{a^2+b^2}\right|}{\left|2 a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-2 b+2 \sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2 \left(a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+b\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*ta
n(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

3.114 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}$$

```
[Out] (a*x)/(a^2 + b^2) + (b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d
)
```

Rubi [A] time = 0.0657701, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3098, 3133}

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d
)
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{ax}{a^2+b^2}+\frac{b\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{ax}{a^2+b^2}+\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

Mathematica [A] time = 0.0661455, size = 41, normalized size = 0.91

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))+a(c+dx)}{d\,(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x) + b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)
```

Maple [A] time = 0.106, size = 74, normalized size = 1.6

$$\frac{b\ln(a+b\tan(dx+c))}{d\,(a^2+b^2)}-\frac{b\ln\left((\tan(dx+c))^2+1\right)}{2\,d\,(a^2+b^2)}+\frac{a\arctan(\tan(dx+c))}{d\,(a^2+b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d*b/(a^2+b^2)*ln(a+b*tan(d*x+c))-1/2/d/(a^2+b^2)*b*ln(tan(d*x+c)^2+1)+1/d
/(a^2+b^2)*a*arctan(tan(d*x+c))
```

Maxima [B] time = 1.7852, size = 167, normalized size = 3.71

$$\frac{\frac{2\,a\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2}+\frac{b\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+b^2}-\frac{b\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^2+b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*a*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2) + b*log(-a - 2*b*s
in(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^
2 + b^2) - b*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A] time = 0.495281, size = 144, normalized size = 3.2

$$\frac{2\,adx + b\log\left(2\,ab\cos\left(dx + c\right)\sin\left(dx + c\right) + \left(a^2 - b^2\right)\cos\left(dx + c\right)^2 + b^2\right)}{2\left(a^2 + b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 + b^2))/((a^2 + b^2)*d)
```

Sympy [A] time = 19.2425, size = 299, normalized size = 6.64

$$\begin{cases} \frac{\tilde{\infty}x\cos(c)}{\sin(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\log(\sin(c+dx))}{bd} & \text{for } a = 0 \\ -\frac{idx\sin(c+dx)}{-2bd\sin(c+dx)+2ibd\cos(c+dx)} - \frac{dx\cos(c+dx)}{-2bd\sin(c+dx)+2ibd\cos(c+dx)} - \frac{i\cos(c+dx)}{-2bd\sin(c+dx)+2ibd\cos(c+dx)} & \text{for } a = -ib \\ -\frac{idx\sin(c+dx)}{2bd\sin(c+dx)+2ibd\cos(c+dx)} + \frac{dx\cos(c+dx)}{2bd\sin(c+dx)+2ibd\cos(c+dx)} - \frac{i\cos(c+dx)}{2bd\sin(c+dx)+2ibd\cos(c+dx)} & \text{for } a = ib \\ \frac{x\cos(c)}{a\cos(c)+b\sin(c)} & \text{for } d = 0 \\ \frac{adx}{a^2d+b^2d} + \frac{b\log\left(\cos(c+dx)+\frac{b\sin(c+dx)}{a}\right)}{a^2d+b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (log(sin(c
 + d*x))/(b*d), Eq(a, 0)), (-I*d*x*sin(c + d*x)/(-2*b*d*sin(c + d*x) + 2*I*
b*d*cos(c + d*x)) - d*x*cos(c + d*x)/(-2*b*d*sin(c + d*x) + 2*I*b*d*cos(c +
 d*x)) - I*cos(c + d*x)/(-2*b*d*sin(c + d*x) + 2*I*b*d*cos(c + d*x)), Eq(a,
 -I*b)), (-I*d*x*sin(c + d*x)/(2*b*d*sin(c + d*x) + 2*I*b*d*cos(c + d*x)) +
```

```
 d*x*cos(c + d*x)/(2*b*d*sin(c + d*x) + 2*I*b*d*cos(c + d*x)) - I*cos(c + d
*x)/(2*b*d*sin(c + d*x) + 2*I*b*d*cos(c + d*x)), Eq(a, I*b)), (x*cos(c)/(a*
cos(c) + b*sin(c)), Eq(d, 0)), (a*d*x/(a**2*d + b**2*d) + b*log(cos(c + d*x
) + b*sin(c + d*x)/a)/(a**2*d + b**2*d), True))
```

Giac [A] time = 1.19466, size = 100, normalized size = 2.22

$$\frac{\frac{2\,b^2\log(|b\tan(dx+c)+a|)}{a^2b+b^3}+\frac{2\,(dx+c)a}{a^2+b^2}-\frac{b\log\left(\tan(dx+c)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*log(abs(b*tan(d*x + c) + a))/(a^2*b + b^3) + 2*(d*x + c)*a/(a^2
+ b^2) - b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

3.115 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rubi [A] time = 0.022887, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3074, 206}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}d}$$

Mathematica [A] time = 0.033069, size = 45, normalized size = 0.96

$$\frac{2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Maple [A] time = 0.116, size = 43, normalized size = 0.9

$$2\frac{1}{d\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a\tan(1/2\,dx+c/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.483834, size = 312, normalized size = 6.64

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.25261, size = 100, normalized size = 2.13

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

3.116 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=41

$$\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}$$

```
[Out] -(Log[Cos[c + d*x]]/(b*d)) + Log[a*Cos[c + d*x] + b*Sin[c + d*x]]/(b*d)
```

Rubi [A] time = 0.0816556, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3102, 3475, 3133}

$$\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(b*d)) + Log[a*Cos[c + d*x] + b*Sin[c + d*x]]/(b*d)
```

Rule 3102

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
```

```
), 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b} + \frac{\int \tan(c+dx)\,dx}{b}$$
$$= -\frac{\log(\cos(c+dx))}{bd} + \frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}$$

Mathematica [A] time = 0.0168481, size = 18, normalized size = 0.44

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Maple [A] time = 0.138, size = 19, normalized size = 0.5

$$\frac{\ln\left(a+b\tan\left(dx+c\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b*ln(a+b*tan(d*x+c))
```

Maxima [B] time = 1.09116, size = 139, normalized size = 3.39

$$\frac{\frac{\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b} - \frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/b - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x +
c)/(cos(d*x + c) + 1) - 1)/b)/d
```

Fricas [A] time = 0.512886, size = 144, normalized size = 3.51

$$\frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) - log(cos(d*x + c)^2))/(b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A] time = 1.15861, size = 26, normalized size = 0.63

$$\frac{\log\left(|b\tan\left(dx+c\right)+a|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(d*x + c) + a))/(b*d)
```

3.117 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c +
d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rubi [A] time = 0.0835684, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3104, 3770, 3074, 206}

$$-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c +
d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec(c+dx)}{bd} - \frac{a\int \sec(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{\sec(c+dx)}{bd} - \frac{\left(a^2+b^2\right)\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c\right.}{b^2d}$$
$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d} + \frac{\sec(c+dx)}{bd}$$

Mathematica [A] time = 0.138254, size = 109, normalized size = 1.36

$$\frac{2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right) + a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

Maple [B] time = 0.174, size = 174, normalized size = 2.2

$$\frac{1}{db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{a}{b^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{1}{db}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{a}{b^2d}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right) + 2\frac{}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/d/b/(tan(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/d/b/(tan(
1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)+2/d/b^2/(a^2+b^2)^(1/2
)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2+2/d/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.590667, size = 471, normalized size = 5.89

$$-\frac{a\cos(dx+c)\log(\sin(dx+c)+1) - a\cos(dx+c)\log(-\sin(dx+c)+1) - \sqrt{a^2+b^2}\cos(dx+c)\log\left(-\frac{2\,ab\cos(dx+c)\sin}{}\right.}{2\,b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x
 + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A] time = 1.3333, size = 184, normalized size = 2.3

$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqr
t(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b^2
+ 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

3.118 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{\left(a^2+b^2\right)\log(\cos(c+dx))}{b^3d}+\frac{\left(a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}$$

```
[Out] -(((a^2 + b^2)*Log[Cos[c + d*x]])/(b^3*d)) + ((a^2 + b^2)*Log[a*Cos[c + d*x
] + b*Sin[c + d*x]])/(b^3*d) + Sec[c + d*x]^2/(2*b*d) - (a*Tan[c + d*x])/(b
^2*d)
```

Rubi [A] time = 0.140972, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3104, 3767, 8, 3102, 3475, 3133}

$$-\frac{\left(a^2+b^2\right)\log(\cos(c+dx))}{b^3d}+\frac{\left(a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)*Log[Cos[c + d*x]])/(b^3*d)) + ((a^2 + b^2)*Log[a*Cos[c + d*x
] + b*Sin[c + d*x]])/(b^3*d) + Sec[c + d*x]^2/(2*b*d) - (a*Tan[c + d*x])/(b
^2*d)
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3102

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= \frac{\sec^2(c+dx)}{2bd} - \frac{a\int \sec^2(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}\\ &= \frac{\sec^2(c+dx)}{2bd} + \frac{\left(a^2+b^2\right)\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^3} + \frac{\left(a^2+b^2\right)\int \tan(c+dx)\,dx}{b^3} + \\ &= -\frac{\left(a^2+b^2\right)\log(\cos(c+dx))}{b^3d} + \frac{\left(a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d} + \frac{\sec^2(}{2}\end{aligned}$$

Mathematica [A] time = 0.14103, size = 52, normalized size = 0.59

$$\frac{\left(a^2+b^2\right)\log(a+b\tan(c+dx)) - ab\tan(c+dx) + \frac{1}{2}b^2\tan^2(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] ((a^2 + b^2)*Log[a + b*Tan[c + d*x]] - a*b*Tan[c + d*x] + (b^2*Tan[c + d*x]
^2)/2)/(b^3*d)
```

Maple [A] time = 0.16, size = 72, normalized size = 0.8

$$\frac{(\tan(dx+c))^2}{2\,db}-\frac{a\tan(dx+c)}{b^2d}+\frac{\ln(a+b\tan(dx+c))\,a^2}{db^3}+\frac{\ln(a+b\tan(dx+c))}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/2/d/b*tan(d*x+c)^2-a*tan(d*x+c)/b^2/d+1/d/b^3*ln(a+b*tan(d*x+c))*a^2+1/d/
b*ln(a+b*tan(d*x+c))
```

Maxima [B] time = 1.10488, size = 321, normalized size = 3.65

$$-\frac{\frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{b^2-\frac{2b^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{b^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\left(a^2+b^2\right)\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^3}+\frac{\left(a^2+b^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^3}+\frac{\left(a^2+b^2\right)\log\left(\frac{\sin(dx+c}{\cos(dx+c)}\right.}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(a*sin(d*x + c)/(cos(d*x + c) + 1) - b*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(b^2 - 2*b^2*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + b^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - (a^2 + b^2)
*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/b^3 + (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^3
+ (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^3)/d
```

Fricas [A] time = 0.524793, size = 294, normalized size = 3.34

$$\frac{\left(a^2+b^2\right)\cos(dx+c)^2\log\left(2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2\right)-\left(a^2+b^2\right)\cos(dx+c)^2\log\left(\mathrm{co}\right.}{2\,b^3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c)^
2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A] time = 1.1557, size = 73, normalized size = 0.83

$$\frac{\frac{b\tan(dx+c)^2-2a\tan(dx+c)}{b^2}+\frac{2\left(a^2+b^2\right)\log(|b\tan(dx+c)+a|)}{b^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*ta
n(d*x + c) + a))/b^3)/d
```

3.119 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{(a^2+b^2)\sec(c+dx)}{b^3d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a\tanh^{-1}(\sin(c+}{2b^2d}$$

```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(2*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]]
)/(b^4*d) - ((a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sq
rt[a^2 + b^2]])/(b^4*d) + ((a^2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]
^3/(3*b*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rubi [A] time = 0.157237, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3104, 3768, 3770, 3074, 206}

$$\frac{(a^2+b^2)\sec(c+dx)}{b^3d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a\tanh^{-1}(\sin(c+}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```


```
[Out] -(a*ArcTanh[Sin[c + d*x]])/(2*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]]
)/(b^4*d) - ((a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sq
rt[a^2 + b^2]])/(b^4*d) + ((a^2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]
^3/(3*b*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```


Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)}{3bd} - \frac{a\int \sec^3(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\left(a^2+b^2\right)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} - \frac{a\int \sec(c+dx)}{2b^2}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\left(a^2+b^2\right)\sec(c+dx)}{b^3d}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{b}{}\right.}{b^4d}$$

Mathematica [B] time = 1.99009, size = 321, normalized size = 2.1

$$\frac{48\left(a^2+b^2\right)^{3/2}\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right) + \sec^3(c+dx)\left(12b\left(a^2+b^2\right)\cos(2(c+dx)) + 9a\left(2a^2+3b^2\right)\cos(c+dx)\left(\log\left(\text{co}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6*a
^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*b^2)
*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

Maple [B] time = 0.202, size = 488, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 2/d/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^
(1/2))*a^4+4/d/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)
/(a^2+b^2)^(1/2))*a^2+2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*
c)-2*b)/(a^2+b^2)^(1/2))+1/3/d/b/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/b^2/(tan(1/
2*d*x+1/2*c)+1)^2*a-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^2+1/d/b^3/(tan(1/2*d*x+1
/2*c)+1)*a^2-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)*a+3/2/d/b/(tan(1/2*d*x+1/2*c)
+1)-1/d*a^3/b^4*ln(tan(1/2*d*x+1/2*c)+1)-3/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+
1)-1/3/d/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2*a-1/
2/d/b/(tan(1/2*d*x+1/2*c)-1)^2-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2
/(tan(1/2*d*x+1/2*c)-1)*a-3/2/d/b/(tan(1/2*d*x+1/2*c)-1)+1/d*a^3/b^4*ln(tan
(1/2*d*x+1/2*c)-1)+3/2/d*a/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.837595, size = 633, normalized size = 4.14

$$\frac{6\left(a^2+b^2\right)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right)-3\left(2\,a^3+3\,ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c)
 + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*a*b^2
*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)/(b^4*
d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A] time = 1.34997, size = 375, normalized size = 2.45

$$-\frac{\frac{3\left(2\,a^3+3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{3\left(2\,a^3+3\,ab^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{6\left(a^4+2\,a^2b^2+b^4\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\right.\right.}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^3
 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 + b
^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(3*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan(1/2*
d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c
)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/2*c)^2
- 1)^3*b^3))/d
```

3.120 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{a\left(a^2+b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(a^2+b^2\right)\sec^2(c+dx)}{2b^3d}-\frac{\left(a^2+b^2\right)^2\log(\cos(c+dx))}{b^5d}+\frac{\left(a^2+b^2\right)^2\log(a\cos(c+dx)+b\sin(c}{b^5d}$$

```
[Out] -(((a^2 + b^2)^2*Log[Cos[c + d*x]])/(b^5*d)) + ((a^2 + b^2)^2*Log[a*Cos[c +
 d*x] + b*Sin[c + d*x]])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^2)/(2*b^3*d) +
 Sec[c + d*x]^4/(4*b*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 + b^2)*Tan[c +
 d*x])/(b^4*d) - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

Rubi [A] time = 0.222416, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3104, 3767, 8, 3102, 3475, 3133}

$$-\frac{a\left(a^2+b^2\right)\tan(c+dx)}{b^4d}+\frac{\left(a^2+b^2\right)\sec^2(c+dx)}{2b^3d}-\frac{\left(a^2+b^2\right)^2\log(\cos(c+dx))}{b^5d}+\frac{\left(a^2+b^2\right)^2\log(a\cos(c+dx)+b\sin(c}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```


```
[Out] -(((a^2 + b^2)^2*Log[Cos[c + d*x]])/(b^5*d)) + ((a^2 + b^2)^2*Log[a*Cos[c +
 d*x] + b*Sin[c + d*x]])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^2)/(2*b^3*d) +
 Sec[c + d*x]^4/(4*b*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 + b^2)*Tan[c +
 d*x])/(b^4*d) - (a*Tan[c + d*x]^3)/(3*b^2*d)
```


Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3102

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^4(c+dx)}{4bd} - \frac{a\int \sec^4(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\left(a^2+b^2\right)\sec^2(c+dx)}{2b^3d} + \frac{\sec^4(c+dx)}{4bd} - \frac{\left(a\left(a^2+b^2\right)\right)\int \sec^2(c+dx)\,dx}{b^4} + \frac{\left(a^2+b\right.}{}$$

$$= \frac{\left(a^2+b^2\right)\sec^2(c+dx)}{2b^3d} + \frac{\sec^4(c+dx)}{4bd} - \frac{a\tan(c+dx)}{b^2d} - \frac{a\tan^3(c+dx)}{3b^2d} + \frac{\left(a^2+\right.}{}$$

$$= -\frac{\left(a^2+b^2\right)^2\log(\cos(c+dx))}{b^5d} + \frac{\left(a^2+b^2\right)^2\log(a\cos(c+dx)+b\sin(c+dx))}{b^5d} + \frac{\left(a\right.}{}$$

Mathematica [A] time = 1.20875, size = 99, normalized size = 0.63

$$\frac{6b^2\left(a^2+b^2\right)\tan^2(c+dx)-12ab\left(a^2+2b^2\right)\tan(c+dx)+12\left(a^2+b^2\right)^2\log(a+b\tan(c+dx))-4ab^3\tan^3(c+dx)+3b^4}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (12*(a^2 + b^2)^2*Log[a + b*Tan[c + d*x]] + 3*b^4*Sec[c + d*x]^4 - 12*a*b*(
a^2 + 2*b^2)*Tan[c + d*x] + 6*b^2*(a^2 + b^2)*Tan[c + d*x]^2 - 4*a*b^3*Tan[
c + d*x]^3)/(12*b^5*d)
```

Maple [A] time = 0.158, size = 162, normalized size = 1.

$$\frac{(\tan(dx+c))^4}{4\,db}-\frac{a(\tan(dx+c))^3}{3\,b^2d}+\frac{(\tan(dx+c))^2a^2}{2\,db^3}+\frac{(\tan(dx+c))^2}{db}-\frac{a^3\tan(dx+c)}{db^4}-2\,\frac{a\tan(dx+c)}{b^2d}+\frac{\ln(a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 1/4/d/b*tan(d*x+c)^4-1/3*a*tan(d*x+c)^3/b^2/d+1/2/d/b^3*tan(d*x+c)^2*a^2+1/
d/b*tan(d*x+c)^2-1/d/b^4*a^3*tan(d*x+c)-2*a*tan(d*x+c)/b^2/d+1/d/b^5*ln(a+b
*tan(d*x+c))*a^4+2/d/b^3*ln(a+b*tan(d*x+c))*a^2+1/d/b*ln(a+b*tan(d*x+c))
```

Maxima [B] time = 1.09426, size = 624, normalized size = 3.95

$$-\frac{\frac{2\left(\frac{3\left(a^3+2ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^2b+2b^3\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(9a^3+14ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\left(a^2b+b^3\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(9a^3+14ab^2\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\left(a^2b+2b^3\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\left(a^3+2ab^2\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{b^4-\frac{4b^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6b^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4b^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{b^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(2*(3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^2*b + 2*b
^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (9*a^3 + 14*a*b^2)*sin(d*x + c)^3
```

```
/(cos(d*x + c) + 1)^3 + 6*(a^2*b + b^3)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + (9*a^3 + 14*a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^2*b + 2*b^
3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*(a^3 + 2*a*b^2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/(b^4 - 4*b^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*b
^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*b^4*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*(a^4 + 2*a^2*b^2 + b
^4)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c)
 + 1) - 1)/b^5)/d
```

Fricas [A] time = 0.568298, size = 439, normalized size = 2.78

$$\frac{6\left(a^4+2\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(a^4+2\,a^2b^2+b^4\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*cos
(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)^2
- 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))
/(b^5*d*cos(d*x + c)^4)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^5\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A] time = 1.19942, size = 162, normalized size = 1.03

$$\frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\left(a^4+2\,a^2b^2+b^4\right)\log(|b\tan(dx+c)+a|)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)
^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))/b
^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d
```

3.121 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=262

$$\frac{(a^2+b^2)\sec^3(c+dx)}{3b^3d}+\frac{(a^2+b^2)^2\sec(c+dx)}{b^5d}-\frac{a(a^2+b^2)^2\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-$$

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(8*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(2*b^4*d) - (a*(a^2 + b^2)^2*ArcTanh[Sin[c + d*x]])/(b^6*d) - ((a^2 + b
^2)^(5/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*
d) + ((a^2 + b^2)^2*Sec[c + d*x])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^3)/(3
*b^3*d) + Sec[c + d*x]^5/(5*b*d) - (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*b^2*d
) - (a*(a^2 + b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d) - (a*Sec[c + d*x]^3
*Tan[c + d*x])/(4*b^2*d)
```

Rubi [A] time = 0.256162, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3104, 3768, 3770, 3074, 206}

$$\frac{(a^2+b^2)\sec^3(c+dx)}{3b^3d}+\frac{(a^2+b^2)^2\sec(c+dx)}{b^5d}-\frac{a(a^2+b^2)^2\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```


```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(8*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(2*b^4*d) - (a*(a^2 + b^2)^2*ArcTanh[Sin[c + d*x]])/(b^6*d) - ((a^2 + b
^2)^(5/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*
d) + ((a^2 + b^2)^2*Sec[c + d*x])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^3)/(3
*b^3*d) + Sec[c + d*x]^5/(5*b*d) - (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*b^2*d
) - (a*(a^2 + b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d) - (a*Sec[c + d*x]^3
*Tan[c + d*x])/(4*b^2*d)
```


Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
```

```
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^5(c+dx)}{5bd} - \frac{a\int \sec^5(c+dx)\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= \frac{\left(a^2+b^2\right)\sec^3(c+dx)}{3b^3d} + \frac{\sec^5(c+dx)}{5bd} - \frac{a\sec^3(c+dx)\tan(c+dx)}{4b^2d} - \frac{(3a)\int \sec^3(c}{4b^2}$$

$$= \frac{\left(a^2+b^2\right)^2\sec(c+dx)}{b^5d} + \frac{\left(a^2+b^2\right)\sec^3(c+dx)}{3b^3d} + \frac{\sec^5(c+dx)}{5bd} - \frac{3a\sec(c+dx)\,\mathrm{ta}}{8b^2d}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(a^2+b^2\right)^2\tanh^{-1}(}{b^6d}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d} - \frac{a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a\left(a^2+b^2\right)^2\tanh^{-1}(}{b^6d}$$

Mathematica [B] time = 5.24765, size = 661, normalized size = 2.52

$$\sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{2b^3(20a^2+29b^2)\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{2b^3(20a^2+29b^2)\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{b^2(20a^2b-60a^3-105ab^2+2\ldots}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+d\ldots\right.\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(240*a^4*b + 520*a^2*b^3 + 298*b^5 + 480*(a^2 + b^2)^(5/2)*Ar
cTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + 30*a*(8*a^4 + 20*a^2*b^2
 + 15*b^4)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 30*a*(8*a^4 + 20*a^2*
b^2 + 15*b^4)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (3*b^4*(-5*a + 2*b
))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + (b^2*(-60*a^3 + 20*a^2*b - 105
*a*b^2 + 29*b^3))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (12*b^5*Sin[(c
+ d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (2*b^3*(20*a^2 + 29*b^
2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b*(120*a^
4 + 260*a^2*b^2 + 149*b^4)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]) - (12*b^5*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5
 + (3*b^4*(5*a + 2*b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - (2*b^3*(20
*a^2 + 29*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 +
(b^2*(60*a^3 + 20*a^2*b + 105*a*b^2 + 29*b^3))/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - (2*b*(120*a^4 + 260*a^2*b^2 + 149*b^4)*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]))*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(240*
b^6*d*(a + b*Tan[c + d*x]))
```

Maple [B] time = 0.201, size = 994, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))-15/8/d*a/b^2*ln(tan(1/2*d*x+1/2*c)+1)+15/8/d*a/b^2*ln(tan(1/2*d*x+1/2*c)
-1)+1/5/d/b/(tan(1/2*d*x+1/2*c)+1)^5-1/2/d/b/(tan(1/2*d*x+1/2*c)+1)^4-1/5/d
/b/(tan(1/2*d*x+1/2*c)-1)^5-1/2/d/b/(tan(1/2*d*x+1/2*c)-1)^4+2/d/b^6/(a^2+b
^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^6+15/
8/d/b/(tan(1/2*d*x+1/2*c)+1)-15/8/d/b/(tan(1/2*d*x+1/2*c)-1)-1/d/b^5/(tan(1
```

```
/2*d*x+1/2*c)-1)*a^4-1/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)*a^3+1/d*a^5/b^6*ln(ta
n(1/2*d*x+1/2*c)-1)+1/d/b^5/(tan(1/2*d*x+1/2*c)+1)*a^4-1/2/d/b^4/(tan(1/2*d
*x+1/2*c)+1)*a^3-1/d*a^5/b^6*ln(tan(1/2*d*x+1/2*c)+1)-1/4/d/b^2/(tan(1/2*d*
x+1/2*c)-1)^4*a-1/3/d/b^3/(tan(1/2*d*x+1/2*c)-1)^3*a^2-1/2/d/b^2/(tan(1/2*d
*x+1/2*c)-1)^3*a+1/4/d/b^2/(tan(1/2*d*x+1/2*c)+1)^4*a+1/3/d/b^3/(tan(1/2*d*
x+1/2*c)+1)^3*a^2-1/2/d/b^2/(tan(1/2*d*x+1/2*c)+1)^3*a+1/2/d/b^4/(tan(1/2*d
*x+1/2*c)+1)^2*a^3-1/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)^2*a^2-1/2/d/b^4/(tan(1/
2*d*x+1/2*c)-1)^2*a^3-1/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)^2*a^2+11/8/d/b^2/(ta
n(1/2*d*x+1/2*c)+1)^2*a+5/2/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a^2-9/8/d/b^2/(tan
(1/2*d*x+1/2*c)+1)*a-5/2/d*a^3/b^4*ln(tan(1/2*d*x+1/2*c)+1)-11/8/d/b^2/(tan
(1/2*d*x+1/2*c)-1)^2*a-5/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a^2-9/8/d/b^2/(tan(
1/2*d*x+1/2*c)-1)*a+5/2/d*a^3/b^4*ln(tan(1/2*d*x+1/2*c)-1)+13/12/d/b/(tan(1
/2*d*x+1/2*c)+1)^3-9/8/d/b/(tan(1/2*d*x+1/2*c)+1)^2-13/12/d/b/(tan(1/2*d*x+
1/2*c)-1)^3-9/8/d/b/(tan(1/2*d*x+1/2*c)-1)^2+6/d/b^2/(a^2+b^2)^(1/2)*arctan
h(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2+6/d/b^4/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^4
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.4155, size = 834, normalized size = 3.18

$$120\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}\cos\left(dx+c\right)^5\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(120*(a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*cos(d*x + c)^5*log(-(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2
*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin
```

```
(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 15*(8*a^5 + 20*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 15*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^5 + 240*(a^4*b + 2*a^2
*b^3 + b^5)*cos(d*x + c)^4 + 80*(a^2*b^3 + b^5)*cos(d*x + c)^2 - 30*(2*a*b^
4*cos(d*x + c) + (4*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(b^6*d
*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38785, size = 748, normalized size = 2.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/b^6 - 15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) -
1))/b^6 + 120*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*a*tan(1/2*d*x +
 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 2*(60*a^3*b*tan(1/2*d*x + 1/2*c)^9
 + 135*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*x + 1/2*c)^8 + 360*
a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*tan(1/2*d*x + 1/2*c)^8 - 120*a^3*b
*tan(1/2*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*a^4*tan(1/
2*d*x + 1/2*c)^6 - 1200*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 720*b^4*tan(1/2*d*
x + 1/2*c)^6 + 720*a^4*tan(1/2*d*x + 1/2*c)^4 + 1600*a^2*b^2*tan(1/2*d*x +
1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*b*tan(1/2*d*x + 1/2*c)
^3 + 150*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 10
40*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 560*b^4*tan(1/2*d*x + 1/2*c)^2 - 60*a^3
*b*tan(1/2*d*x + 1/2*c) - 135*a*b^3*tan(1/2*d*x + 1/2*c) + 120*a^4 + 280*a^
2*b^2 + 184*b^4)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*b^5))/d
```

3.122 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{b^4}{ad\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}-\frac{\sin^2(c+dx)\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)}{2d\left(a^2+b^2\right)^2}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+$$

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + b^4/(a*(a^2 + b^2)^2*d*(b
 + a*Cot[c + d*x])) + (4*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2
+ b^2)^3*d) - ((2*a*b - (a^2 - b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*(a^2 +
 b^2)^2*d)
```

Rubi [A] time = 0.292613, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3088, 1647, 1629, 635, 203, 260}

$$\frac{b^4}{ad\left(a^2+b^2\right)^2(a\cot(c+dx)+b)}-\frac{\sin^2(c+dx)\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)}{2d\left(a^2+b^2\right)^2}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + b^4/(a*(a^2 + b^2)^2*d*(b
 + a*Cot[c + d*x])) + (4*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2
+ b^2)^3*d) - ((2*a*b - (a^2 - b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*(a^2 +
 b^2)^2*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(b+ax)^2(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{-\frac{b^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}-\frac{2abx}{a^2+b^2}-\frac{\left(a^4+5a^2b^2+\right.}{\left(a^2+b^2\right.}}{(b+ax)^2\left(1+x^2\right)}\right.}{2d}$$

$$= -\frac{\left(2ab-\left(a^2-b^2\right)\cot(c+dx)\right)\sin^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{\text{Subst}\left(\int\left(-\frac{2b^4}{\left(a^2+b^2\right)^2(b+ax)^2}+\frac{8}{\left(a^2+b\right.}\right.\right.}{}$$

$$= \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+a\cot(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{\left(2ab-\left(a^2-b\right.\right.}{2}$$

$$= \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+a\cot(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{\left(2ab-\left(a^2-b\right.\right.}{2}$$

$$= \frac{\left(a^4+6a^2b^2-3b^4\right)x}{2\left(a^2+b^2\right)^3} + \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+a\cot(c}{\left(a^2+b^2\right)^3 d}$$

Mathematica [A] time = 0.896495, size = 149, normalized size = 1.03

$$\frac{2\left(6a^2b^2+a^4-3b^4\right)(c+dx)+\left(a^2-b^2\right)\left(a^2+b^2\right)\sin(2(c+dx))+2ab\left(a^2+b^2\right)\cos(2(c+dx))+\frac{4b^4\left(a^2+b^2\right)\sin(c+dx)}{a(a\cos(c+dx)+b\sin(c+dx))}+}{4d\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 + 6*a^2*b^2 - 3*b^4)*(c + d*x) + 2*a*b*(a^2 + b^2)*Cos[2*(c + d*x)]
 + 16*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + (4*b^4*(a^2 + b^2)*Sin[c
 + d*x])/(a*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (a^2 - b^2)*(a^2 + b^2)*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^3*d)
```

Maple [B] time = 0.161, size = 292, normalized size = 2.

$$-\frac{b^3}{d\left(a^2+b^2\right)^2\left(a+b\tan\left(dx+c\right)\right)}+4\,\frac{ab^3\ln\left(a+b\tan\left(dx+c\right)\right)}{d\left(a^2+b^2\right)^3}+\frac{a^4\tan\left(dx+c\right)}{2\,d\left(a^2+b^2\right)^3\left(\left(\tan\left(dx+c\right)\right)^2+1\right)}-\frac{\tan}{2\,d\left(a^2+b^2\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d*b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4/d*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+
c))+1/2/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)*tan(d*x+c)*a^4-1/2/d/(a^2+b^2)^3/(ta
n(d*x+c)^2+1)*tan(d*x+c)*b^4+1/d/(a^2+b^2)^3/(tan(d*x+c)^2+1)*a^3*b+1/d/(a^
2+b^2)^3/(tan(d*x+c)^2+1)*a*b^3-2/d/(a^2+b^2)^3*a*b^3*ln(tan(d*x+c)^2+1)+3/
d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^2*b^2-3/2/d/(a^2+b^2)^3*arctan(tan(d*x+c
))*b^4+1/2/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^4
```

Maxima [B] time = 1.76085, size = 381, normalized size = 2.63

$$\frac{\frac{8\,ab^3\log\left(b\tan\left(dx+c\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,ab^3\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)\left(dx+c\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,a^2b-2\,b^3+\left(a^2b-3\,b^3\right)\tan\left(dx+c\right)^2+\left(a^3+ab^2\right)\tan\left(dx\right.}{a^5+2\,a^3b^2+ab^4+\left(a^4b+2\,a^2b^3+b^5\right)\tan\left(dx+c\right)^3+\left(a^5+2\,a^3b^2+ab^4\right)\tan\left(dx\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^4
+ 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*a^2
*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (a^5 +
 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)
))/d
```

Fricas [A] time = 0.565004, size = 614, normalized size = 4.23

$$\frac{\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^3-\left(a^2b^3+3\,b^5-\left(a^5+6\,a^3b^2-3\,ab^4\right)dx\right)\cos\left(dx+c\right)+4\left(a^2b^3\cos\left(dx+c\right)+ab^4\sin\left(\right.\right.}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 + 6
*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*sin
(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x +
c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13898, size = 338, normalized size = 2.33

$$\frac{\frac{8\,ab^4\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{4\,ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{\left(a^4+6\,a^2b^2-3\,b^4\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{a^2b\tan(dx+c)^2-3\,b^3\tan(dx+c)^2+a^3\tan(dx+c)+ab^2\tan(dx+c)+2\,a^2b-2\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\tan(dx+c)^3+a\tan(dx+c)^2+b\tan(dx+c)+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*ta
n(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)^3 +
a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

3.123 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$\frac{\left(a^2-b^2\right)\sin(c+dx)}{d\left(a^2+b^2\right)^2}+\frac{2ab\cos(c+dx)}{d\left(a^2+b^2\right)^2}-\frac{b^3}{d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}-\frac{3ab^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+d}{\sqrt{a^2+b^2}}\right.}{d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] (-3*a*b^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2
 + b^2)^(5/2)*d) + (2*a*b*Cos[c + d*x])/((a^2 + b^2)^2*d) + ((a^2 - b^2)*Si
n[c + d*x])/((a^2 + b^2)^2*d) - b^3/((a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Si
n[c + d*x]))
```

Rubi [A] time = 1.04833, antiderivative size = 231, normalized size of antiderivative = 1.67, number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {6742, 639, 203, 638, 618, 206}

$$-\frac{2b^3\left(a+b\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2\left(\left(a^2-b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+2ab\right)}{d\left(a^2+b^2\right)^2\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)}+\frac{2b^4\tanh^{-1}\left(\frac{b-}{}\right.}{ad\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^4*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5
/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b
^2]])/(a*(a^2 + b^2)^(5/2)*d) + (2*(2*a*b + (a^2 - b^2)*Tan[(c + d*x)/2]))/
((a^2 + b^2)^2*d*(1 + Tan[(c + d*x)/2]^2)) - (2*b^3*(a + b*Tan[(c + d*x)/2]
))/(a*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
```

```
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{2\,\mathrm{Subst}\left(\int \frac{(1-x^2)^3}{(1+x^2)^2(a+2bx-ax^2)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2\,\mathrm{Subst}\left(\int \left(\frac{2(a^2-b^2-2abx)}{(a^2+b^2)^2(1+x^2)^2} + \frac{-a^2+b^2}{(a^2+b^2)^2(1+x^2)} - \frac{2b^3x}{a(a^2+b^2)(-a-2bx+ax^2)^2} - \frac{b^2(3a^2+b}{a(a^2+b^2)^2(-a-2}\right.\right.}{d}$$

$$= \frac{4\,\mathrm{Subst}\left(\int \frac{a^2-b^2-2abx}{(1+x^2)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2+b^2)^2 d} - \frac{(2(a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{(a^2+b^2)^2 d}$$

$$= -\frac{(a^2-b^2)x}{(a^2+b^2)^2} + \frac{2\left(2ab+(a^2-b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2+b^2)^2 d\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{2b^3(a+}{a(a^2+b^2)^2 d\,(a+2b\tan}$$

$$= -\frac{2b^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}d} + \frac{2\left(2ab+(a^2-b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2+b^2)^2 d\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}d} - \frac{2b^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a(a^2+b^2)^{5/2}d} + \frac{2(2a}{(a^2+}$$

Mathematica [A] time = 0.774981, size = 130, normalized size = 0.94

$$\frac{\frac{a(a^2+b^2)\sin(2(c+dx))+b(a^2+b^2)\cos(2(c+dx))+3b(a^2-b^2)}{(a^2+b^2)^2(a\cos(c+dx)+b\sin(c+dx))} + \frac{12ab^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^
(5/2) + (3*b*(a^2 - b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*S
in[2*(c + d*x)])/((a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

Maple [A] time = 0.185, size = 172, normalized size = 1.3

$$\frac{1}{d}\left(-2\,\frac{b^2}{\left(a^2+b^2\right)^2}\left(\frac{1}{\left(\tan\left(1/2\,dx+c/2\right)\right)^2 a-2\,\tan\left(1/2\,dx+c/2\right)b-a}\left(-\frac{b^2\tan\left(1/2\,dx+c/2\right)}{a}-b\right)-3\,\frac{a}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(tan(1/2*d*x+1/2*c)^
2*a-2*tan(1/2*d*x+1/2*c)*b-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2
*d*x+1/2*c)-2*a*b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.553538, size = 697, normalized size = 5.05

$$\frac{2\,a^4b-2\,a^2b^3-4\,b^5+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^2+2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(a^2b^2\cos\left(\right.\right.}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)d\cos\left(dx+c\right)+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)
^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*cos
(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*si
n(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(
```

```
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(
d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28279, size = 386, normalized size = 2.8

$$-\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\cdots\right.}{\left(a^5+2\,a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\cdots\right.\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 - a^
4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x +
 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*
c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

3.124 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$-\frac{b}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}$$

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (2*a*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)^2*d) - b/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.136527, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3086, 3483, 3531, 3530}

$$-\frac{b}{d\left(a^2+b^2\right)(a+b\tan(c+dx))}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^2}+\frac{x\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (2*a*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)^2*d) - b/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \int \frac{1}{(a+b\tan(c+dx))^2}\,dx$$
$$= -\frac{b}{\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{\int \frac{a-b\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2}$$
$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} - \frac{b}{\left(a^2+b^2\right)d(a+b\tan(c+dx))} + \frac{(2ab)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{\left(a^2+b^2\right)^2}$$
$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^2} + \frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^2 d} - \frac{b}{\left(a^2+b^2\right)d(a+b\tan(c+$$

Mathematica [C] time = 0.389831, size = 192, normalized size = 2.34

$$\frac{b\sin(c+dx)\left(a^2b\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)+(a+ib)\left(a^2(c+dx)+ab(ic+idx+1)-ib^2\right)\right)+a^2\cos(c+dx)}{ad\left(a^2+b^2\right)^2(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*((a + I*b)^2*(c + d*x) + a*b*Log[(a*Cos[c + d*x] + b*Sin[
c + d*x])^2]) + b*((a + I*b)*((-I)*b^2 + a*b*(1 + I*c + I*d*x) + a^2*(c + d
*x)) + a^2*b*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Sin[c + d*x] - (2*I)
*a^2*b*ArcTan[Tan[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a*(a^2 + b^
```

```
2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A] time = 0.155, size = 130, normalized size = 1.6

$$-\frac{b}{\left(a^{2}+b^{2}\right) d \left(a+b \tan \left(d x+c\right)\right)}+2 \frac{a b \ln \left(a+b \tan \left(d x+c\right)\right)}{d \left(a^{2}+b^{2}\right)^{2}}+\frac{\arctan \left(\tan \left(d x+c\right)\right) a^{2}}{d \left(a^{2}+b^{2}\right)^{2}}-\frac{\arctan \left(\tan \left(d x+c\right)\right) b^{2}}{d \left(a^{2}+b^{2}\right)^{2}}-\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -b/(a^2+b^2)/d/(a+b*tan(d*x+c))+2/d*a*b/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/d/
(a^2+b^2)^2*arctan(tan(d*x+c))*a^2-1/d/(a^2+b^2)^2*arctan(tan(d*x+c))*b^2-1
/d/(a^2+b^2)^2*a*b*ln(tan(d*x+c)^2+1)
```

Maxima [A] time = 1.79684, size = 177, normalized size = 2.16

$$\frac{\frac{2\,ab\log\left(b\tan\left(dx+c\right)+a\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{ab\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{\left(a^{2}-b^{2}\right)\left(dx+c\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{b}{a^{3}+ab^{2}+\left(a^{2}b+b^{3}\right)\tan\left(dx+c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a*b*log(tan(d*x +
c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2
+ b^4) - b/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

Fricas [B] time = 0.518388, size = 389, normalized size = 4.74

$$-\frac{\left(b^{3}-\left(a^{3}-ab^{2}\right)dx\right)\cos\left(dx+c\right)-\left(a^{2}b\cos\left(dx+c\right)+ab^{2}\sin\left(dx+c\right)\right)\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\right.}{\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)d\cos\left(dx+c\right)+\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((b^3 - (a^3 - a*b^2)*d*x)*cos(d*x + c) - (a^2*b*cos(d*x + c) + a*b^2*sin(
d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - (a*b^2 + (a^2*b - b^3)*d*x)*sin(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^
4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.13989, size = 215, normalized size = 2.62

$$\frac{\frac{2\,ab^2\log\left(|b\tan(dx+c)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^2-b^2\right)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab^2\tan(dx+c)+3\,a^2b+b^3}{\left(a^4+2\,a^2b^2+b^4\right)(b\tan(dx+c)+a)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*log(t
an(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2
*a^2*b^2 + b^4) - (2*a*b^2*tan(d*x + c) + 3*a^2*b + b^3)/((a^4 + 2*a^2*b^2
+ b^4)*(b*tan(d*x + c) + a)))/d
```

3.125 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{b}{d\left(a^2+b^2\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -((a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d)) - b/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0665358, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.115, Rules used = {3155, 3074, 206}

$$-\frac{b}{d\left(a^2+b^2\right)\left(a\cos(c+dx)+b\sin(c+dx)\right)}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d)) - b/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))} + \frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= -\frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))} - \frac{a\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c\right.}{(a^2+b^2)\,d}$$

$$= -\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))}$$

Mathematica [A] time = 0.213311, size = 79, normalized size = 0.95

$$\frac{\frac{2a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b}{(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)
 - b/((a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])))/d
```

Maple [A] time = 0.167, size = 118, normalized size = 1.4

$$\frac{1}{d}\left(-2\,\frac{1}{(\tan(1/2\,dx+c/2))^2\,a-2\,\tan(1/2\,dx+c/2)\,b-a}\left(-\frac{b^2\tan(1/2\,dx+c/2)}{(a^2+b^2)\,a}-\frac{b}{a^2+b^2}\right)+2\,\frac{a}{(a^2+b^2)^{3/2}}\mathrm{Artanh}\left(1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/d*(-2*(-b^2/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(tan(1/2*d*x+1/2*
c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.507247, size = 502, normalized size = 6.05

$$-\frac{2a^2b+2b^3-\left(a^2\cos(dx+c)+ab\sin(dx+c)\right)\sqrt{a^2+b^2}\log\left(-\frac{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2a^2-b^2+2\sqrt{a^2+b^2}(b\cos(\cdots}{2ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2}\right.}{2\left(\left(a^5+2a^3b^2+ab^4\right)d\cos(dx+c)+\left(a^4b+2a^2b^3+b^5\right)d\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a^3*b
^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26548, size = 186, normalized size = 2.24

$$-\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{\left(a^3+ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*ta
n(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*ta
n(1/2*d*x + 1/2*c) - a)))/d
```

3.126 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rubi [A] time = 0.0167772, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3075}

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]

[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Mathematica [A] time = 0.0324883, size = 32, normalized size = 1.

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A] time = 0.142, size = 21, normalized size = 0.7

$$-\frac{1}{db\left(a+b\tan\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] -1/d/b/(a+b*tan(d*x+c))
```

Maxima [A] time = 1.06713, size = 28, normalized size = 0.88

$$-\frac{1}{\left(b^{2}\tan\left(dx+c\right)+ab\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b^2*tan(d*x + c) + a*b)*d)
```

Fricas [A] time = 0.49695, size = 132, normalized size = 4.12

$$-\frac{b\cos\left(dx+c\right)-a\sin\left(dx+c\right)}{\left(a^{3}+ab^{2}\right)d\cos\left(dx+c\right)+\left(a^{2}b+b^{3}\right)d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11405, size = 27, normalized size = 0.84

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

3.127 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Si
n[c + d*x]))
```

Rubi [A] time = 0.0794156, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3094, 3770, 3074, 206}

$$\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Si
n[c + d*x]))
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\int \sec(c+dx)\,dx}{b^2}-\frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}$$

$$=\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,c\right.}{}$$

$$=\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\frac{1}{bd(a\cos(c+dx)+b\sin}$$

Mathematica [A] time = 0.73498, size = 120, normalized size = 1.3

$$-\frac{\frac{2a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{b\sec(c+dx)}{a+b\tan(c+dx)}+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -((((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
 + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

Maple [A] time = 0.22, size = 174, normalized size = 1.9

$$2\,\frac{\tan\left(1/2\,dx+c/2\right)}{d\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-2\,\tan\left(1/2\,dx+c/2\right)b-a\right)a}+2\,\frac{1}{db\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-2\,\tan\left(1/2\,dx+c/2\right)b-a\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)/a*tan(1/2*d*x+1/2*c)+
2/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)-2/d/b^2*a/(a^2+b^2)
^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+1/d/b^2*ln
(tan(1/2*d*x+1/2*c)+1)-1/d/b^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.598545, size = 699, normalized size = 7.6

$$-\frac{2\,a^{2}b+2\,b^{3}-\left(a^{2}\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}-2\,a^{2}-b^{2}-2\,\sqrt{a^{2}+b^{2}}(b\cos(}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}+b^{2}}\right.}{2\left(\left(a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b^2)*
```

```
cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a^3 +
a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.28809, size = 224, normalized size = 2.43

$$\frac{\frac{a\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(ab
s(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 +
 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*
x + 1/2*c) - a)*a*b))/d
```

3.128 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{\frac{a}{b^2}+\frac{1}{a}}{d(a\cot(c+dx)+b)}-\frac{2a\log(\tan(c+dx))}{b^3d}-\frac{2a\log(a\cot(c+dx)+b)}{b^3d}+\frac{\tan(c+dx)}{b^2d}$$

```
[Out] (a^(-1) + a/b^2)/(d*(b + a*Cot[c + d*x])) - (2*a*Log[b + a*Cot[c + d*x]])/(
b^3*d) - (2*a*Log[Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d)
```

Rubi [A] time = 0.0962912, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{\frac{a}{b^2}+\frac{1}{a}}{d(a\cot(c+dx)+b)}-\frac{2a\log(\tan(c+dx))}{b^3d}-\frac{2a\log(a\cot(c+dx)+b)}{b^3d}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^(-1) + a/b^2)/(d*(b + a*Cot[c + d*x])) - (2*a*Log[b + a*Cot[c + d*x]])/(
b^3*d) - (2*a*Log[Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x^2(b+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2a}{b^3x}+\frac{a^2+b^2}{b^2(b+ax)^2}+\frac{2a^2}{b^3(b+ax)}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{\frac{1}{a}+\frac{a}{b^2}}{d(b+a\cot(c+dx))}-\frac{2a\log(b+a\cot(c+dx))}{b^3d}-\frac{2a\log(\tan(c+dx))}{b^3d}+\frac{\tan(c}{b^2}$$

Mathematica [A] time = 0.25982, size = 51, normalized size = 0.68

$$\frac{-\frac{a^2+b^2}{a+b\tan(c+dx)}-2a\log(a+b\tan(c+dx))+b\tan(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*Log[a + b*Tan[c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c +
 d*x]))/(b^3*d)
```

Maple [A] time = 0.204, size = 78, normalized size = 1.

$$\frac{\tan(dx+c)}{b^2d}-2\,\frac{a\ln(a+b\tan(dx+c))}{b^3d}-\frac{a^2}{b^3d(a+b\tan(dx+c))}-\frac{1}{db(a+b\tan(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] tan(d*x+c)/b^2/d-2/d*a/b^3*ln(a+b*tan(d*x+c))-1/d/b^3/(a+b*tan(d*x+c))*a^2-
1/d/b/(a+b*tan(d*x+c))
```

Maxima [A] time = 1.19236, size = 81, normalized size = 1.08

$$-\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3}+\frac{2\,a\log(b\tan(dx+c)+a)}{b^3}-\frac{\tan(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
- tan(d*x + c)/b^2)/d
```

Fricas [B] time = 0.535613, size = 440, normalized size = 5.87

$$-\frac{2\,b^2\cos(dx+c)^2-2\,ab\cos(dx+c)\sin(dx+c)-b^2+\left(a^2\cos(dx+c)^2+ab\cos(dx+c)\sin(dx+c)\right)\log\left(2\,ab\cos(c\right.}{ab^3d\cos(dx+c)^2+b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(d
*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(d*x
 + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*d*co
s(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.13786, size = 96, normalized size = 1.28

$$-\frac{\frac{2\,a\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^3}-\frac{\tan\left(dx+c\right)}{b^2}-\frac{2\,ab\tan\left(dx+c\right)+a^2-b^2}{\left(b\tan\left(dx+c\right)+a\right)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
+ c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

3.129 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))}+\frac{3a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b}{\ }\right.}{b^4d}$$

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/(b^4*d) + ArcTanh[Sin[c + d*x]]/(2*b^2*d) + (
(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(
b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*d) - (2*a*Sec[c + d
*x])/(b^3*d) - (a^2 + b^2)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (Sec
[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rubi [A] time = 0.23961, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3106, 3094, 3770, 3074, 206, 3768, 3104}

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))}+\frac{3a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b}{\ }\right.}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```


```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/(b^4*d) + ArcTanh[Sin[c + d*x]]/(2*b^2*d) + (
(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(
b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*d) - (2*a*Sec[c + d
*x])/(b^3*d) - (a^2 + b^2)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (Sec
[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```


Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\int \sec^3(c+dx)\,dx}{b^2} - \frac{(2a)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin}}{b^2}$$

$$= -\frac{2a\sec(c+dx)}{b^3d} - \frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))} + \frac{\sec(c+dx)\tan(c+dx}{2b^2d}$$

$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx}{b^4d}$$

$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx}{b^4d}$$

Mathematica [C] time = 6.11163, size = 709, normalized size = 3.96

$$-\frac{3\left(2a^2+b^2\right)\sec^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{2b^4d(a+b\tan(c+dx))^2} + \frac{3\left(2a^2+b^2\right)\sec^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(b
^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcTanh
[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c
 + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x
])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a + b*Tan[c
 + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a + b*Tan[c + d*
x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) + (2*a*Sec[c +
d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Maple [B] time = 0.247, size = 440, normalized size = 2.5

$$2\frac{a\tan(1/2\,dx+c/2)}{b^2d\left((\tan(1/2\,dx+c/2))^2a-2\tan(1/2\,dx+c/2)\,b-a\right)}+2\frac{\tan(1/2\,dx+c/2)}{d\left((\tan(1/2\,dx+c/2))^2a-2\tan(1/2\,dx+c/2)\,b-a\right)a}+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 2/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)*a*tan(1/2*d*x+1/2
*c)+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)/a*tan(1/2*d*x+1/2
*c)+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)*a^2+2/d/b/(ta
n(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)-6/d/b^4*(a^2+b^2)^(1/2)*a*ar
ctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-1/2/d/b^2/(tan(1/2*
d*x+1/2*c)+1)^2-2/d/b^3/(tan(1/2*d*x+1/2*c)+1)*a+1/2/d/b^2/(tan(1/2*d*x+1/2
*c)+1)+3/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)*a^2+3/2/d/b^2*ln(tan(1/2*d*x+1/2*c)
+1)+1/2/d/b^2/(tan(1/2*d*x+1/2*c)-1)^2+2/d/b^3/(tan(1/2*d*x+1/2*c)-1)*a+1/2
/d/b^2/(tan(1/2*d*x+1/2*c)-1)-3/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)*a^2-3/2/d/b^
2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.697012, size = 860, normalized size = 4.8

$$-\frac{6\,ab^2\cos(dx+c)\sin(dx+c)-2\,b^3+6\left(2\,a^2b+b^3\right)\cos(dx+c)^2-6\left(a^2\cos(dx+c)^3+ab\cos(dx+c)^2\sin(dx+c)\right)\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*x
 + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a^2
 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*
cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2*a^3
+ a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c))*log(
sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos
(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x + c)^3 +
 b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.32164, size = 378, normalized size = 2.11

$$\frac{\frac{3\left(2a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3\left(2a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}+\frac{6\left(a^3+ab^2\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b^4}+\frac{2\left(b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a\cdots\right.}{\left(\tan\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c
```

```
)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*
x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/
2*c) - a)*a*b^3))/d
```

3.130 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=141

$$\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+b^2)^2}{ab^4d(a\cot(c+dx)+b)}-\frac{4a(a^2+b^2)\log(\tan(c+dx))}{b^5d}-\frac{4a(a^2+b^2)\log(a\cot(c+dx)+b}{b^5d}$$

```
[Out] (a^2 + b^2)^2/(a*b^4*d*(b + a*Cot[c + d*x])) - (4*a*(a^2 + b^2)*Log[b + a*C
ot[c + d*x]])/(b^5*d) - (4*a*(a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) + ((3*a
^2 + 2*b^2)*Tan[c + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*
x]^3/(3*b^2*d)
```

Rubi [A] time = 0.149899, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+b^2)^2}{ab^4d(a\cot(c+dx)+b)}-\frac{4a(a^2+b^2)\log(\tan(c+dx))}{b^5d}-\frac{4a(a^2+b^2)\log(a\cot(c+dx)+b}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```


```
[Out] (a^2 + b^2)^2/(a*b^4*d*(b + a*Cot[c + d*x])) - (4*a*(a^2 + b^2)*Log[b + a*C
ot[c + d*x]])/(b^5*d) - (4*a*(a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) + ((3*a
^2 + 2*b^2)*Tan[c + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*
x]^3/(3*b^2*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4(b+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x^4}-\frac{2a}{b^3x^3}+\frac{3a^2+2b^2}{b^4x^2}-\frac{4a\left(a^2+b^2\right)}{b^5x}+\frac{\left(a^2+b^2\right)^2}{b^4(b+ax)^2}+\frac{4a^2\left(a^2+b^2\right)}{b^5(b+ax)}\right)dx, x, \cot(c$$

$$= \frac{\left(a^2+b^2\right)^2}{ab^4d(b+a\cot(c+dx))} - \frac{4a\left(a^2+b^2\right)\log(b+a\cot(c+dx))}{b^5d} - \frac{4a\left(a^2+b^2\right)\log(}{b^5d}$$

Mathematica [A] time = 2.81513, size = 122, normalized size = 0.87

$$\frac{4b\left(2a^2+b^2\right)\tan(c+dx)+\frac{b^4\sec^4(c+dx)-4\left(a^2+b^2\right)\left(3a^2\log(a+b\tan(c+dx))+a^2+3ab\tan(c+dx)\log(a+b\tan(c+dx))+b^2\right)}{a+b\tan(c+dx)}-2ab^2\tan^2(c+dx)}{3b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (4*b*(2*a^2 + b^2)*Tan[c + d*x] - 2*a*b^2*Tan[c + d*x]^2 + (b^4*Sec[c + d*x
]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a^2*Log[a + b*Tan[c + d*x]] + 3*a*b*Log[
a + b*Tan[c + d*x]]*Tan[c + d*x]))/(a + b*Tan[c + d*x]))/(3*b^5*d)
```

Maple [A] time = 0.216, size = 174, normalized size = 1.2

$$\frac{(\tan(dx+c))^3}{3\,b^2d} - \frac{a\,(\tan(dx+c))^2}{b^3d} + 3\,\frac{a^2\tan(dx+c)}{db^4} + 2\,\frac{\tan(dx+c)}{b^2d} - 4\,\frac{a^3\ln(a+b\tan(dx+c))}{db^5} - 4\,\frac{a\ln(a+b\tan(}{b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/b^2/d-a*tan(d*x+c)^2/b^3/d+3/d/b^4*a^2*tan(d*x+c)+2*tan(d*
x+c)/b^2/d-4/d*a^3/b^5*ln(a+b*tan(d*x+c))-4/d*a/b^3*ln(a+b*tan(d*x+c))-1/d/
b^5/(a+b*tan(d*x+c))*a^4-2/d/b^3/(a+b*tan(d*x+c))*a^2-1/d/b/(a+b*tan(d*x+c)
)
```

Maxima [A] time = 1.13593, size = 155, normalized size = 1.1

$$-\frac{\frac{3\left(a^4+2a^2b^2+b^4\right)}{b^6\tan(dx+c)+ab^5}-\frac{b^2\tan(dx+c)^3-3ab\tan(dx+c)^2+3\left(3a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{12\left(a^3+ab^2\right)\log(b\tan(dx+c)+a)}{b^5}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x +
 c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(a^
3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [B] time = 0.58217, size = 656, normalized size = 4.65

$$-\frac{4\left(3a^2b^2+2b^4\right)\cos\left(dx+c\right)^4-b^4-2\left(3a^2b^2+2b^4\right)\cos\left(dx+c\right)^2+6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4+\left(a^3b+ab^3\right)\cos\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*co
s(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(
d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos
(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c) - 2*(
3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4 +
b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A] time = 1.14558, size = 201, normalized size = 1.43

$$-\frac{\frac{12\left(a^3+ab^2\right)\log(|b\tan(dx+c)+a|)}{b^5}-\frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6}-\frac{3\left(4\,a^3b\tan(dx+c)+4\,ab^3\tan(dx+c)+3\,a^4+2\,a^2b^2}{(b\tan(dx+c)+a)b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c)
^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c))/
b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2 -
b^4)/((b*tan(d*x + c) + a)*b^5))/d
```

3.131 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=216

$$\frac{a\left(a^2-3b^2\right)\sin(c+dx)}{d\left(a^2+b^2\right)^3}+\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d\left(a^2+b^2\right)^3}+\frac{b^4\sin(c+dx)}{2ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\ldots}{2ad\left(a^2+b^2\right)^3(\ldots}$$

```
[Out] (-3*b^2*(4*a^2 - b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((
a^2 + b^2)^(7/2)*d) + (b*(3*a^2 - b^2)*Cos[c + d*x])/((a^2 + b^2)^3*d) + (a
*(a^2 - 3*b^2)*Sin[c + d*x])/((a^2 + b^2)^3*d) + (b^4*Sin[c + d*x])/(2*a*(a
^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^3*(8*a^2 + b^2))/(2
*a*(a^2 + b^2)^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rubi [B] time = 1.74227, antiderivative size = 492, normalized size of antiderivative = 2.28, number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {6742, 639, 203, 638, 618, 206, 614}

$$-\frac{3b^4\left(a^2+2b^2\right)\left(b-a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d\left(a^2+b^2\right)^3\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2b^4\left(\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+ab\right)}{a^3d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^4*(a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a
^2*(a^2 + b^2)^(7/2)*d) + (4*b^4*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(c + d*
x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(7/2)*d) - (2*b^2*(6*a^4 + 3*a^2*
b^2 + b^4)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b
^2)^(7/2)*d) + (2*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[(c + d*x)/2]))/((a
^2 + b^2)^3*d*(1 + Tan[(c + d*x)/2]^2)) + (2*b^4*(a*b + (a^2 + 2*b^2)*Tan[(
c + d*x)/2]))/(a^3*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d
*x)/2]^2)^2) - (3*b^4*(a^2 + 2*b^2)*(b - a*Tan[(c + d*x)/2]))/(a^3*(a^2 + b
^2)^3*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)) - (4*b^3*(2*a^4
- b^4 + a*b*(3*a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2)^3*d*(a + 2*
b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rule 639

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*
a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 638

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p +
 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a
*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 614

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x
)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p +
 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
```

egerQ[4*p]

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(1+x^2\right)^2\left(a+2bx-ax^2\right)^3}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(\frac{2\left(a\left(a^2-3b^2\right)-b\left(3a^2-b^2\right)x\right)}{\left(a^2+b^2\right)^3\left(1+x^2\right)^2} - \frac{a\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3\left(1+x^2\right)} + \frac{4b^3\left(-b\left(a^2+b^2\right)-a\left(2a^2+b^2\right)x\right)}{a^3\left(a^2+b^2\right)^2\left(a+2bx-ax^2\right)^2} - \cdots\right.\right.}{d}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{a\left(a^2-3b^2\right)-b\left(3a^2-b^2\right)x}{\left(1+x^2\right)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2+b^2\right)^3 d} - \frac{\left(2a\left(a^2-3b^2\right)\right)\text{Subst}\left(\cdots\right.}{\left(a\cdots\right.}$$

$$= -\frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{2\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2+b^2\right)^3 d\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{\cdots}{a^3\left(a^2+b^2\cdots\right.}$$

$$= -\frac{2b^2\left(6a^4+3a^2b^2+b^4\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} + \frac{2\left(b\left(3a^2-b^2\right)+a\left(a^2-3\cdots\right.\right.}{\left(a^2+b^2\right)^3 d\left(1+\tan\cdots\right.}$$

$$= \frac{4b^4\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} - \frac{2b^2\left(6a^4+3a^2b^2+b^4\right)\tanh^{-1}\left(\frac{b-a\cdots}{\cdots}\right.}{a^2\left(a^2+b^2\right)^{7/2}d}$$

$$= -\frac{3b^4\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} + \frac{4b^4\left(3a^2+2b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c\cdots\right.}{\sqrt{a^2+b^2}}\right.}{a^2\left(a^2+b^2\right)^{7/2}d}$$

Mathematica [C] time = 1.03486, size = 211, normalized size = 0.98

$$\frac{\frac{2a\left(a^2-3b^2\right)\sin(c+dx)}{\left(a^2+b^2\right)^3} - \frac{2b\left(b^2-3a^2\right)\cos(c+dx)}{\left(a^2+b^2\right)^3} - \frac{b^3\left(8a^2+b^2\right)}{a\left(a^2+b^2\right)^3(a\cos(c+dx)+b\sin(c+dx))} - \frac{6b^2\left(b^2-4a^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} + \frac{b^4\sin\cdots}{a(a-ib)^2(a+ib)^2(a\cos\cdots}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])
/(a^2 + b^2)^(7/2) - (2*b*(-3*a^2 + b^2)*Cos[c + d*x])/(a^2 + b^2)^3 + (2*a
*(a^2 - 3*b^2)*Sin[c + d*x])/(a^2 + b^2)^3 + (b^4*Sin[c + d*x])/(a*(a - I*b
)^2*(a + I*b)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^3*(8*a^2 + b^2))/
(a*(a^2 + b^2)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

Maple [A] time = 0.213, size = 283, normalized size = 1.3

$$\frac{1}{d}\left(-2\,\frac{b^2}{\left(a^2+b^2\right)^3}\left(\frac{1}{\left((\tan(1/2\,dx+c/2))^2\,a-2\,\tan(1/2\,dx+c/2)\,b-a\right)^2}\left(-1/2\,\frac{b^2\left(9\,a^2+2\,b^2\right)(\tan(1/2\,dx+c/2))^3}{a}-1/2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2
*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(23*a^2+2*b^2)
/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*
x+1/2*c)*b-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*
x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*
b^2)*tan(1/2*d*x+1/2*c)-3*a^2*b+b^3)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.609099, size = 1073, normalized size = 4.97

$$\frac{4\left(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}\right)\cos\left(dx+c\right)^{3}-3\left(4\,a^{2}b^{4}-b^{6}+\left(4\,a^{4}b^{2}-5\,a^{2}b^{4}+b^{6}\right)\cos\left(dx+c\right)^{2}+2\left(4\,a^{3}b^{3}-ab^{5}\right)\cos}{4\left(\left(a^{10}+3\,a^{8}b^{2}+2\,a^{6}b^{4}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
- b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b^5
)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*co
s(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b^7)*co
s(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*
a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b
^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3
+ 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4
*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37751, size = 539, normalized size = 2.5

$$-\frac{\frac{3\left(4\,a^{2}b^{2}-b^{4}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}-\frac{4\left(a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^{2}b-b^{3}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}-\frac{2\left(9\,a^{3}b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+2\,ab^{6}\tan\left(\frac{1}{2}\,dx+\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/2*c
) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 -
15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 23*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3 - a^2*b^
5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.132 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=122

$$-\frac{2ab}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+$$

```
[Out] (a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3 + (b*(3*a^2 - b^2)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/((a^2 + b^2)^3*d) - b/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (2*a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.212299, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3483, 3529, 3531, 3530}

$$-\frac{2ab}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))}-\frac{b}{2d\left(a^2+b^2\right)(a+b\tan(c+dx))^2}+\frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3 + (b*(3*a^2 - b^2)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/((a^2 + b^2)^3*d) - b/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (2*a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```


Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{1}{(a+b\tan(c+dx))^3}\,dx$$

$$= -\frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2}$$

$$= -\frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{2ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))} + \frac{\int \frac{a^2-b^2-2\ldots}{a+b\,\mathrm{t}\ldots}}{\left(a^2\ldots\right.}$$

$$= \frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} - \frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{2ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+d\ldots}$$

$$= \frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{\ldots}{2\left(a^2+b^2\right)d(a\ldots}$$

Mathematica [C] time = 1.15967, size = 154, normalized size = 1.26

$$\frac{\frac{2a\left(a^2-3b^2\right)(c+dx)}{\left(a^2+b^2\right)^3}+\frac{6b^2\sin(c+dx)}{\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}-\frac{2b\left(b^2-3a^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3}-\frac{b^3}{(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a*(a^2 - 3*b^2)*(c + d*x))/(a^2 + b^2)^3 - (2*b*(-3*a^2 + b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/(a^2 + b^2)^3 - b^3/((a - I*b)^2*(a + I*b)^2*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (6*b^2*Sin[c + d*x])/((a^2 + b^2)^2*
(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

Maple [A] time = 0.184, size = 219, normalized size = 1.8

$$-\frac{b}{\left(2a^2+2b^2\right)d\left(a+b\tan(dx+c)\right)^2}+3\frac{b\ln(a+b\tan(dx+c))a^2}{d\left(a^2+b^2\right)^3}-\frac{b^3\ln(a+b\tan(dx+c))}{d\left(a^2+b^2\right)^3}-2\frac{ab}{\left(a^2+b^2\right)^2 d(a+b\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2+3/d*b/(a^2+b^2)^3*ln(a+b*tan(d*x+c))*
a^2-1/d*b^3/(a^2+b^2)^3*ln(a+b*tan(d*x+c))-2*a*b/(a^2+b^2)^2/d/(a+b*tan(d*x
+c))-3/2/d/(a^2+b^2)^3*ln(tan(d*x+c)^2+1)*a^2*b+1/2/d/(a^2+b^2)^3*ln(tan(d*
x+c)^2+1)*b^3+1/d/(a^2+b^2)^3*arctan(tan(d*x+c))*a^3-3/d/(a^2+b^2)^3*arctan
(tan(d*x+c))*a*b^2
```

Maxima [B] time = 1.65487, size = 649, normalized size = 5.32

$$\frac{\frac{2\left(a^3-3ab^2\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\left(3a^2b-b^3\right)\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(3a^2b-b^3\right)\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{\cdots}{a^8+2a^6b^2+a^4b^4+\frac{4\left(a^7b+2a^5b^3\cdots\right)}{\cos\cdots}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*(a^3 - 3*a*b^2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + (3*a^2*b - b^3)*log(-a - 2*b*sin(d*x + c)/(cos(d*x +
c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6) - (3*a^2*b - b^3)*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*((3*a^3*b^2 + a*b^4)*sin(d*x + c)/(co
s(d*x + c) + 1) + (5*a^2*b^3 + b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (
3*a^3*b^2 + a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 +
a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) -
 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a
^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*
a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Fricas [B] time = 0.547234, size = 748, normalized size = 6.13

$$\frac{5\,a^{2}b^{3}-b^{5}+2\left(a^{3}b^{2}-3\,ab^{4}\right)dx-2\left(6\,a^{2}b^{3}-\left(a^{5}-4\,a^{3}b^{2}+3\,ab^{4}\right)dx\right)\cos\left(dx+c\right)^{2}+2\left(3\,a^{3}b^{2}-3\,ab^{4}+2\left(a^{4}b-3\,a^{2}b^{3}\right)\right.}{2\left(\left(a^{8}+2\,a^{6}b^{2}-2\,a^{2}b^{6}-b^{8}\right)d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^2*b^3 - b^5 + 2*(a^3*b^2 - 3*a*b^4)*d*x - 2*(6*a^2*b^3 - (a^5 - 4*
a^3*b^2 + 3*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 3*a*b^4 + 2*(a^4*b
- 3*a^2*b^3)*d*x)*cos(d*x + c)*sin(d*x + c) + (3*a^2*b^3 - b^5 + (3*a^4*b -
 4*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c)*sin(d
*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3
*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^6*b^2 + 3*a^
4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.26319, size = 358, normalized size = 2.93

$$\frac{\frac{2\left(a^3-3\,ab^2\right)(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} - \frac{\left(3\,a^2b-b^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6} + \frac{2\left(3\,a^2b^2-b^4\right)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7} - \frac{9\,a^2b^3\tan(dx+c)^2-3\,b^5\tan(dx+c)^2+22\,a^3b^2\tan(dx+c)-2}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)(b\tan(dx}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a
^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2
*(3*a^2*b^2 - b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) - (9*a^2*b^3*tan(d*x + c)^2 - 3*b^5*tan(d*x + c)^2 + 22*a^3*b^2*
tan(d*x + c) - 2*a*b^4*tan(d*x + c) + 14*a^4*b + 3*a^2*b^3 + b^5)/((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) + a)^2))/d
```

3.133 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=119

$$\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}-\frac{b\left(\left(4a^2+b^2\right)\cos(c+dx)+3ab\sin(c+dx)\right)}{2d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[Out] ((2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d) - (b*((4*a^2 + b^2)*Cos[c + d*x] + 3*a*b*Sin[c + d*x]))/(2*(a
^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Rubi [A] time = 0.587705, antiderivative size = 225, normalized size of antiderivative = 1.89, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 12, 618, 206}

$$\frac{2b^2\left(\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+ab\right)}{a^3d\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{b\left(ab\left(5a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+3a^2b^2+4a^4+2b\right.}{a^3d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 +
 b^2)^(5/2)*d)) + (2*b^2*(a*b + (a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2
+ b^2)*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)^2) - (b*(4*a^4 +
 3*a^2*b^2 + 2*b^4 + a*b*(5*a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2
)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
```

```
- 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+2bx-ax^2\right)^3}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{8\left(a^4+2\right.}{a^3}}{\ldots}\right.}{\ldots}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b\left(4a^4+3a^2b\ldots\right.}{a^3\left(a^2+b^2\right)^2 d\left(\ldots\right.}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b\left(4a^4+3a^2b\ldots\right.}{a^3\left(a^2+b^2\right)^2 d\left(\ldots\right.}$$

$$= \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b\left(4a^4+3a^2b\ldots\right.}{a^3\left(a^2+b^2\right)^2 d\left(\ldots\right.}$$

$$= -\frac{\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d} + \frac{2b^2\left(ab+\left(a^2+2b^2\right)\tan\left(\frac{1}{2}(c\ldots\right.\right.}{a^3\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-\ldots\right.}$$

Mathematica [A] time = 0.655404, size = 119, normalized size = 1.

$$\frac{\frac{2\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{b\left(\left(4a^2+b^2\right)\cos(c+dx)+3ab\sin(c+dx)\right)}{\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(5/2) - (b*((4*a^2 + b^2)*Cos[c + d*x] + 3*a*b*Sin[c + d*x]))/((a^2
+ b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] time = 0.206, size = 280, normalized size = 2.4

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-2\,\tan\left(1/2\,dx+c/2\right)b-a\right)^{2}}\left(-1/2\,\frac{b^{2}\left(5\,a^{2}+2\,b^{2}\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^{3}}{a\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}-1/2\,\frac{b\left(4\,a^{4}-7\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-
1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^2+
1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4*a^
2+b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-
a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.586519, size = 795, normalized size = 6.68

$$-\frac{\left(2\,a^{2}b^{2}-b^{4}+\left(2\,a^{4}-3\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}+2\left(2\,a^{3}b-ab^{3}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)\sin}{}\right.}{4\left(\left(a^{8}+2\,a^{6}b^{2}-2\,a^{2}b^{6}-b^{8}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b+3\,a^{5}b^{3}+3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^
3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cos(d*x
 + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b
^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35814, size = 396, normalized size = 3.33

$$-\frac{\frac{(2a^2-b^2)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(5a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b^5\tan\left(\frac{1}{2}\right.\right.}{(a^6+2a^4b^2+a^2b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}d\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b
^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2
*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*tan(1
/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*d*x +
 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2*a^4*b
^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))
/d
```

3.134 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[Out] -1/(2*b*d*(a + b*Tan[c + d*x])^2)
```

Rubi [A] time = 0.0312509, antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3088, 37}

$$-\frac{\cot^2(c+dx)}{2bd(a\cot(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -Cot[c + d*x]^2/(2*b*d*(b + a*Cot[c + d*x])^2)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x}{(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\cot^2(c+dx)}{2bd(b+a\cot(c+dx))^2}$$

Mathematica [B] time = 0.118288, size = 57, normalized size = 2.59

$$\frac{a\sin(2(c+dx)) - b\cos(2(c+dx))}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(b*Cos[2*(c + d*x)]) + a*Sin[2*(c + d*x)])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.168, size = 21, normalized size = 1.

$$-\frac{1}{2\,db\,(a+b\tan(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Maxima [B] time = 1.1391, size = 231, normalized size = 10.5

$$\frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1} + \frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a^4 + \frac{4\,a^3b\sin(dx+c)}{\cos(dx+c)+1} - \frac{4\,a^3b\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{2\left(a^4-2\,a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2*(a*sin(d*x + c)/(cos(d*x + c) + 1) + b*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a^4 + 4*a^3*b*sin(d*x + c)/(co
s(d*x + c) + 1) - 4*a^3*b*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 - 2*(a^4 - 2*a^2*b^2)*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*d)
```

Fricas [B] time = 0.482848, size = 313, normalized size = 14.23

$$-\frac{4\,a^2b\cos\left(dx+c\right)^2-a^2b+b^3-2\left(a^3-ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^4b^2+2\,a^2b^4+b^6\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*s
in(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^
6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23095, size = 27, normalized size = 1.23

$$-\frac{1}{2\left(b\tan\left(dx+c\right)+a\right)^2bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

3.135 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

```
[Out] -ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^
(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0494178, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3076, 3074, 206}

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] -ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*(a^2 + b^2)^
(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx &= -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{2\left(a^2+b^2\right)}\\ &= -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c\right.}{2\left(a^2+b^2\right)}\\ &= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}\end{aligned}$$

Mathematica [C] time = 0.262403, size = 132, normalized size = 1.28

$$\frac{\left(a^2+b^2\right)(a\sin(c+dx)-b\cos(c+dx))+2\sqrt{a^2+b^2}(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{2d(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [A] time = 0.184, size = 191, normalized size = 1.9

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^2a-2\,\tan\left(1/2\,dx+c/2\right)b-a\right)^2}\left(-1/2\,\frac{\left(a^2+2\,b^2\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^3}{\left(a^2+b^2\right)a}-1/2\,\frac{b\left(a^2-2\,b^2\right)(\tan}{(a^2+}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.516769, size = 679, normalized size = 6.59

$$\frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2-2\,a^2-b^2+}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)c}\right.}{4\left(\left(a^6+a^4b^2-a^2b^4-b^6\right)d\cos\left(dx+c\right)^2+2\left(a^5b+2\,a^3b^3+ab^5\right)d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26274, size = 298, normalized size = 2.89

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^2\tan\right.}{\left(a^4+a^2b^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

3.136 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(a\cot(c+dx)+b)}-\frac{\frac{b}{a^2}+\frac{1}{b}}{2d(a\cot(c+dx)+b)^2}+\frac{\log(a\cot(c+dx)+b)}{b^3d}+\frac{\log(\tan(c+dx))}{b^3d}$$

```
[Out] -(b^(-1) + b/a^2)/(2*d*(b + a*Cot[c + d*x])^2) + (a^(-2) - b^(-2))/(d*(b +
a*Cot[c + d*x])) + Log[b + a*Cot[c + d*x]]/(b^3*d) + Log[Tan[c + d*x]]/(b^3
*d)
```

Rubi [A] time = 0.103137, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {3088, 894}

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(a\cot(c+dx)+b)}-\frac{\frac{b}{a^2}+\frac{1}{b}}{2d(a\cot(c+dx)+b)^2}+\frac{\log(a\cot(c+dx)+b)}{b^3d}+\frac{\log(\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(b^(-1) + b/a^2)/(2*d*(b + a*Cot[c + d*x])^2) + (a^(-2) - b^(-2))/(d*(b +
a*Cot[c + d*x])) + Log[b + a*Cot[c + d*x]]/(b^3*d) + Log[Tan[c + d*x]]/(b^3
*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
```

```
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x}+\frac{-a^2-b^2}{ab(b+ax)^3}+\frac{-a^2+b^2}{ab^2(b+ax)^2}-\frac{a}{b^3(b+ax)}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\frac{1}{b}+\frac{b}{a^2}}{2d(b+a\cot(c+dx))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(b+a\cot(c+dx))}+\frac{\log(b+a\cot(c+dx))}{b^3d}+\frac{\log(\ldots}{}$$

Mathematica [A] time = 0.524883, size = 57, normalized size = 0.66

$$\frac{-\frac{a^2+b^2}{2(a+b\tan(c+dx))^2}+\frac{2a}{a+b\tan(c+dx)}+\log(a+b\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Log[a + b*Tan[c + d*x]] - (a^2 + b^2)/(2*(a + b*Tan[c + d*x])^2) + (2*a)/(
a + b*Tan[c + d*x]))/(b^3*d)
```

Maple [A] time = 0.234, size = 84, normalized size = 1.

$$\frac{\ln(a+b\tan(dx+c))}{db^3}+2\,\frac{a}{db^3(a+b\tan(dx+c))}-\frac{a^2}{2\,db^3(a+b\tan(dx+c))^2}-\frac{1}{2\,db\,(a+b\tan(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/d/b^3*ln(a+b*tan(d*x+c))+2/d*a/b^3/(a+b*tan(d*x+c))-1/2/d/b^3/(a+b*tan(d*
x+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2
```

Maxima [B] time = 1.19974, size = 425, normalized size = 4.94

$$-\frac{\frac{2\left(\frac{(a^3-ab^2)\sin(dx+c)}{\cos(dx+c)+1}+\frac{(3a^2b-b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3-ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4b^2+\frac{4a^3b^3\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a^3b^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^4b^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(a^4b^2-2a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}-\frac{\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^3}+\frac{\log\left(\cdots\right.}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2*((a^3 - a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + (3*a^2*b - b^3)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 - a*b^2)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(d*x + c)/(cos(d*x + c) + 1) - 4*a^3*b^3*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*b^2*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 2*(a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - log(-
a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) +
1)^2)/b^3 + log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^3 + log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/b^3)/d
```

Fricas [B] time = 0.562974, size = 647, normalized size = 7.52

$$\frac{4a^2b^2\cos(dx+c)^2-3a^2b^2-b^4-2(a^3b-ab^3)\cos(dx+c)\sin(dx+c)+\left(a^2b^2+b^4+(a^4-b^4)\cos(dx+c)^2+2(a^3b\cdots\right.}{2\left(\left(a^4\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x
 + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3*b
 + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x +
c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/((
a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*sin(d*
x + c) + (a^2*b^5 + b^7)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27015, size = 84, normalized size = 0.98

$$\frac{\frac{2 \log(|b \tan(dx+c)+a|)}{b^3} - \frac{3 b \tan(dx+c)^2+2 a \tan(dx+c)+b}{(b \tan(dx+c)+a)^2 b^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*x
 + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

3.137 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=260

$$-\frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\sqrt{a^2+b^2}}-\frac{3a\tanh$$

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) - (2*a^2*ArcTanh[(b*Cos[c + d*x] - a*S
in[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - ArcTanh[(b*Cos[c +
 d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*b^2*Sqrt[a^2 + b^2]*d) - (Sqrt[
a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4
*d) + Sec[c + d*x]/(b^3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*b^2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2) + (2*a)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c
+ d*x]))
```

Rubi [A] time = 0.28615, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3106, 3076, 3074, 206, 3104, 3770, 3094}

$$-\frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\sqrt{a^2+b^2}}-\frac{3a\tanh$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```



```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) - (2*a^2*ArcTanh[(b*Cos[c + d*x] - a*S
in[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - ArcTanh[(b*Cos[c +
 d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*b^2*Sqrt[a^2 + b^2]*d) - (Sqrt[
a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4
*d) + Sec[c + d*x]/(b^3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*b^2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2) + (2*a)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c
+ d*x]))
```


Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
```

```
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
```

```
& LtQ[n, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} - \frac{(2a)\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{}{(a\cos(}}{}$$

$$= \frac{\sec(c+dx)}{b^3 d} - \frac{b\cos(c+dx)-a\sin(c+dx)}{2b^2 d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{2a}{b^3 d(a\cos(c+dx)+b\sin}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4 d} + \frac{\sec(c+dx)}{b^3 d} - \frac{b\cos(c+dx)-a\sin(c+dx)}{2b^2 d(a\cos(c+dx)+b\sin(c+dx))^2}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4 d} - \frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}\,d} - \frac{\tanh^{-1}\left(\frac{b\cos(c}{}\right.}{2b^2\sqrt{a^2}}$$

Mathematica [A] time = 2.44481, size = 396, normalized size = 1.52

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2\left(a^2+b^2\right)\sin(c+dx)}{a} + \frac{6\left(2a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}} + \frac{2b\sin}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c +
 d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a + 2*
b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/Sqrt[
a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c + d*x)/2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
)/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

Maple [B] time = 0.29, size = 611, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] -3/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a*tan(1/2*d*x+
1/2*c)^3+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a*tan(1/2*
d*x+1/2*c)^3-4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^
2*tan(1/2*d*x+1/2*c)^2+9/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b
-a)^2*tan(1/2*d*x+1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*
c)*b-a)^2/a^2*tan(1/2*d*x+1/2*c)^2+13/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1
/2*d*x+1/2*c)*b-a)^2*a*tan(1/2*d*x+1/2*c)-2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan
(1/2*d*x+1/2*c)*b-a)^2/a*tan(1/2*d*x+1/2*c)+4/d/b^3/(tan(1/2*d*x+1/2*c)^2*a
-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^2-1/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*
x+1/2*c)*b-a)^2+6/d/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)
-2*b)/(a^2+b^2)^(1/2))*a^2+3/d/b^2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+1/d/b^3/(tan(1/2*d*x+1/2*c)+1)-3/d*a/b^4*
ln(tan(1/2*d*x+1/2*c)+1)-1/d/b^3/(tan(1/2*d*x+1/2*c)-1)+3/d*a/b^4*ln(tan(1/
2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.718939, size = 1175, normalized size = 4.52

$$\frac{4\,a^{2}b^{3}+4\,b^{5}+6\left(2\,a^{4}b+a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{2}+18\left(a^{3}b^{2}+ab^{4}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(\left(2\,a^{4}-a^{2}b^{2}-b^{4}\right)\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(a
^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*cos(
d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^2 +
 b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x +
c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*co
s(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin(d*x + c
) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos(d*x + c)
^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x + c)^2*sin
(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.61047, size = 424, normalized size = 1.63

$$-\frac{\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{3\left(2\,a^2+b^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b^3}+\frac{2\left(3\,a^3b\tan\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b
```

```
 - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2
)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3*a^3
*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*
d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2
*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) - 4*a^
4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a
^2*b^3))/d
```

3.138 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{\left(a^2+b^2\right)^2}{2a^2b^3d(a\cot(c+dx)+b)^2}-\frac{\left(3a^2-b^2\right)\left(a^2+b^2\right)}{a^2b^4d(a\cot(c+dx)+b)}+\frac{2\left(3a^2+b^2\right)\log(\tan(c+dx))}{b^5d}+\frac{2\left(3a^2+b^2\right)\log(a\cot(c+d$$

```
[Out] -(a^2 + b^2)^2/(2*a^2*b^3*d*(b + a*Cot[c + d*x])^2) - ((3*a^2 - b^2)*(a^2 +
 b^2))/(a^2*b^4*d*(b + a*Cot[c + d*x])) + (2*(3*a^2 + b^2)*Log[b + a*Cot[c
+ d*x]])/(b^5*d) + (2*(3*a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) - (3*a*Tan[c
 + d*x])/(b^4*d) + Tan[c + d*x]^2/(2*b^3*d)
```

Rubi [A] time = 0.167825, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$-\frac{\left(a^2+b^2\right)^2}{2a^2b^3d(a\cot(c+dx)+b)^2}-\frac{\left(3a^2-b^2\right)\left(a^2+b^2\right)}{a^2b^4d(a\cot(c+dx)+b)}+\frac{2\left(3a^2+b^2\right)\log(\tan(c+dx))}{b^5d}+\frac{2\left(3a^2+b^2\right)\log(a\cot(c+d$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] -(a^2 + b^2)^2/(2*a^2*b^3*d*(b + a*Cot[c + d*x])^2) - ((3*a^2 - b^2)*(a^2 +
 b^2))/(a^2*b^4*d*(b + a*Cot[c + d*x])) + (2*(3*a^2 + b^2)*Log[b + a*Cot[c
+ d*x]])/(b^5*d) + (2*(3*a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) - (3*a*Tan[c
 + d*x])/(b^4*d) + Tan[c + d*x]^2/(2*b^3*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^3(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x^3}-\frac{3a}{b^4x^2}+\frac{2\left(3a^2+b^2\right)}{b^5x}-\frac{\left(a^2+b^2\right)^2}{ab^3(b+ax)^3}+\frac{-3a^4-2a^2b^2+b^4}{ab^4(b+ax)^2}-\frac{2a\left(3a^2+b^2\right)}{b^5(b+ax)}\right)dx, x,\right.}{d}$$
$$= -\frac{\left(a^2+b^2\right)^2}{2a^2b^3d(b+a\cot(c+dx))^2}-\frac{\left(3a^2-b^2\right)\left(a^2+b^2\right)}{a^2b^4d(b+a\cot(c+dx))}+\frac{2\left(3a^2+b^2\right)\log(b+a}{b^5d}$$

Mathematica [A] time = 3.10517, size = 140, normalized size = 0.87

$$\frac{-2a\left(-\frac{a^2+b^2}{a+b\tan(c+dx)}-2a\log(a+b\tan(c+dx))+b\tan(c+dx)\right)+2\left(a^2+b^2\right)\left(\frac{3a^2+4ab\tan(c+dx)-b^2}{2(a+b\tan(c+dx))^2}+\log(a+b\tan(c+dx)\right.}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((b^4*Sec[c + d*x]^4)/(2*(a + b*Tan[c + d*x])^2) - 2*a*(-2*a*Log[a + b*Tan[
c + d*x]] + b*Tan[c + d*x] - (a^2 + b^2)/(a + b*Tan[c + d*x])) + 2*(a^2 + b
^2)*(Log[a + b*Tan[c + d*x]] + (3*a^2 - b^2 + 4*a*b*Tan[c + d*x])/(2*(a + b
*Tan[c + d*x])^2)))/(b^5*d)
```

Maple [A] time = 0.267, size = 184, normalized size = 1.1

$$\frac{(\tan(dx+c))^2}{2b^3d}-3\frac{a\tan(dx+c)}{b^4d}+6\frac{\ln(a+b\tan(dx+c))a^2}{db^5}+2\frac{\ln(a+b\tan(dx+c))}{b^3d}-\frac{a^4}{2db^5(a+b\tan(dx+c))^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/2*tan(d*x+c)^2/b^3/d-3*a*tan(d*x+c)/b^4/d+6/d/b^5*ln(a+b*tan(d*x+c))*a^2+
2/d/b^3*ln(a+b*tan(d*x+c))-1/2/d/b^5/(a+b*tan(d*x+c))^2*a^4-1/d/b^3/(a+b*ta
n(d*x+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2+4/d*a^3/b^5/(a+b*tan(d*x+c))+4/d
*a/b^3/(a+b*tan(d*x+c))
```

Maxima [B] time = 1.32987, size = 880, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2*(((6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) + (18*a^4*
b + 6*a^2*b^3 - b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (18*a^5 - 2*a^3*
b^2 - 3*a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*(18*a^4*b + 8*a^2*b^
3 - b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (18*a^5 - 2*a^3*b^2 - 3*a*b^
4)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (18*a^4*b + 6*a^2*b^3 - b^5)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - (6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/(a^4*b^4 + 4*a^3*b^5*sin(d*x + c)/(cos(d*x + c) + 1)
 - 12*a^3*b^5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 12*a^3*b^5*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 4*a^3*b^5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a
^4*b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*(a^4*b^4 - a^2*b^6)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 2*(3*a^4*b^4 - 4*a^2*b^6)*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 4*(a^4*b^4 - a^2*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6) - (3*a^2 + b^2)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2)/b^5 + (3*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/b^5 + (3*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/b^5)/d
```

Fricas [B] time = 0.598297, size = 814, normalized size = 5.06

$$\frac{24\,a^2b^2\cos\left(dx+c\right)^4 + b^4 - 2\left(9\,a^2b^2 + b^4\right)\cos\left(dx+c\right)^2 + 2\left(\left(3\,a^4 - 2\,a^2b^2 - b^4\right)\cos\left(dx+c\right)^4 + 2\left(3\,a^3b + ab^3\right)\cos\left(dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 +
 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^2*b^
2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x + c) +
 (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos(d*x +
 c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos(d*x +
c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*x + c)^4
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.343, size = 189, normalized size = 1.17

$$\frac{\frac{4\left(3\,a^{2}+b^{2}\right)\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^{5}}+\frac{b^{3}\tan\left(dx+c\right)^{2}-6\,ab^{2}\tan\left(dx+c\right)}{b^{6}}-\frac{18\,a^{2}b^{2}\tan\left(dx+c\right)^{2}+6\,b^{4}\tan\left(dx+c\right)^{2}+28\,a^{3}b\tan\left(dx+c\right)+4\,ab^{3}\tan\left(dx+c\right)+11\,a^{4}+b^{4}}{\left(b\tan\left(dx+c\right)+a\right)^{2}b^{5}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^2
 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x +
 c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b*ta
n(d*x + c) + a)^2*b^5))/d
```

3.139 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=383

$$\frac{4a^2\sec(c+dx)}{b^5d}+\frac{2\left(a^2+b^2\right)\sec(c+dx)}{b^5d}-\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{6a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{4}{b^5d(a\cos(c}$$

```
[Out] (-4*a^3*ArcTanh[Sin[c + d*x]])/(b^6*d) - (3*a*ArcTanh[Sin[c + d*x]])/(2*b^4
*d) - (6*a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) - (8*a^2*Sqrt[a^2 + b
^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*d) - (
Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(2*b^4*d) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])
/Sqrt[a^2 + b^2]])/(b^6*d) + (4*a^2*Sec[c + d*x])/(b^5*d) + (2*(a^2 + b^2)*
Sec[c + d*x])/(b^5*d) + Sec[c + d*x]^3/(3*b^3*d) - ((a^2 + b^2)*(b*Cos[c +
d*x] - a*Sin[c + d*x]))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (4*
a*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) - (3*a*Sec[c + d*x
]*Tan[c + d*x])/(2*b^4*d)
```

Rubi [A] time = 0.785488, antiderivative size = 383, normalized size of antiderivative = 1., number of steps used = 31, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3106, 3076, 3074, 206, 3104, 3770, 3094, 3768}

$$\frac{4a^2\sec(c+dx)}{b^5d}+\frac{2\left(a^2+b^2\right)\sec(c+dx)}{b^5d}-\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{6a\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{4}{b^5d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] (-4*a^3*ArcTanh[Sin[c + d*x]])/(b^6*d) - (3*a*ArcTanh[Sin[c + d*x]])/(2*b^4
*d) - (6*a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) - (8*a^2*Sqrt[a^2 + b
^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*d) - (
Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(2*b^4*d) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])
/Sqrt[a^2 + b^2]])/(b^6*d) + (4*a^2*Sec[c + d*x])/(b^5*d) + (2*(a^2 + b^2)*
Sec[c + d*x])/(b^5*d) + Sec[c + d*x]^3/(3*b^3*d) - ((a^2 + b^2)*(b*Cos[c +
d*x] - a*Sin[c + d*x]))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (4*
a*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) - (3*a*Sec[c + d*x
]*Tan[c + d*x])/(2*b^4*d)
```

Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} - \frac{(2a)\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{}{(a\cos(}}{}$$

$$= \frac{\sec^3(c+dx)}{3b^3d} - \frac{a\int \sec^3(c+dx)\,dx}{b^4} - \frac{(2a)\int \sec^3(c+dx)\,dx}{b^4} + \frac{\left(4a^2\right)\int \frac{\sec}{a\cos(c+d}}{b^4}$$

$$= \frac{4a^2\sec(c+dx)}{b^5d} + \frac{\sec^3(c+dx)}{3b^3d} - \frac{\left(a^2+b^2\right)(b\cos(c+dx)-a\sin(c+dx))}{2b^4d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{3a}{}$$

$$= -\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d} - \frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{4a^2\sec(c+dx)}{b^5d} + \frac{\sec^3(}{3}$$

$$= -\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d} - \frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{4a^2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b}{}\right.}{b^6d}$$

Mathematica [C] time = 2.29544, size = 688, normalized size = 1.8

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{6b^2\left(a^2+b^2\right)^2\sin(c+dx)}{a} + 2b\left(36a^2+13b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2 + \frac{2b(3}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Sin
[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b*S
in[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 -
30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/2]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2
*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]*(a*Cos[c +
d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (b^2*(9
*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b^6*d*(a + b*Tan[
c + d*x])^3)
```

Maple [B] time = 0.308, size = 1125, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 9/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^2*tan(1/2*d*x
+1/2*c)^2-2/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a^2*tan
(1/2*d*x+1/2*c)^2+23/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a
)^2*a*tan(1/2*d*x+1/2*c)+25/d/b^4/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^2-3/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)^2*a-6
/d/b^5/(tan(1/2*d*x+1/2*c)-1)*a^2-3/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)*a+10/d*a
^3/b^6*ln(tan(1/2*d*x+1/2*c)-1)+1/3/d/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2/d/b^
3/(tan(1/2*d*x+1/2*c)+1)^2-1/3/d/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/b^3/(ta
n(1/2*d*x+1/2*c)-1)^2+25/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)
*b-a)^2*a^3*tan(1/2*d*x+1/2*c)+20/d/b^6/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*ta
n(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))*a^4-8/d/b^5/(tan(1/2*d*x+1/2*c)^2*a-
2*tan(1/2*d*x+1/2*c)*b-a)^2*a^4*tan(1/2*d*x+1/2*c)^2-7/d/b^4/(tan(1/2*d*x+1
/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^3*tan(1/2*d*x+1/2*c)^3+2/d/(tan(1/2
*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a*tan(1/2*d*x+1/2*c)^3+15/d/b/(
```

```
tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*tan(1/2*d*x+1/2*c)^2-2/d
/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2/a*tan(1/2*d*x+1/2*c)+7
/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^2+5/d/b^2/(a^2
+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-15/2/
d*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)+15/2/d*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)+8/d/
b^5/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a^4+3/2/d/b^4/(tan(
1/2*d*x+1/2*c)+1)^2*a+6/d/b^5/(tan(1/2*d*x+1/2*c)+1)*a^2-3/2/d/b^4/(tan(1/2
*d*x+1/2*c)+1)*a-10/d*a^3/b^6*ln(tan(1/2*d*x+1/2*c)+1)-5/d/b^2/(tan(1/2*d*x
+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2*a*tan(1/2*d*x+1/2*c)^3-1/d/b/(tan(1
/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^2+5/2/d/b^3/(tan(1/2*d*x+1/2*c)
+1)-5/2/d/b^3/(tan(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.856379, size = 1299, normalized size = 3.39

$$\frac{4\,b^5 + 30\left(4\,a^4b + a^2b^3 - b^5\right)\cos\left(dx + c\right)^4 + 20\left(2\,a^2b^3 + b^5\right)\cos\left(dx + c\right)^2 + 15\left(\left(4\,a^4 - 3\,a^2b^2 - b^4\right)\cos\left(dx + c\right)^5 + 2\left(4\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3 +
 b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(4*
a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x + c)
^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x +
 c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
 - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a^2*b^3)
```

```
*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3)*log(si
n(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4
*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x
 + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*a^3*b^2 + 2
*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4*sin(d*x + c
) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)

[Out] Timed out
```

Giac [A] time = 1.41592, size = 689, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] -1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a
^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*b^
2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) +
2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*d*x + 1
/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3*tan(1/2
*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*
c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 - 7*a^4*b^2 +
 a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a^2*
b^5))/d
```

3.140 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$\frac{3\left(2a^2+b^2\right)\tan^2(c+dx)}{2b^5d}-\frac{a\left(10a^2+9b^2\right)\tan(c+dx)}{b^6d}-\frac{\left(5a^2-b^2\right)\left(a^2+b^2\right)^2}{a^2b^6d(a\cot(c+dx)+b)}-\frac{\left(a^2+b^2\right)^3}{2a^2b^5d(a\cot(c+dx)+b)^2}+\frac{3\left(a\right.}{}$$

```
[Out] -(a^2 + b^2)^3/(2*a^2*b^5*d*(b + a*Cot[c + d*x])^2) - ((5*a^2 - b^2)*(a^2 +
 b^2)^2)/(a^2*b^6*d*(b + a*Cot[c + d*x])) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Lo
g[b + a*Cot[c + d*x]])/(b^7*d) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[Tan[c + d
*x]])/(b^7*d) - (a*(10*a^2 + 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b^2
)*Tan[c + d*x]^2)/(2*b^5*d) - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4/(
4*b^3*d)
```

Rubi [A] time = 0.243583, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{3\left(2a^2+b^2\right)\tan^2(c+dx)}{2b^5d}-\frac{a\left(10a^2+9b^2\right)\tan(c+dx)}{b^6d}-\frac{\left(5a^2-b^2\right)\left(a^2+b^2\right)^2}{a^2b^6d(a\cot(c+dx)+b)}-\frac{\left(a^2+b^2\right)^3}{2a^2b^5d(a\cot(c+dx)+b)^2}+\frac{3\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```


```
[Out] -(a^2 + b^2)^3/(2*a^2*b^5*d*(b + a*Cot[c + d*x])^2) - ((5*a^2 - b^2)*(a^2 +
 b^2)^2)/(a^2*b^6*d*(b + a*Cot[c + d*x])) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Lo
g[b + a*Cot[c + d*x]])/(b^7*d) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[Tan[c + d
*x]])/(b^7*d) - (a*(10*a^2 + 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b^2
)*Tan[c + d*x]^2)/(2*b^5*d) - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4/(
4*b^3*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x^5} - \frac{3a}{b^4x^4} + \frac{3(2a^2+b^2)}{b^5x^3} + \frac{-10a^3-9ab^2}{b^6x^2} + \frac{3(5a^4+6a^2b^2+b^4)}{b^7x} - \frac{(a^2+b^2)^3}{ab^5(b+ax)^3} - \frac{(5a\cdots}{}\right.\right.}{d}$$

$$= -\frac{(a^2+b^2)^3}{2a^2b^5d(b+a\cot(c+dx))^2} - \frac{(5a^2-b^2)(a^2+b^2)^2}{a^2b^6d(b+a\cot(c+dx))} + \frac{3(a^2+b^2)(5a^2+b^2)\cdots}{b\cdots}$$

Mathematica [A] time = 1.33063, size = 272, normalized size = 1.17

$$\frac{4a^2b^4\tan^4(c+dx) - 20ab^3(a^2+b^2)\tan^3(c+dx) + 4b^2\tan^2(c+dx)\left(3\left(6a^2b^2+5a^4+b^4\right)\log(a+b\tan(c+dx)) - 10a^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (2*(a^2 + b^2)*(19*a^4 + 16*a^2*b^2 - 3*b^4 + 6*a^2*(5*a^2 + b^2)*Log[a + b
*Tan[c + d*x]]) + b^6*Sec[c + d*x]^6 + 4*a*b*(4*a^4 + 17*a^2*b^2 + 11*b^4 +
 6*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 4*b^2*
(-13*a^4 - 10*a^2*b^2 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a + b*Tan[c + d*x]]
)*Tan[c + d*x]^2 - 20*a*b^3*(a^2 + b^2)*Tan[c + d*x]^3 + 4*a^2*b^4*Tan[c +
d*x]^4 + b^4*Sec[c + d*x]^4*(a^2 + 3*b^2 - 2*a*b*Tan[c + d*x]))/(4*b^7*d*(a
 + b*Tan[c + d*x])^2)
```

Maple [A] time = 0.269, size = 321, normalized size = 1.4

$$\frac{(\tan(dx+c))^4}{4b^3d} - \frac{a(\tan(dx+c))^3}{b^4d} + 3\frac{(\tan(dx+c))^2a^2}{db^5} + \frac{3(\tan(dx+c))^2}{2b^3d} - 10\frac{a^3\tan(dx+c)}{db^6} - 9\frac{a\tan(dx+c)}{b^4d} + 15\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x)
```

```
[Out] 1/4*tan(d*x+c)^4/b^3/d-a*tan(d*x+c)^3/b^4/d+3/d/b^5*tan(d*x+c)^2*a^2+3/2*ta
n(d*x+c)^2/b^3/d-10/d/b^6*a^3*tan(d*x+c)-9*a*tan(d*x+c)/b^4/d+15/d/b^7*ln(a
+b*tan(d*x+c))*a^4+18/d/b^5*ln(a+b*tan(d*x+c))*a^2+3/d/b^3*ln(a+b*tan(d*x+c
))-1/2/d/b^7/(a+b*tan(d*x+c))^2*a^6-3/2/d/b^5/(a+b*tan(d*x+c))^2*a^4-3/2/d/
b^3/(a+b*tan(d*x+c))^2*a^2-1/2/b/d/(a+b*tan(d*x+c))^2+6/d*a^5/b^7/(a+b*tan(
d*x+c))+12/d*a^3/b^5/(a+b*tan(d*x+c))+6/d*a/b^3/(a+b*tan(d*x+c))
```

Maxima [B] time = 1.45028, size = 1422, normalized size = 6.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2*((15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)/(cos(d*x + c) +
 1) + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - (75*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 2*(90*a^6*b + 113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 2*(75*a^7 + 60*a^5*b^2 - 17*a^3*b^4 - 5*a*b^6)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*(135*a^6*b + 172*a^4*b^3 + 35*a^2*
b^5 - 3*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*(75*a^7 + 60*a^5*b^2 -
 17*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 2*(90*a^6*b +
113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (75
*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - (15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11)/(a^4*b^6 + 4*a^3*b^7*sin(d*x + c)/(cos(d*x + c) + 1) - 20*
a^3*b^7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 40*a^3*b^7*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 40*a^3*b^7*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 20*a^3
*b^7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 4*a^3*b^7*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + a^4*b^6*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 2*(3*a^4*
b^6 - 2*a^2*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (15*a^4*b^6 - 16*a^2
*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(5*a^4*b^6 - 6*a^2*b^8)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + (15*a^4*b^6 - 16*a^2*b^8)*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 - 2*(3*a^4*b^6 - 2*a^2*b^8)*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10) - 3*(5*a^4 + 6*a^2*b^2 + b^4)*log(-a - 2*b*sin(d*x + c)/(cos(d*
x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/b^7 + 3*(5*a^4 + 6*a^2
```

```
*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^7 + 3*(5*a^4 + 6*a^2
*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^7)/d
```

Fricas [B] time = 0.689576, size = 1076, normalized size = 4.64

$$\frac{8\left(15\,a^4b^2+13\,a^2b^4\right)\cos\left(dx+c\right)^6+b^6-2\left(45\,a^4b^2+44\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^4+\left(5\,a^2b^4+3\,b^6\right)\cos\left(dx+c\right)^2+6\left(\left(5\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44*
a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*((
5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b^3
+ a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*cos(d*
x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b
 + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4
+ b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c) + 2*(15*
a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5)*cos(d*x
 + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b^9*d*cos(d
*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31064, size = 328, normalized size = 1.41

$$\frac{\frac{12\left(5\,a^4+6\,a^2b^2+b^4\right)\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{b^7}-\frac{2\left(45\,a^4b^2\tan\left(dx+c\right)^2+54\,a^2b^4\tan\left(dx+c\right)^2+9\,b^6\tan\left(dx+c\right)^2+78\,a^5b\tan\left(dx+c\right)+84\,a^3b^3\tan\left(dx+c\right)+6\,ab^5\tan\left(dx+\right.\right.}{\left(b\tan\left(dx+c\right)+a\right)^2b^7}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45*
a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2 +
 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 3
4*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x + c)^
4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x + c)
^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

3.141 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=165

$$-\frac{b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{ab}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{b}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}+\frac{4ab\left(a^2-b^2\right)l}{}$$

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4 + (4*a*b*(a^2 - b^2)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - b/(3*(a^2 + b^2)*d*(a + b*Tan
[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^2
- b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rubi [A] time = 0.30323, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {3086, 3483, 3529, 3531, 3530}

$$-\frac{b\left(3a^2-b^2\right)}{d\left(a^2+b^2\right)^3(a+b\tan(c+dx))}-\frac{ab}{d\left(a^2+b^2\right)^2(a+b\tan(c+dx))^2}-\frac{b}{3d\left(a^2+b^2\right)(a+b\tan(c+dx))^3}+\frac{4ab\left(a^2-b^2\right)l}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4 + (4*a*b*(a^2 - b^2)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - b/(3*(a^2 + b^2)*d*(a + b*Tan
[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^2
- b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3529

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*(a + b*Tan[e + f*x])^(m + 1))
/(f*(m + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \int \frac{1}{(a+b\tan(c+dx))^4}\,dx$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx}{a^2+b^2}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} + \frac{\int \frac{a^2-b^2-}{(a+b}}{(a}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))^2} - \frac{}{\left(a^2+b^2\right.}$$

$$= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4} - \frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3} - \frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan}$$

$$= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4} + \frac{4ab\left(a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^4 d} - \frac{}{3\left(a^2+\right.}$$

Mathematica [C] time = 6.21098, size = 419, normalized size = 2.54

$$\frac{\left(a^2-2ab-b^2\right)\left(a^2+2ab-b^2\right)(c+dx)}{d(a-ib)^4(a+ib)^4} + \frac{4\left(a^9b^2+2ia^8b^3+2a^7b^4-2ia^4b^7-2a^3b^8-ia^2b^9+ia^{10}b-ab^{10}\right)(c+dx)}{d(a-ib)^8(a+ib)^7} - \frac{4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*(a^2 + 2*a*b - b^2)*(c + d*x))/((a - I*b)^4*(a + I*b)^
4*d) + (4*(I*a^10*b + a^9*b^2 + (2*I)*a^8*b^3 + 2*a^7*b^4 - (2*I)*a^4*b^7 -
 2*a^3*b^8 - I*a^2*b^9 - a*b^10)*(c + d*x))/((a - I*b)^8*(a + I*b)^7*d) - (
(4*I)*(a^3*b - a*b^3)*ArcTan[Tan[c + d*x]])/((a^2 + b^2)^4*d) + (2*(a^3*b -
 a*b^3)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])/((a^2 + b^2)^4*d) + (b^4*
Sin[c + d*x])/(3*a*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*
x])^3) - (b^3*(6*a^2 + b^2))/(3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^2) + (2*(9*a^2*b^2*Sin[c + d*x] - 2*b^4*Sin[c + d*x]))/(
3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A] time = 0.212, size = 304, normalized size = 1.8

$$-\frac{b}{\left(3\,a^2+3\,b^2\right)d\left(a+b\tan\left(dx+c\right)\right)^3}-3\,\frac{a^2b}{d\left(a^2+b^2\right)^3\left(a+b\tan\left(dx+c\right)\right)}+\frac{b^3}{d\left(a^2+b^2\right)^3\left(a+b\tan\left(dx+c\right)\right)}-\frac{}{\left(a^2+b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] -1/3*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-3/d*b/(a^2+b^2)^3/(a+b*tan(d*x+c))*a^
2+1/d*b^3/(a^2+b^2)^3/(a+b*tan(d*x+c))-a*b/(a^2+b^2)^2/d/(a+b*tan(d*x+c))^2
+4/d*a^3*b/(a^2+b^2)^4*ln(a+b*tan(d*x+c))-4/d*a*b^3/(a^2+b^2)^4*ln(a+b*tan(
d*x+c))-2/d/(a^2+b^2)^4*ln(tan(d*x+c)^2+1)*a^3*b+2/d/(a^2+b^2)^4*ln(tan(d*x
+c)^2+1)*a*b^3+1/d/(a^2+b^2)^4*arctan(tan(d*x+c))*a^4-6/d/(a^2+b^2)^4*arcta
n(tan(d*x+c))*a^2*b^2+1/d/(a^2+b^2)^4*arctan(tan(d*x+c))*b^4
```

Maxima [B] time = 1.82541, size = 520, normalized size = 3.15

$$\frac{\frac{3\left(a^4-6\,a^2b^2+b^4\right)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\left(a^3b-ab^3\right)\log(b\tan(dx+c)+a)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\left(a^3b-ab^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{13\,a^4b+2\,a^2b^3}{a^9+3\,a^7b^2+3\,a^5b^4+a^3b^6+\left(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9\right)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) + 12*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (13*a^4*b + 2*a^2*b^3 +
b^5 + 3*(3*a^2*b^3 - b^5)*tan(d*x + c)^2 + 3*(7*a^3*b^2 - a*b^4)*tan(d*x +
c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b
^7 + b^9)*tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(
d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(d*x + c)))/d
```

Fricas [B] time = 0.610633, size = 1256, normalized size = 7.61

$$-\frac{\left(54\,a^4b^3-30\,a^2b^5+4\,b^7-3\left(a^7-9\,a^5b^2+19\,a^3b^4-3\,ab^6\right)dx\right)\cos\left(dx+c\right)^3-3\left(10\,a^4b^3-11\,a^2b^5+b^7+3\left(a^5b^2-6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*((54*a^4*b^3 - 30*a^2*b^5 + 4*b^7 - 3*(a^7 - 9*a^5*b^2 + 19*a^3*b^4 -
3*a*b^6)*d*x)*cos(d*x + c)^3 - 3*(10*a^4*b^3 - 11*a^2*b^5 + b^7 + 3*(a^5*b^
2 - 6*a^3*b^4 + a*b^6)*d*x)*cos(d*x + c) - 6*((a^6*b - 4*a^4*b^3 + 3*a^2*b^
5)*cos(d*x + c)^3 + 3*(a^4*b^3 - a^2*b^5)*cos(d*x + c) + (a^3*b^4 - a*b^6 +
 (3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*co
s(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (13*a^3*b^4 -
 9*a*b^6 + 3*(a^4*b^3 - 6*a^2*b^5 + b^7)*d*x + (18*a^5*b^2 - 58*a^3*b^4 + 1
2*a*b^6 + 3*(3*a^6*b - 19*a^4*b^3 + 9*a^2*b^5 - b^7)*d*x)*cos(d*x + c)^2)*s
in(d*x + c))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^6 - 11*a^3*b^8 - 3*a*b
^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*
b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a
^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*
b^9 + b^11)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17555, size = 500, normalized size = 3.03

$$\frac{\frac{3\left(a^4-6a^2b^2+b^4\right)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{6\left(a^3b-ab^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{12\left(a^3b^2-ab^4\right)\log(|b\tan(dx+c)+a|)}{a^8b+4a^6b^3+6a^4b^5+4a^2b^7+b^9}-\frac{22a^3b^4\tan(dx+c)^3-22ab^6\tan(dx+c)^3+75a^4b^3\tan\cdots}{\cdots}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
```

```
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c
) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - (22*a^3*b^4*tan
(d*x + c)^3 - 22*a*b^6*tan(d*x + c)^3 + 75*a^4*b^3*tan(d*x + c)^2 - 60*a^2*
b^5*tan(d*x + c)^2 - 3*b^7*tan(d*x + c)^2 + 87*a^5*b^2*tan(d*x + c) - 48*a^
3*b^4*tan(d*x + c) - 3*a*b^6*tan(d*x + c) + 35*a^6*b - 7*a^4*b^3 + 3*a^2*b^
5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) +
 a)^3))/d
```

3.142 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=157

$$\frac{\frac{1}{2}b\left(b^2-9a^2\right)\left(2\left(a^2+b^2\right)+3ab\sin(2(c+dx))\right)-3\left(-a^2b^3+3a^4b+b^5\right)\cos(2(c+dx))}{6d\left(a^2+b^2\right)^3(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a^2+}}\right.}{d\left(a^2+b^2\right)^{7/2}}$$

```
[Out] (a*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^
2 + b^2)^(7/2)*d) + (-3*(3*a^4*b - a^2*b^3 + b^5)*Cos[2*(c + d*x)] + (b*(-9
*a^2 + b^2)*(2*(a^2 + b^2) + 3*a*b*Sin[2*(c + d*x)]))/2)/(6*(a^2 + b^2)^3*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Rubi [B] time = 1.17309, antiderivative size = 362, normalized size of antiderivative = 2.31, number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1660, 12, 618, 206}

$$-\frac{8b^3\left(b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)+a\left(a^2+2b^2\right)\right)}{3a^5d\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{2b^2\left(a\left(30a^2b^2+9a^4+16b^4\right)\tan\left(\frac{1}{2}(c+dx)\right)+b\left(18\right.\right.}{3a^5d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a
^2 + b^2)^(7/2)*d)) - (8*b^3*(a*(a^2 + 2*b^2) + b*(3*a^2 + 4*b^2)*Tan[(c +
d*x)/2]))/(3*a^5*(a^2 + b^2)*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/
2]^2)^3) + (2*b^2*(b*(15*a^4 + 18*a^2*b^2 + 8*b^4) + a*(9*a^4 + 30*a^2*b^2
+ 16*b^4)*Tan[(c + d*x)/2]))/(3*a^5*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/
2] - a*Tan[(c + d*x)/2]^2)^2) - (b*(6*a^6 + 9*a^4*b^2 + 12*a^2*b^4 + 4*b^6
+ a*b*(9*a^4 + 6*a^2*b^2 + 2*b^4)*Tan[(c + d*x)/2]))/(a^4*(a^2 + b^2)^3*d*(
a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 1660

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
```

```
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{2\,\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a+2bx-ax^2\right)^4}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{\text{Subst}\left(\int -\frac{4(3}{\ldots}\right.}{\ldots}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\left(15a^4+\ldots\right.\right.}{3a^5\left(a^2+\ldots\right.}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\left(15a^4+\ldots\right.\right.}{3a^5\left(a^2+\ldots\right.}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\left(15a^4+\ldots\right.\right.}{3a^5\left(a^2+\ldots\right.}$$

$$= -\frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{2b^2\left(b\left(15a^4+\ldots\right.\right.}{3a^5\left(a^2+\ldots\right.}$$

$$= -\frac{a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}d} - \frac{8b^3\left(a\left(a^2+2b^2\right)+b\left(3a^2+4b^2\ldots\right.\right.}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)\ldots\right.}$$

Mathematica [C] time = 1.05464, size = 165, normalized size = 1.05

$$\frac{\frac{6a\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}+\frac{\frac{1}{2}b\left(b^2-9a^2\right)\left(2\left(a^2+b^2\right)+3ab\sin(2(c+dx))\right)-3\left(-a^2b^3+3a^4b+b^5\right)\cos(2(c+dx))}{(a-ib)^3(a+ib)^3(a\cos(c+dx)+b\sin(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*a*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(
a^2 + b^2)^(7/2) + (-3*(3*a^4*b - a^2*b^3 + b^5)*Cos[2*(c + d*x)] + (b*(-9*
a^2 + b^2)*(2*(a^2 + b^2) + 3*a*b*Sin[2*(c + d*x)]))/2)/((a - I*b)^3*(a + I
*b)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

Maple [B] time = 0.234, size = 494, normalized size = 3.2

$$\frac{1}{d}\left(-2\,\frac{1}{\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-2\,\tan\left(1/2\,dx+c/2\right)b-a\right)^{3}}\left(-1/2\,\frac{b^{2}\left(9\,a^{4}+6\,a^{2}b^{2}+2\,b^{4}\right)\left(\tan\left(1/2\,dx+c/2\right)\right)^{5}}{a\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}-1/2\,\frac{b\,(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
an(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*a^
4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2-4*
a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^3+1/a^2*b*(
6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x
+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*
tan(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6))/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3+a*(2*a^2-3*b^2)/(
a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1
/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.595758, size = 1173, normalized size = 7.47

$$-\frac{22\,a^4b^3+14\,a^2b^5-8\,b^7+12\left(3\,a^6b+2\,a^4b^3+b^7\right)\cos\left(dx+c\right)^2+6\left(9\,a^5b^2+8\,a^3b^4-ab^6\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3}{12\left(\left(a^{11}+a^9b^2-6\,a^7b^4-14\,a^5b^6-11\,a^3b^8-3\,ab^{10}\right)d\cos\left(dx+c\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/12*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 + 12*(3*a^6*b + 2*a^4*b^3 + b^7)*cos
(d*x + c)^2 + 6*(9*a^5*b^2 + 8*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) +
 3*((2*a^6 - 9*a^4*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 - 3*a^2*b
^4)*cos(d*x + c) + (2*a^3*b^3 - 3*a*b^5 + (6*a^5*b - 11*a^3*b^3 + 3*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*c
os(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 + b^2)))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^6 - 11*a
^3*b^8 - 3*a*b^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 +
4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 +
 6*a^4*b^7 - a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6*b^5 + 6*a^
4*b^7 + 4*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27717, size = 707, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(27*a^6*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 18*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^6*tan(1/2*d*x + 1/
2*c)^5 + 18*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 81*a^5*b^3*tan(1/2*d*x + 1/2*c)^
4 - 36*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 1
08*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 42*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 8*a
^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*a^7*b*tan
(1/2*d*x + 1/2*c)^2 + 120*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 18*a^3*b^5*tan(1
/2*d*x + 1/2*c)^2 + 12*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 81*a^6*b^2*tan(1/2*d*
x + 1/2*c) + 12*a^4*b^4*tan(1/2*d*x + 1/2*c) + 6*a^2*b^6*tan(1/2*d*x + 1/2*
c) + 18*a^7*b + 5*a^5*b^3 + 2*a^3*b^5)/((a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*
b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^3))/d
```

3.143 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot^3(c+dx)}{3bd(a\cot(c+dx)+b)^3}$$

```
[Out] -Cot[c + d*x]^3/(3*b*d*(b + a*Cot[c + d*x])^3)
```

Rubi [A] time = 0.0528595, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 37}

$$-\frac{\cot^3(c+dx)}{3bd(a\cot(c+dx)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -Cot[c + d*x]^3/(3*b*d*(b + a*Cot[c + d*x])^3)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\cot^3(c+dx)}{3bd(b+a\cot(c+dx))^3}$$

Mathematica [B] time = 0.662484, size = 124, normalized size = 4.13

$$\frac{-6ab\left(a^2+b^2\right)\cos(c+dx)+\left(2ab^3-6a^3b\right)\cos(3(c+dx))+2\left(a^2-b^2\right)\sin(c+dx)\left(\left(3a^2-b^2\right)\cos(2(c+dx))+3a^2+b\right.}{12ad\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-6*a*b*(a^2 + b^2)*Cos[c + d*x] + (-6*a^3*b + 2*a*b^3)*Cos[3*(c + d*x)] +
2*(a^2 - b^2)*(3*a^2 + b^2 + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(12*a*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A] time = 0.2, size = 21, normalized size = 0.7

$$-\frac{1}{3\,db\,(a+b\tan(dx+c))^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] -1/3/d/b/(a+b*tan(d*x+c))^3
```

Maxima [A] time = 1.17786, size = 72, normalized size = 2.4

$$-\frac{1}{3\left(b^4\tan(dx+c)^3+3\,ab^3\tan(dx+c)^2+3\,a^2b^2\tan(dx+c)+a^3b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3/((b^4*tan(d*x + c)^3 + 3*a*b^3*tan(d*x + c)^2 + 3*a^2*b^2*tan(d*x + c)
 + a^3*b)*d)
```

Fricas [B] time = 0.526365, size = 552, normalized size = 18.4

$$-\frac{\left(9\,a^{4}b-6\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{3}-3\left(a^{4}b-3\,a^{2}b^{3}\right)\cos\left(dx+c\right)-\left(a^{3}b^{2}-3\,ab^{4}+\left(3\,a^{5}-1\right.\right.}{3\left(\left(a^{9}-6\,a^{5}b^{4}-8\,a^{3}b^{6}-3\,ab^{8}\right)d\cos\left(dx+c\right)^{3}+3\left(a^{7}b^{2}+3\,a^{5}b^{4}+3\,a^{3}b^{6}+ab^{8}\right)d\cos\left(dx+c\right)+\left(\left(3\,a^{8}b+8\,a^{6}b^{3}+6\,a^{4}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*((9*a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^3 - 3*(a^4*b - 3*a^2*b^3)*co
s(d*x + c) - (a^3*b^2 - 3*a*b^4 + (3*a^5 - 10*a^3*b^2 + 3*a*b^4)*cos(d*x +
c)^2)*sin(d*x + c))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x + c)
^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a^8*b
 + 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^5 + 3
*a^2*b^7 + b^9)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.11551, size = 27, normalized size = 0.9

$$-\frac{1}{3\left(b\tan\left(dx+c\right)+a\right)^{3}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3/((b*tan(d*x + c) + a)^3*b*d)
```

3.144 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=141

$$-\frac{b}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{5/2}}$$

```
[Out] -(a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*(a^2 + b
^2)^(5/2)*d) - b/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a
*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(2*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^2)
```

Rubi [A] time = 0.110976, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3158, 12, 3076, 3074, 206}

$$-\frac{b}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -(a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*(a^2 + b
^2)^(5/2)*d) - b/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) - (a
*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(2*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*
Sin[c + d*x])^2)
```

Rule 3158

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> -Simp[((c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x])*(a + b*Cos[d + e*x] + c*Sin[d +
 e*x])^(n + 1))/(e*(n + 1)*(a^2 - b^2 - c^2)), x] + Dist[1/((n + 1)*(a^2 -
b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n + 1)
*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 - b
^2 - c^2, 0] && NeQ[n, -2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{\int \frac{3a}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx}{3\left(a^2+b^2\right)}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx}{a^2+b^2}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a\sin(}{2\left(a^2+b^2\right)^2 d(a\cos(c+dx)+}$$

$$= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a\sin(}{2\left(a^2+b^2\right)^2 d(a\cos(c+dx)+}$$

$$= -\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d}-\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}$$

Mathematica [A] time = 0.699489, size = 128, normalized size = 0.91

$$\frac{\frac{3\left(a^3-ab^2\right)\sin(2(c+dx))-4b\left(a^2+b^2\right)-6a^2b\cos(2(c+dx))}{2\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{6a\tanh^{-1}\left(\frac{a\tan\left(\frac{1}{2}(c+dx)\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
 + (-4*b*(a^2 + b^2) - 6*a^2*b*Cos[2*(c + d*x)] + 3*(a^3 - a*b^2)*Sin[2*(c
+ d*x)])/(2*(a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

Maple [B] time = 0.233, size = 383, normalized size = 2.7

$$\frac{1}{d}\left(-2\frac{1}{\left((\tan(1/2\,dx+c/2))^2a-2\tan(1/2\,dx+c/2)\,b-a\right)^3}\left(-1/2\frac{\left(a^4+4a^2b^2+2b^4\right)(\tan(1/2\,dx+c/2))^5}{a\left(a^4+2a^2b^2+b^4\right)}-1/2\frac{b\left(a^4-8\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-2*(-1/2*(a^4+4*a^2*b^2+2*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c
)^5-1/2*b*(a^4-8*a^2*b^2-4*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^
4+1/3/a^3*b^2*(15*a^4-4*a^2*b^2-4*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*
c)^3+1/a^2*b*(2*a^4-5*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)
^2+1/2/a*(a^4-6*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-1/6*b
*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4))/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1
/2*c)*b-a)^3+a/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.566791, size = 945, normalized size = 6.7

$$\frac{2\,a^4b-2\,a^2b^3-4\,b^5-12\left(a^4b+a^2b^3\right)\cos\left(dx+c\right)^2+6\left(a^5-ab^4\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\left(3\,a^2b^2\cos\left(dx+c\right)+\left(a\right.\right.}{12\left(\left(a^9-6\,a^5b^4-8\,a^3b^6-3\,ab^8\right)d\cos\left(dx+c\right)^3+3\left(a^7b^2+3\,a^5b^4+3\,a^3b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*a^4*b - 2*a^2*b^3 - 4*b^5 - 12*(a^4*b + a^2*b^3)*cos(d*x + c)^2 + 6
*(a^5 - a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(3*a^2*b^2*cos(d*x + c) + (a^4
 - 3*a^2*b^2)*cos(d*x + c)^3 + (a*b^3 + (3*a^3*b - a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*
sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2)))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x + c)^3 + 3*(a
^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a^8*b + 8*a^6*
b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7
+ b^9)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

<u>Giac</u> [B] time = 1.30217, size = 575, normalized size = 4.08

$$-\frac{\frac{3\,a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,a^3b^3\tan\left(\frac{1}{2}\,dx \cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(3*a^6*tan(1/2*d*x + 1/2*c)^5 + 12*a^4*b^2*tan(1/2*d
*x + 1/2*c)^5 + 6*a^2*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*a^5*b*tan(1/2*d*x + 1/
2*c)^4 - 24*a^3*b^3*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^5*tan(1/2*d*x + 1/2*c)^
4 - 30*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 +
8*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 30*a^3*b^3
*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^5*tan(1/2*d*x + 1/2*c)^2 - 3*a^6*tan(1/2*d
*x + 1/2*c) + 18*a^4*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^4*tan(1/2*d*x + 1/2
*c) + 5*a^5*b + 2*a^3*b^3)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/
2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^3))/d
```

3.145 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x]))
```

Rubi [A] time = 0.0404252, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3076, 3075}

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] -(b*Cos[c + d*x] - a*Sin[c + d*x])/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin
[c + d*x]))
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{3\left(a^2+b^2\right)}$$
$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\sin(c+dx)}{3a\left(a^2+b^2\right)d(a\cos(c+dx)+b$$

Mathematica [A] time = 0.289512, size = 85, normalized size = 0.87

$$\frac{\sin(c+dx)\left(\left(a^2-b^2\right)\cos(2(c+dx))+2a^2+b^2\right)-ab\cos(3(c+dx))}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] (-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Sin
[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A] time = 0.202, size = 64, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{1}{b^3\left(a+b\tan\left(dx+c\right)\right)}+\frac{a}{b^3\left(a+b\tan\left(dx+c\right)\right)^2}-\frac{a^2+b^2}{3\,b^3\left(a+b\tan\left(dx+c\right)\right)^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/d*(-1/b^3/(a+b*tan(d*x+c))+a/b^3/(a+b*tan(d*x+c))^2-1/3*(a^2+b^2)/b^3/(a+
b*tan(d*x+c))^3)
```

Maxima [A] time = 1.18775, size = 115, normalized size = 1.17

$$-\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\left(b^6\tan\left(dx+c\right)^3+3\,ab^5\tan\left(dx+c\right)^2+3\,a^2b^4\tan\left(dx+c\right)+a^3b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
+ c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

Fricas [B] time = 0.501956, size = 470, normalized size = 4.8

$$-\frac{2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^3-3\left(a^2b-b^3\right)\cos\left(dx+c\right)-\left(a^3+3\,ab^2+2\left(a^3-3\,ab^2\right)\cos\left(\right.\right.}{3\left(\left(a^7-a^5b^2-5\,a^3b^4-3\,ab^6\right)d\cos\left(dx+c\right)^3+3\left(a^5b^2+2\,a^3b^4+ab^6\right)d\cos\left(dx+c\right)+\left(\left(3\,a^6b+5\,a^4b^3+a^2b^5-b^7\right)d\,c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a^
3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5*b
^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c)^2
+ (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15292, size = 68, normalized size = 0.69

$$-\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\left(b\tan\left(dx+c\right)+a\right)^3b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
c) + a)^3*b^3*d)
```

3.146 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=231

$$\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2b^2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\left(a^2+b^2\right)^{3/2}}-\frac{}{b^3}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(2*b^2*(a^2 + b^2)^(3/2)*d) + (a*ArcTanh[(b*Cos[c + d*x
] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - 1/(3*b*d*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(b*Cos[c + d*x] - a*Sin[c + d*x]))/
(2*b^2*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - 1/(b^3*d*(a*Cos
[c + d*x] + b*Sin[c + d*x]))
```

Rubi [A] time = 0.169594, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3094, 3770, 3074, 206, 3076}

$$\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2b^2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\left(a^2+b^2\right)^{3/2}}-\frac{}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(2*b^2*(a^2 + b^2)^(3/2)*d) + (a*ArcTanh[(b*Cos[c + d*x
] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - 1/(3*b*d*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(b*Cos[c + d*x] - a*Sin[c + d*x]))/
(2*b^2*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - 1/(b^3*d*(a*Cos
[c + d*x] + b*Sin[c + d*x]))
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
```

```
& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2}-\frac{a\int \frac{}{(a\cos(c+}$$

$$= -\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2b^2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a(b\cos(c+}{2b^2\left(a^2+b^2\right)d(a\,c}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2\left(a^2+b^2\right)^{3/2}d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\,s}{\sqrt{a^2+b^2}}\right.}{b^4\sqrt{a^2+b^2}d}$$

Mathematica [A] time = 3.2569, size = 290, normalized size = 1.26

$$-\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{3b\left(2a^2+b^2\right)\cos(c+dx)(a+b\tan(c+dx))^2}{a^2+b^2}+\frac{6a\left(2a^2+3b^2\right)\cos^2(c+dx)(a+b\tan(c+dx))^3\tanh^{-1}\left(\cdots\right)}{\left(a^2+b^2\right)^{3/2}}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(2*b^3*Sec[c + d*x] + 3*
b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*Tan[c + d*x] + (3*b*(2*a^2 + b^2)*Cos
[c + d*x]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2) + (6*a*(2*a^2 + 3*b^2)*ArcTan
h[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]^2*(a + b*Tan[c +
d*x])^3)/(a^2 + b^2)^(3/2) + 6*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]]*(a + b*Tan[c + d*x])^3 - 6*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3))/(6*b^4*d*(a + b*Tan[c + d*x])^
4)
```

Maple [B] time = 0.316, size = 1367, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] -4/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/(a^2+b^2)*tan(1/
2*d*x+1/2*c)^4+2/3/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/
(a^2+b^2)+14/d*b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/(a^2+b
^2)*tan(1/2*d*x+1/2*c)^2+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*
c)*b-a)^3/(a^2+b^2)*a^4+5/3/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c
)*b-a)^3/(a^2+b^2)*a^2-2/d/b^4*a^3/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2
*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))-3/d/b^2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*
a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*ta
n(1/2*d*x+1/2*c)*b-a)^3/(a^2+b^2)*a*tan(1/2*d*x+1/2*c)^5-2/d/(tan(1/2*d*x+1
/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3+8/d/
(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a/(a^2+b^2)*tan(1/2*d*x
+1/2*c)-1/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/d/b^4*ln(tan(1/2*d*x+1/2*c)+1)+4
```

```
/d*b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/a^2/(a^2+b^2)*ta
n(1/2*d*x+1/2*c)^2+11/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-
a)^3*a^3/(a^2+b^2)*tan(1/2*d*x+1/2*c)+2/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan
(1/2*d*x+1/2*c)*b-a)^3/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)+1/d/b^2/(tan(1/2*d*x+
1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/(a^2+b^2)*a^3*tan(1/2*d*x+1/2*c)^5+2
/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/(a^2+b^2)/a*tan(
1/2*d*x+1/2*c)^5+2/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^
3/(a^2+b^2)*a^4*tan(1/2*d*x+1/2*c)^4-3/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/
2*d*x+1/2*c)*b-a)^3/(a^2+b^2)*a^2*tan(1/2*d*x+1/2*c)^4-4/d*b^3/(tan(1/2*d*x
+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^4-
12/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^3/(a^2+b^2)*
tan(1/2*d*x+1/2*c)^3+8/3/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)
*b-a)^3/a/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3+8/3/d*b^4/(tan(1/2*d*x+1/2*c)^2*a-
2*tan(1/2*d*x+1/2*c)*b-a)^3/a^3/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3-4/d/b^3/(tan
(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^4/(a^2+b^2)*tan(1/2*d*x+1
/2*c)^2+16/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^2/(a^2
+b^2)*tan(1/2*d*x+1/2*c)^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.03503, size = 1655, normalized size = 7.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/12*(22*a^4*b^3 + 38*a^2*b^5 + 16*b^7 + 12*(a^6*b - 2*a^2*b^5 - b^7)*cos(
d*x + c)^2 + 6*(5*a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d*x + c)*sin(d*x + c)
 - 3*((2*a^6 - 3*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*
b^4)*cos(d*x + c) + (2*a^3*b^3 + 3*a*b^5 + (6*a^5*b + 7*a^3*b^3 - 3*a*b^5)*
```

```
cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*c
os(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 + b^2)) - 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*cos(d*
x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4*b^3 + 2*a^2*
b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(sin(d*x + c) + 1) + 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*cos(d
*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4*b^3 + 2*a^2
*b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1))/((a^7*b^4 - a^5*b^6 - 5*a^3*b^8 - 3*a*b^10)*d
*cos(d*x + c)^3 + 3*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^6
*b^5 + 5*a^4*b^7 + a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^4*b^7 + 2*a^2*b^9
+ b^11)*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3403, size = 711, normalized size = 3.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^
2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^2*b^
4 + b^6)*sqrt(a^2 + b^2)) + 2*(3*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^4*b^3*t
an(1/2*d*x + 1/2*c)^5 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*a^7*tan(1/2*d*
x + 1/2*c)^4 - 9*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^3*b^4*tan(1/2*d*x +
1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 36*a^6*b*tan(1/2*d*x + 1/2*c)^
3 - 6*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 8
```

```
*b^7*tan(1/2*d*x + 1/2*c)^3 - 12*a^7*tan(1/2*d*x + 1/2*c)^2 + 48*a^5*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 42*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^6*tan(1/2
*d*x + 1/2*c)^2 + 33*a^6*b*tan(1/2*d*x + 1/2*c) + 24*a^4*b^3*tan(1/2*d*x +
1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 6*a^7 + 5*a^5*b^2 + 2*a^3*b^4)/((
a^5*b^3 + a^3*b^5)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a
)^3) + 6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/b^4)/d
```

3.147 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{\left(a^2+b^2\right)^2}{3a^3b^2d(a\cot(c+dx)+b)^3}+\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(a\cot(c+dx)+b)}+\frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(a\cot(c+dx)+b)^2}-\frac{4a\log(\tan(c+dx))}{b^5d}-\frac{4a\log(a\cot(c+dx}{b^5d}$$

```
[Out] (a^2 + b^2)^2/(3*a^3*b^2*d*(b + a*Cot[c + d*x])^3) + (a/b^3 - b/a^3)/(d*(b
+ a*Cot[c + d*x])^2) + (a^(-3) + (3*a)/b^4)/(d*(b + a*Cot[c + d*x])) - (4*a
*Log[b + a*Cot[c + d*x]])/(b^5*d) - (4*a*Log[Tan[c + d*x]])/(b^5*d) + Tan[c
 + d*x]/(b^4*d)
```

Rubi [A] time = 0.16002, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{\left(a^2+b^2\right)^2}{3a^3b^2d(a\cot(c+dx)+b)^3}+\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(a\cot(c+dx)+b)}+\frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(a\cot(c+dx)+b)^2}-\frac{4a\log(\tan(c+dx))}{b^5d}-\frac{4a\log(a\cot(c+dx}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^2 + b^2)^2/(3*a^3*b^2*d*(b + a*Cot[c + d*x])^3) + (a/b^3 - b/a^3)/(d*(b
+ a*Cot[c + d*x])^2) + (a^(-3) + (3*a)/b^4)/(d*(b + a*Cot[c + d*x])) - (4*a
*Log[b + a*Cot[c + d*x]])/(b^5*d) - (4*a*Log[Tan[c + d*x]])/(b^5*d) + Tan[c
 + d*x]/(b^4*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^4x^2} - \frac{4a}{b^5x} + \frac{\left(a^2+b^2\right)^2}{a^2b^2(b+ax)^4} + \frac{2\left(a^4-b^4\right)}{a^2b^3(b+ax)^3} + \frac{3a^4+b^4}{a^2b^4(b+ax)^2} + \frac{4a^2}{b^5(b+ax)}\right) dx, x, \text{co}\right.}{d}$$

$$= \frac{\left(a^2+b^2\right)^2}{3a^3b^2d(b+a\cot(c+dx))^3} + \frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(b+a\cot(c+dx))^2} + \frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(b+a\cot(c+dx))} - \frac{4a}{}$$

Mathematica [A] time = 2.20459, size = 133, normalized size = 0.96

$$\frac{-4\left(a^2+b^2\right)\left(a^2+3ab\tan(c+dx)+3b^2\tan^2(c+dx)+b^2\right)+6a(a+b\tan(c+dx))\left(a^2-4a(a+b\tan(c+dx))-2(a+b\right.}{3b^5d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*b^4*Sec[c + d*x]^4 - 4*(a^2 + b^2)*(a^2 + b^2 + 3*a*b*Tan[c + d*x] + 3*b
^2*Tan[c + d*x]^2) + 6*a*(a + b*Tan[c + d*x])*(a^2 + b^2 - 4*a*(a + b*Tan[c
 + d*x]) - 2*Log[a + b*Tan[c + d*x]]*(a + b*Tan[c + d*x])^2))/(3*b^5*d*(a +
 b*Tan[c + d*x])^3)
```

Maple [A] time = 0.285, size = 188, normalized size = 1.4

$$\frac{\tan(dx+c)}{b^4d} - 4\,\frac{a\ln(a+b\tan(dx+c))}{b^5d} + 2\,\frac{a^3}{b^5d(a+b\tan(dx+c))^2} + 2\,\frac{a}{db^3(a+b\tan(dx+c))^2} - 6\,\frac{a^2}{b^5d(a+b\tan(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] tan(d*x+c)/b^4/d-4/d*a/b^5*ln(a+b*tan(d*x+c))+2/d*a^3/b^5/(a+b*tan(d*x+c))^
2+2/d*a/b^3/(a+b*tan(d*x+c))^2-6/d/b^5/(a+b*tan(d*x+c))*a^2-2/d/b^3/(a+b*ta
n(d*x+c))-1/3/d/b^5/(a+b*tan(d*x+c))^3*a^4-2/3/d/b^3/(a+b*tan(d*x+c))^3*a^2
-1/3/d/b/(a+b*tan(d*x+c))^3
```

Maxima [A] time = 1.27017, size = 194, normalized size = 1.41

$$-\frac{\frac{13\,a^4+2\,a^2b^2+b^4+6\left(3\,a^2b^2+b^4\right)\tan(dx+c)^2+6\left(5\,a^3b+ab^3\right)\tan(dx+c)}{b^8\tan(dx+c)^3+3\,ab^7\tan(dx+c)^2+3\,a^2b^6\tan(dx+c)+a^3b^5}+\frac{12\,a\log(b\tan(dx+c)+a)}{b^5}-\frac{3\,\tan(dx+c)}{b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((13*a^4 + 2*a^2*b^2 + b^4 + 6*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 6*(5
*a^3*b + a*b^3)*tan(d*x + c))/(b^8*tan(d*x + c)^3 + 3*a*b^7*tan(d*x + c)^2
+ 3*a^2*b^6*tan(d*x + c) + a^3*b^5) + 12*a*log(b*tan(d*x + c) + a)/b^5 - 3*
tan(d*x + c)/b^4)/d
```

Fricas [B] time = 0.702726, size = 1191, normalized size = 8.63

$$\frac{3\,a^2b^4+3\,b^6-4\left(9\,a^4b^2+3\,a^2b^4-2\,b^6\right)\cos\left(dx+c\right)^4+6\left(5\,a^4b^2+a^2b^4-2\,b^6\right)\cos\left(dx+c\right)^2-6\left(\left(a^6-2\,a^4b^2-3\,a^2b^4\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*b^4 + 3*b^6 - 4*(9*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*cos(d*x + c)^4 +
 6*(5*a^4*b^2 + a^2*b^4 - 2*b^6)*cos(d*x + c)^2 - 6*((a^6 - 2*a^4*b^2 - 3*a
^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + ((3*a^5*b +
 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 + (a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*
x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2) + 6*((a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + a^2*b
^4)*cos(d*x + c)^2 + ((3*a^5*b + 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 + (a^3*b
^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(2*(3*a^5*b
 - 7*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^3 + (11*a^3*b^3 + 9*a*b^5)*cos(d*x + c
))*sin(d*x + c))/((a^5*b^5 - 2*a^3*b^7 - 3*a*b^9)*d*cos(d*x + c)^4 + 3*(a^3
*b^7 + a*b^9)*d*cos(d*x + c)^2 + ((3*a^4*b^6 + 2*a^2*b^8 - b^10)*d*cos(d*x
```

```
+ c)^3 + (a^2*b^8 + b^10)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19932, size = 186, normalized size = 1.35

$$-\frac{\frac{12\,a\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^5}-\frac{3\,\tan\left(dx+c\right)}{b^4}-\frac{22\,ab^3\tan\left(dx+c\right)^3+48\,a^2b^2\tan\left(dx+c\right)^2-6\,b^4\tan\left(dx+c\right)^2+36\,a^3b\tan\left(dx+c\right)-6\,ab^3\tan\left(dx+c\right)+9\,a^4-2\,a^2b^2-b^4}{\left(b\tan\left(dx+c\right)+a\right)^3b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*a*log(abs(b*tan(d*x + c) + a))/b^5 - 3*tan(d*x + c)/b^4 - (22*a*b^
3*tan(d*x + c)^3 + 48*a^2*b^2*tan(d*x + c)^2 - 6*b^4*tan(d*x + c)^2 + 36*a^
3*b*tan(d*x + c) - 6*a*b^3*tan(d*x + c) + 9*a^4 - 2*a^2*b^2 - b^4)/((b*tan(
d*x + c) + a)^3*b^5))/d
```

3.148 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=400

$$\frac{8a^2\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{2\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{4a^2}{b^5d(a\cos(c+dx)+b\sin(c+dx))}-\frac{2\left(a^2+b\right.}{b^5d(a\cos(c+dx)+}$$

```
[Out] (8*a^2*ArcTanh[Sin[c + d*x]])/(b^6*d) + ArcTanh[Sin[c + d*x]]/(2*b^4*d) + (
2*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) + (4*a^3*ArcTanh[(b*Cos[c + d*
x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*Sqrt[a^2 + b^2]*d) + (3*a*ArcTa
nh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*b^4*Sqrt[a^2 + b^
2]*d) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt
[a^2 + b^2]])/(b^6*d) - (4*a*Sec[c + d*x])/(b^5*d) - (a^2 + b^2)/(3*b^3*d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*(b*Cos[c + d*x] - a*Sin[c + d*x]
))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (4*a^2)/(b^5*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])) - (2*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[
c + d*x])) + (Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d)
```

Rubi [A] time = 0.796426, antiderivative size = 400, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3106, 3094, 3770, 3074, 206, 3076, 3768, 3104}

$$\frac{8a^2\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{2\left(a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{4a^2}{b^5d(a\cos(c+dx)+b\sin(c+dx))}-\frac{2\left(a^2+b\right.}{b^5d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```


```
[Out] (8*a^2*ArcTanh[Sin[c + d*x]])/(b^6*d) + ArcTanh[Sin[c + d*x]]/(2*b^4*d) + (
2*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) + (4*a^3*ArcTanh[(b*Cos[c + d*
x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*Sqrt[a^2 + b^2]*d) + (3*a*ArcTa
nh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*b^4*Sqrt[a^2 + b^
2]*d) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt
[a^2 + b^2]])/(b^6*d) - (4*a*Sec[c + d*x])/(b^5*d) - (a^2 + b^2)/(3*b^3*d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*(b*Cos[c + d*x] - a*Sin[c + d*x]
))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (4*a^2)/(b^5*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])) - (2*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[
c + d*x])) + (Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d)
```

Rule 3106

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[(2
*a)/b^2, Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3094

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3076

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3104

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} - \frac{(2a)\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx}{b^2} + \frac{\left(a^2+b^2\right)\int \frac{}{(a\,c}}{}$$

$$= -\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{\int \sec^3(c+dx)\,dx}{b^4} - 2\frac{(2a)\int \frac{\sec^2(}{a\cos(c+dx)}}{b^4}$$

$$= -\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{3a(b\cos(c+dx)-a\sin(c+dx))}{2b^4d(a\cos(c+dx)+b\sin(c+dx))^2} -$$

$$= \frac{4a^2\tanh^{-1}(\sin(c+dx))}{b^6d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c}$$

$$= \frac{4a^2\tanh^{-1}(\sin(c+dx))}{b^6d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{4a^3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c}{\sqrt{a^2+b^2}}\right.}{b^6\sqrt{a^2+b^2}d}$$

Mathematica [A] time = 3.20565, size = 538, normalized size = 1.34

$$-\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(18b^2\left(a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b\sin(c+dx))+6b\left(12a^2+b^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -(Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(4*b^3*(a^2 + b^2) + 18*
b^2*(a^2 + b^2)*Sin[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + 6*b*(12*a^
2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 48*a*b*(a*Cos[c + d*x] + b*S
in[c + d*x])^3 + (60*a*(4*a^2 + 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sq
rt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/Sqrt[a^2 + b^2] + 30*(4
*a^2 + b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Si
n[c + d*x])^3 - 30*(4*a^2 + b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x
])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (48*a*b*Sin[(c + d*x)/2]*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) +
(3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] + Sin[(c + d*
x)/2])^2 - (48*a*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(C
os[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b^6*d*(a + b*Tan[c + d*x])^4)
```

Maple [B] time = 0.344, size = 1255, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 9/d/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^3*tan(1/2*d*x
+1/2*c)^5-5/2/d/b^4*ln(tan(1/2*d*x+1/2*c)-1)+5/2/d/b^4*ln(tan(1/2*d*x+1/2*c
)+1)+18/d/b/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*tan(1/2*d*x
+1/2*c)^2+12/d/b^5/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^4+
5/3/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^2+10/d/b^6*
ln(tan(1/2*d*x+1/2*c)+1)*a^2+4/d/b^5/(tan(1/2*d*x+1/2*c)-1)*a-10/d/b^6*ln(t
an(1/2*d*x+1/2*c)-1)*a^2+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b
-a)^3/a*tan(1/2*d*x+1/2*c)^5+8/3/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/
2*c)*b-a)^3/a*tan(1/2*d*x+1/2*c)^3+2/d/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*
x+1/2*c)*b-a)^3/a*tan(1/2*d*x+1/2*c)-4/d/b^5/(tan(1/2*d*x+1/2*c)+1)*a-1/2/d
/b^4/(tan(1/2*d*x+1/2*c)+1)^2+1/2/d/b^4/(tan(1/2*d*x+1/2*c)+1)+1/2/d/b^4/(t
an(1/2*d*x+1/2*c)-1)^2+1/2/d/b^4/(tan(1/2*d*x+1/2*c)-1)+2/3/d/b/(tan(1/2*d*
x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3+12/d/b^5/(tan(1/2*d*x+1/2*c)^2*a-2
*tan(1/2*d*x+1/2*c)*b-a)^3*a^4*tan(1/2*d*x+1/2*c)^4-39/d/b^3/(tan(1/2*d*x+1
/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^2*tan(1/2*d*x+1/2*c)^4-4/d*b/(tan(1
/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/a^2*tan(1/2*d*x+1/2*c)^4-72/d
/b^4/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^3*tan(1/2*d*x+1/
2*c)^3+38/d/b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a*tan(1
```

```
/2*d*x+1/2*c)^3+8/3/d*b^2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)
^3/a^3*tan(1/2*d*x+1/2*c)^3-24/d/b^5/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+
1/2*c)*b-a)^3*a^4*tan(1/2*d*x+1/2*c)^2+100/d/b^3/(tan(1/2*d*x+1/2*c)^2*a-2*
tan(1/2*d*x+1/2*c)*b-a)^3*a^2*tan(1/2*d*x+1/2*c)^2+4/d*b/(tan(1/2*d*x+1/2*c
)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3/a^2*tan(1/2*d*x+1/2*c)^2+63/d/b^4/(tan(1/
2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a^3*tan(1/2*d*x+1/2*c)+10/d/b^
2/(tan(1/2*d*x+1/2*c)^2*a-2*tan(1/2*d*x+1/2*c)*b-a)^3*a*tan(1/2*d*x+1/2*c)-
20/d/b^6*a^3/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+
b^2)^(1/2))-15/d/b^4*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-
2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.917428, size = 1843, normalized size = 4.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^2*b^5 + 6*b^7 - 30*(4*a^6*b - 3*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d*
x + c)^4 - 20*(11*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 15*((4*a^6
 - 9*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^5 + 3*(4*a^4*b^2 + 3*a^2*b^4)*cos(d*
x + c)^3 + ((12*a^5*b + 5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 + (4*a^3*b^3 +
3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((4*a^7 - 7*a^5*b^2 - 14*a^3*b^4 -
3*a*b^6)*cos(d*x + c)^5 + 3*(4*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*x + c)^3
+ ((12*a^6*b + 11*a^4*b^3 - 2*a^2*b^5 - b^7)*cos(d*x + c)^4 + (4*a^4*b^3 +
5*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(
```

```
(4*a^7 - 7*a^5*b^2 - 14*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^5 + 3*(4*a^5*b^2 +
5*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + ((12*a^6*b + 11*a^4*b^3 - 2*a^2*b^5 - b
^7)*cos(d*x + c)^4 + (4*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1) - 30*(10*(a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 +
(a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^6 - 2*a^3*b^8 - 3*a*b
^10)*d*cos(d*x + c)^5 + 3*(a^3*b^8 + a*b^10)*d*cos(d*x + c)^3 + ((3*a^4*b^7
 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^4 + (a^2*b^9 + b^11)*d*cos(d*x + c)^2)*
sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)

[Out] Timed out
```

Giac [A] time = 1.35717, size = 740, normalized size = 1.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")

[Out] 1/6*(15*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a^2 +
b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^3 + 3*a*b^2)*log(abs(
2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1
/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 6*(b*tan(1/2*d*x
+ 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 8*a)/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*b^5) + 2*(27*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*
a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^7*tan(1/2*d*x + 1/2*c)^4 - 117*a^5*b^
2*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 216*a^6*b*tan(
1/2*d*x + 1/2*c)^3 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^5*tan(1/2
*d*x + 1/2*c)^3 + 8*b^7*tan(1/2*d*x + 1/2*c)^3 - 72*a^7*tan(1/2*d*x + 1/2*c
)^2 + 300*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 + 54*a^3*b^4*tan(1/2*d*x + 1/2*c)^
2 + 12*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 189*a^6*b*tan(1/2*d*x + 1/2*c) + 30*a
```

```
^4*b^3*tan(1/2*d*x + 1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 36*a^7 + 5*a
^5*b^2 + 2*a^3*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) -
 a)^3*a^3*b^5))/d
```

3.149 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$\frac{(10a^2+3b^2)\tan(c+dx)}{b^6d}+\frac{(a^2+b^2)^3}{3a^3b^4d(a\cot(c+dx)+b)^3}+\frac{9a^4b^2+10a^6+b^6}{a^3b^6d(a\cot(c+dx)+b)}+\frac{3a^4b^2+2a^6-b^6}{a^3b^5d(a\cot(c+dx)+b)^2}-\frac{4a(5a^2$$

```
[Out] (a^2 + b^2)^3/(3*a^3*b^4*d*(b + a*Cot[c + d*x])^3) + (2*a^6 + 3*a^4*b^2 - b
^6)/(a^3*b^5*d*(b + a*Cot[c + d*x])^2) + (10*a^6 + 9*a^4*b^2 + b^6)/(a^3*b^
6*d*(b + a*Cot[c + d*x])) - (4*a*(5*a^2 + 3*b^2)*Log[b + a*Cot[c + d*x]])/(
b^7*d) - (4*a*(5*a^2 + 3*b^2)*Log[Tan[c + d*x]])/(b^7*d) + ((10*a^2 + 3*b^2
)*Tan[c + d*x])/(b^6*d) - (2*a*Tan[c + d*x]^2)/(b^5*d) + Tan[c + d*x]^3/(3*
b^4*d)
```

Rubi [A] time = 0.248932, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3088, 894}

$$\frac{(10a^2+3b^2)\tan(c+dx)}{b^6d}+\frac{(a^2+b^2)^3}{3a^3b^4d(a\cot(c+dx)+b)^3}+\frac{9a^4b^2+10a^6+b^6}{a^3b^6d(a\cot(c+dx)+b)}+\frac{3a^4b^2+2a^6-b^6}{a^3b^5d(a\cot(c+dx)+b)^2}-\frac{4a(5a^2$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^2 + b^2)^3/(3*a^3*b^4*d*(b + a*Cot[c + d*x])^3) + (2*a^6 + 3*a^4*b^2 - b
^6)/(a^3*b^5*d*(b + a*Cot[c + d*x])^2) + (10*a^6 + 9*a^4*b^2 + b^6)/(a^3*b^
6*d*(b + a*Cot[c + d*x])) - (4*a*(5*a^2 + 3*b^2)*Log[b + a*Cot[c + d*x]])/(
b^7*d) - (4*a*(5*a^2 + 3*b^2)*Log[Tan[c + d*x]])/(b^7*d) + ((10*a^2 + 3*b^2
)*Tan[c + d*x])/(b^6*d) - (2*a*Tan[c + d*x]^2)/(b^5*d) + Tan[c + d*x]^3/(3*
b^4*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^4x^4} - \frac{4a}{b^5x^3} + \frac{10a^2+3b^2}{b^6x^2} - \frac{4(5a^3+3ab^2)}{b^7x} + \frac{(a^2+b^2)^3}{a^2b^4(b+ax)^4} + \frac{2(2a^6+3a^4b^2-b^6)}{a^2b^5(b+ax)^3} + \right.\right.}{d}$$

$$= \frac{(a^2+b^2)^3}{3a^3b^4d(b+a\cot(c+dx))^3} + \frac{2a^6+3a^4b^2-b^6}{a^3b^5d(b+a\cot(c+dx))^2} + \frac{10a^6+9a^4b^2+b}{a^3b^6d(b+a\cot(c+}$$

Mathematica [A] time = 2.0133, size = 295, normalized size = 1.27

$$\frac{-2\left(-6a^2b^4\tan^4(c+dx)+6ab^3\tan^3(c+dx)\left(\left(5a^2+3b^2\right)\log(a+b\tan(c+dx))-3a^2\right)+6b^2\tan^2(c+dx)\left(3a^2\left(5a^2+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^6*Sec[c + d*x]^6 + 3*b^4*Sec[c + d*x]^4*(a^2 + 2*b^2 - a*b*Tan[c + d*x])
 - 2*(37*a^6 + 36*a^4*b^2 + 3*a^2*b^4 + 4*b^6 + 6*a^4*(5*a^2 + 3*b^2)*Log[a
 + b*Tan[c + d*x]] + 3*a*b*(27*a^4 + 30*a^2*b^2 + b^4 + 6*a^2*(5*a^2 + 3*b^
2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x] + 6*b^2*(6*a^4 + 11*a^2*b^2 + 2*b^
4 + 3*a^2*(5*a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x]^2 + 6*a*b^3
*(-3*a^2 + (5*a^2 + 3*b^2)*Log[a + b*Tan[c + d*x]])*Tan[c + d*x]^3 - 6*a^2*
b^4*Tan[c + d*x]^4))/(3*b^7*d*(a + b*Tan[c + d*x])^3)
```

Maple [A] time = 0.306, size = 330, normalized size = 1.4

$$\frac{(\tan(dx+c))^3}{3\,b^4d} - 2\,\frac{a\,(\tan(dx+c))^2}{b^5d} + 10\,\frac{a^2\tan(dx+c)}{db^6} + 3\,\frac{\tan(dx+c)}{b^4d} - 20\,\frac{a^3\ln(a+b\tan(dx+c))}{db^7} - 12\,\frac{a\ln(a+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x)
```

```
[Out] 1/3*tan(d*x+c)^3/b^4/d-2*a*tan(d*x+c)^2/b^5/d+10/d/b^6*a^2*tan(d*x+c)+3*tan
(d*x+c)/b^4/d-20/d*a^3/b^7*ln(a+b*tan(d*x+c))-12/d*a/b^5*ln(a+b*tan(d*x+c))
+3/d*a^5/b^7/(a+b*tan(d*x+c))^2+6/d*a^3/b^5/(a+b*tan(d*x+c))^2+3/d*a/b^3/(a
+b*tan(d*x+c))^2-15/d/b^7/(a+b*tan(d*x+c))*a^4-18/d/b^5/(a+b*tan(d*x+c))*a^
2-3/d/b^3/(a+b*tan(d*x+c))-1/3/d/b^7/(a+b*tan(d*x+c))^3*a^6-1/d/b^5/(a+b*ta
n(d*x+c))^3*a^4-1/d/b^3/(a+b*tan(d*x+c))^3*a^2-1/3/d/b/(a+b*tan(d*x+c))^3
```

Maxima [A] time = 1.19471, size = 293, normalized size = 1.26

$$-\frac{\frac{37\,a^6+39\,a^4b^2+3\,a^2b^4+b^6+9\left(5\,a^4b^2+6\,a^2b^4+b^6\right)\tan(dx+c)^2+9\left(9\,a^5b+10\,a^3b^3+ab^5\right)\tan(dx+c)}{b^{10}\tan(dx+c)^3+3\,ab^9\tan(dx+c)^2+3\,a^2b^8\tan(dx+c)+a^3b^7}-\frac{b^2\tan(dx+c)^3-6\,ab\tan(dx+c)^2+3\left(10\,a^2+3\,b^2\right)\tan(dx+c)}{b^6}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((37*a^6 + 39*a^4*b^2 + 3*a^2*b^4 + b^6 + 9*(5*a^4*b^2 + 6*a^2*b^4 + b
^6)*tan(d*x + c)^2 + 9*(9*a^5*b + 10*a^3*b^3 + a*b^5)*tan(d*x + c))/(b^10*t
an(d*x + c)^3 + 3*a*b^9*tan(d*x + c)^2 + 3*a^2*b^8*tan(d*x + c) + a^3*b^7)
- (b^2*tan(d*x + c)^3 - 6*a*b*tan(d*x + c)^2 + 3*(10*a^2 + 3*b^2)*tan(d*x +
 c))/b^6 + 12*(5*a^3 + 3*a*b^2)*log(b*tan(d*x + c) + a)/b^7)/d
```

Fricas [B] time = 0.703292, size = 1251, normalized size = 5.39

$$-\frac{4\left(45\,a^4b^2-3\,a^2b^4-4\,b^6\right)\cos\left(dx+c\right)^6-b^6-6\left(25\,a^4b^2-5\,a^2b^4-4\,b^6\right)\cos\left(dx+c\right)^4-3\left(5\,a^2b^4+2\,b^6\right)\cos\left(dx+c\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(45*a^4*b^2 - 3*a^2*b^4 - 4*b^6)*cos(d*x + c)^6 - b^6 - 6*(25*a^4*b
^2 - 5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 3*(5*a^2*b^4 + 2*b^6)*cos(d*x + c)
^2 + 6*((5*a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 + 3*
```

```
a^2*b^4)*cos(d*x + c)^4 + ((15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^5
+ (5*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))*log(2*a*b*cos(d*x + c
)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*((5*a^6 - 12*a^4*b^2
 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^4 + (
(15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^5 + (5*a^3*b^3 + 3*a*b^5)*cos
(d*x + c)^3)*sin(d*x + c))*log(cos(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*
(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(d*x + c)^5 - 2*(55*a^3*b^3 + 21*a*b^
5)*cos(d*x + c)^3)*sin(d*x + c))/(3*a*b^9*d*cos(d*x + c)^4 + (a^3*b^7 - 3*a
*b^9)*d*cos(d*x + c)^6 + (b^10*d*cos(d*x + c)^3 + (3*a^2*b^8 - b^10)*d*cos(
d*x + c)^5)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13594, size = 336, normalized size = 1.45

$$-\frac{\frac{12\left(5a^3+3ab^2\right)\log(|b\tan(dx+c)+a|)}{b^7}-\frac{110a^3b^3\tan(dx+c)^3+66ab^5\tan(dx+c)^3+285a^4b^2\tan(dx+c)^2+144a^2b^4\tan(dx+c)^2-9b^6\tan(dx+c)^2+249a^5b\tan}{(b\tan(dx+c)+a)^3b^7}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(5*a^3 + 3*a*b^2)*log(abs(b*tan(d*x + c) + a))/b^7 - (110*a^3*b^3*
tan(d*x + c)^3 + 66*a*b^5*tan(d*x + c)^3 + 285*a^4*b^2*tan(d*x + c)^2 + 144
*a^2*b^4*tan(d*x + c)^2 - 9*b^6*tan(d*x + c)^2 + 249*a^5*b*tan(d*x + c) + 1
08*a^3*b^3*tan(d*x + c) - 9*a*b^5*tan(d*x + c) + 73*a^6 + 27*a^4*b^2 - 3*a^
2*b^4 - b^6)/((b*tan(d*x + c) + a)^3*b^7) - (b^8*tan(d*x + c)^3 - 6*a*b^7*t
an(d*x + c)^2 + 30*a^2*b^6*tan(d*x + c) + 9*b^8*tan(d*x + c))/b^12)/d
```

3.150 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{i\cos^6(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{5\sin(c+dx)\cos(c+dx)}{16ad}+\frac{5x}{16a}$$

```
[Out] (5*x)/(16*a) + ((I/6)*Cos[c + d*x]^6)/(a*d) + (5*Cos[c + d*x]*Sin[c + d*x])
/(16*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[
c + d*x])/(6*a*d)
```

Rubi [A] time = 0.150734, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 2635, 8, 2565, 30}

$$\frac{i\cos^6(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{5\sin(c+dx)\cos(c+dx)}{16ad}+\frac{5x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(16*a) + ((I/6)*Cos[c + d*x]^6)/(a*d) + (5*Cos[c + d*x]*Sin[c + d*x])
/(16*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[
c + d*x])/(6*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\cos^6(c+dx)+a\cos^5(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \cos^5(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^6(c+dx)\,dx}{a}\\ &= \frac{\cos^5(c+dx)\sin(c+dx)}{6ad}+\frac{5\int \cos^4(c+dx)\,dx}{6a}+\frac{i\,\mathrm{Subst}\left(\int x^5\,dx,x,\cos(c+dx\right.}{ad}\\ &= \frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}+\frac{5\int \cos}{}\\ &= \frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+\frac{\cos^5(c}{}\\ &= \frac{5x}{16a}+\frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+\end{aligned}$$

Mathematica [A] time = 0.135858, size = 82, normalized size = 0.83

$$\frac{45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))+60c+}{192ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (60*c + 60*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[6
*(c + d*x)] + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])/
(192*a*d)
```

Maple [A] time = 0.119, size = 137, normalized size = 1.4

$$\frac{-\frac{5i}{32}\ln\left(\tan\left(dx+c\right)-i\right)}{ad}-\frac{\frac{3i}{32}}{ad\left(\tan\left(dx+c\right)-i\right)^{2}}-\frac{1}{24\,ad\left(\tan\left(dx+c\right)-i\right)^{3}}+\frac{3}{16\,ad\left(\tan\left(dx+c\right)-i\right)}+\frac{\frac{i}{32}}{ad\left(\tan\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] -5/32*I/a/d*ln(tan(d*x+c)-I)-3/32*I/a/d/(tan(d*x+c)-I)^2-1/24/a/d/(tan(d*x+
c)-I)^3+3/16/a/d/(tan(d*x+c)-I)+1/32*I/a/d/(tan(d*x+c)+I)^2+5/32*I/a/d*ln(t
an(d*x+c)+I)+1/8/a/d/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.472815, size = 242, normalized size = 2.44

$$\frac{\left(120\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(10i\,dx+10i\,c)}-30i\,e^{(8i\,dx+8i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8*
I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I)
*e^(-6*I*d*x - 6*I*c)/(a*d)
```

Sympy [A] time = 0.861229, size = 221, normalized size = 2.23

$$\begin{cases} \frac{\left(-50331648ia^4d^4e^{16ic}e^{4idx}-503316480ia^4d^4e^{14ic}e^{2idx}+1006632960ia^4d^4e^{10ic}e^{-2idx}+251658240ia^4d^4e^{8ic}e^{-4idx}+33554432ia^4d^4e^{6ic}e^{-6idx}\right)e^{-12ic}}{6442450944a^5d^5} & \text{for } 6442 \\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-6ic}}{32a}-\frac{5}{16a}\right) & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a*
*4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp(-
2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a**4
*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne(644
2450944*a**5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*ex
p(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a))
, True)) + 5*x/(16*a)
```

Giac [A] time = 1.13472, size = 157, normalized size = 1.59

$$-\frac{-\frac{30i\log(\tan(dx+c)+i)}{a}+\frac{30i\log(\tan(dx+c)-i)}{a}+\frac{3\left(-15i\tan(dx+c)^2+38\tan(dx+c)+25i\right)}{a(-i\tan(dx+c)+1)^2}-\frac{55i\tan(dx+c)^3+201\tan(dx+c)^2-255i\tan(dx+c)-117}{a(\tan(dx+c)-i)^3}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(-
15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2) -
 (55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(a*(
tan(d*x + c) - I)^3))/d
```

3.151 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\sin^5(c+dx)}{5ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^5(c+dx)}{5ad}$$

```
[Out] ((I/5)*Cos[c + d*x]^5)/(a*d) + Sin[c + d*x]/(a*d) - (2*Sin[c + d*x]^3)/(3*a
*d) + Sin[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.127816, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 2633, 2565, 30}

$$\frac{\sin^5(c+dx)}{5ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/5)*Cos[c + d*x]^5)/(a*d) + Sin[c + d*x]/(a*d) - (2*Sin[c + d*x]^3)/(3*a
*d) + Sin[c + d*x]^5/(5*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2} \\
&= -\frac{i\int \left(ia\cos^5(c+dx)+a\cos^4(c+dx)\sin(c+dx)\right)\,dx}{a^2} \\
&= -\frac{i\int \cos^4(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^5(c+dx)\,dx}{a} \\
&= \frac{i\,\text{Subst}\left(\int x^4\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{ad} \\
&= \frac{i\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)}{ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [A] time = 0.0677495, size = 111, normalized size = 1.59

$$
\frac{5\sin(c+dx)}{8ad}+\frac{5\sin(3(c+dx))}{48ad}+\frac{\sin(5(c+dx))}{80ad}+\frac{i\cos(c+dx)}{8ad}+\frac{i\cos(3(c+dx))}{16ad}+\frac{i\cos(5(c+dx))}{80ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/8)*Cos[c + d*x])/(a*d) + ((I/16)*Cos[3*(c + d*x)])/(a*d) + ((I/80)*Cos[
5*(c + d*x)])/(a*d) + (5*Sin[c + d*x])/(8*a*d) + (5*Sin[3*(c + d*x)])/(48*a
*d) + Sin[5*(c + d*x)]/(80*a*d)
```

Maple [B] time = 0.112, size = 141, normalized size = 2.

$$2\frac{1}{ad}\left(\frac{-i/2}{(\tan(1/2\,dx+c/2)-i)^4}+\frac{3/4\,i}{(\tan(1/2\,dx+c/2)-i)^2}+1/5\,(\tan(1/2\,dx+c/2)-i)^{-5}-5/6\,(\tan(1/2\,dx+c/2)-i)^{-3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/2*I/(tan(1/2*d*x+1/2*c)-I)^4+3/4*I/(tan(1/2*d*x+1/2*c)-I)^2+1/5/(
tan(1/2*d*x+1/2*c)-I)^5-5/6/(tan(1/2*d*x+1/2*c)-I)^3+11/16/(tan(1/2*d*x+1/2
*c)-I)-1/8*I/(tan(1/2*d*x+1/2*c)+I)^2-1/12/(tan(1/2*d*x+1/2*c)+I)^3+5/16/(t
an(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.467378, size = 200, normalized size = 2.86

$$\frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d*
x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [A] time = 0.761387, size = 197, normalized size = 2.81

$$\begin{cases} \frac{\left(-30720ia^4d^4e^{12ic}e^{3idx}-368640ia^4d^4e^{10ic}e^{idx}+552960ia^4d^4e^{8ic}e^{-idx}+122880ia^4d^4e^{6ic}e^{-3idx}+18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for } 1474560a^5d^5e^{9ic} \neq 0 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d**
4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 1228
80*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*exp(
-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(1474560*a**5*d**5*exp(9*I*c)
, 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp
(-5*I*c)/(16*a), True))
```

Giac [A] time = 1.10948, size = 161, normalized size = 2.3

$$\frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(t
an(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2*d
*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c) + 1
13)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.152 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{i\cos^4(c+dx)}{4ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

```
[Out] (3*x)/(8*a) + ((I/4)*Cos[c + d*x]^4)/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.128129, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 2635, 8, 2565, 30}

$$\frac{i\cos^4(c+dx)}{4ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + ((I/4)*Cos[c + d*x]^4)/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2} \\
&= -\frac{i\int \left(ia\cos^4(c+dx)+a\cos^3(c+dx)\sin(c+dx)\right)\,dx}{a^2} \\
&= -\frac{i\int \cos^3(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^4(c+dx)\,dx}{a} \\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int \cos^2(c+dx)\,dx}{4a}+\frac{i\,\mathrm{Subst}\left(\int x^3\,dx,x,\cos(c+dx\right.}{ad} \\
&= \frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int 1\,dx}{8a} \\
&= \frac{3x}{8a}+\frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}
\end{aligned}$$

Mathematica [A] time = 0.10056, size = 60, normalized size = 0.8

$$\frac{8\sin(2(c+dx))+\sin(4(c+dx))+4i\cos(2(c+dx))+i\cos(4(c+dx))+12c+12dx}{32ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (12*c + 12*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 8*Sin[2*(c +
 d*x)] + Sin[4*(c + d*x)])/(32*a*d)
```

Maple [A] time = 0.115, size = 98, normalized size = 1.3

$$\frac{-\frac{3i}{16}\ln\left(\tan\left(dx+c\right)-i\right)}{ad}-\frac{\frac{i}{8}}{ad\left(\tan\left(dx+c\right)-i\right)^{2}}+\frac{1}{4\,ad\left(\tan\left(dx+c\right)-i\right)}+\frac{\frac{3i}{16}\ln\left(\tan\left(dx+c\right)+i\right)}{ad}+\frac{1}{8\,ad\left(\tan\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] -3/16*I/a/d*ln(tan(d*x+c)-I)-1/8*I/a/d/(tan(d*x+c)-I)^2+1/4/a/d/(tan(d*x+c)
-I)+3/16*I/a/d*ln(tan(d*x+c)+I)+1/8/a/d/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.467277, size = 159, normalized size = 2.12

$$\frac{\left(12\,dxe^{(4i\,dx+4i\,c)}-2i\,e^{(6i\,dx+6i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [A] time = 0.438954, size = 153, normalized size = 2.04

$$\begin{cases} \frac{\left(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx}\right)e^{-6ic}}{8192a^3d^3} & \text{for } 8192a^3d^3e^{6ic} \neq 0 \\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise} \end{cases} + \frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*exp
(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*
c)/(8192*a**3*d**3), Ne(8192*a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3
*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/
(8*a)
```

Giac [A] time = 1.12508, size = 134, normalized size = 1.79

$$-\frac{\frac{6i\log(i\tan(dx+c)+1)}{a}-\frac{6i\log(i\tan(dx+c)-1)}{a}+\frac{2(3\tan(dx+c)+5i)}{a(-i\tan(dx+c)+1)}+\frac{-9i\tan(dx+c)^2-26\tan(dx+c)+21i}{a(\tan(dx+c)-i)^2}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*I*log(I*tan(d*x + c) + 1)/a - 6*I*log(I*tan(d*x + c) - 1)/a + 2*(3
*tan(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*
tan(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

3.153 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^3(c+dx)}{3ad}$$

```
[Out] ((I/3)*Cos[c + d*x]^3)/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.120334, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 2633, 2565, 30}

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/3)*Cos[c + d*x]^3)/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\cos^3(c+dx)+a\cos^2(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \cos^2(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^3(c+dx)\,dx}{a}\\ &= \frac{i\,\text{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{ad}\\ &= \frac{i\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.0712707, size = 73, normalized size = 1.4

$$\frac{3\sin(c+dx)}{4ad}+\frac{\sin(3(c+dx))}{12ad}+\frac{i\cos(c+dx)}{4ad}+\frac{i\cos(3(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a*d) + ((I/12)*Cos[3*(c + d*x)])/(a*d) + (3*Sin[c + d
*x])/(4*a*d) + Sin[3*(c + d*x)]/(12*a*d)
```

Maple [A] time = 0.11, size = 75, normalized size = 1.4

$$2\,\frac{1}{ad}\left(-1/3\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-3}+\frac{i/2}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{2}}+3/4\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-1}+1/4\,\left(\tan\left(1/2\,dx+c/2\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 2/d/a*(-1/3/(tan(1/2*d*x+1/2*c)-I)^3+1/2*I/(tan(1/2*d*x+1/2*c)-I)^2+3/4/(ta
n(1/2*d*x+1/2*c)-I)+1/4/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.465694, size = 122, normalized size = 2.35

$$\frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x -
 3*I*c)/(a*d)
```

Sympy [A] time = 0.441612, size = 128, normalized size = 2.46

$$\begin{cases} \frac{\left(-24ia^{2}d^{2}e^{5ic}e^{idx}+48ia^{2}d^{2}e^{3ic}e^{-idx}+8ia^{2}d^{2}e^{ic}e^{-3idx}\right)e^{-4ic}}{96a^{3}d^{3}} & \text{for } 96a^{3}d^{3}e^{4ic}\neq 0 \\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I*
c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a**3
*d**3), Ne(96*a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)
*exp(-3*I*c)/(4*a), True))
```

Giac [A] time = 1.12533, size = 90, normalized size = 1.73

$$\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*ta
n(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

3.154 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] x/(2*a) + ((I/2)*Cos[c + d*x])/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rubi [A] time = 0.0294502, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3082, 8}

$$\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + ((I/2)*Cos[c + d*x])/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 3082

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n)/(2*a*d*n*Cos[c + d*x]^n), x] + Dist[1/(2*a), Int[(a*Cos[c + d*
x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= \frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}\end{aligned}$$

Mathematica [A] time = 0.0679494, size = 38, normalized size = 0.83

$$\frac{2(c+dx)+\sin(2(c+dx))+i\cos(2(c+dx))}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (2*(c + d*x) + I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])/(4*a*d)
```

Maple [A] time = 0.098, size = 59, normalized size = 1.3

$$\frac{-\frac{i}{4}\ln\left(\tan\left(dx+c\right)-i\right)}{ad}+\frac{1}{2\,ad\left(\tan\left(dx+c\right)-i\right)}+\frac{\frac{i}{4}\ln\left(\tan\left(dx+c\right)+i\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] -1/4*I/a/d*ln(tan(d*x+c)-I)+1/2/a/d/(tan(d*x+c)-I)+1/4*I/a/d*ln(tan(d*x+c)+
I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.459964, size = 86, normalized size = 1.87

$$\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A] time = 0.544379, size = 61, normalized size = 1.33

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } 4ade^{2ic} \neq 0 \\ x\left(\frac{\left(e^{2ic}+1\right)e^{-2ic}}{2a} - \frac{1}{2a}\right) & \text{otherwise} \end{cases} + \frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(4*a*d*exp(2*I*c), 0)), (
x*((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

Giac [A] time = 1.14433, size = 81, normalized size = 1.76

$$-\frac{\frac{i \log(\tan(dx+c)-i)}{a} - \frac{i \log(-i \tan(dx+c)+1)}{a} + \frac{-i \tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*log(tan(d*x + c) - I)/a - I*log(-I*tan(d*x + c) + 1)/a + (-I*tan(d*
x + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

3.155 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rubi [A] time = 0.015677, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx=\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Mathematica [A] time = 0.0339486, size = 29, normalized size = 1.

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Maple [A] time = 0.09, size = 23, normalized size = 0.8

$$2\,\frac{1}{ad\left(\tan\left(1/2\,dx+c/2\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 2/d/a/(tan(1/2*d*x+1/2*c)-I)
```

Maxima [A] time = 1.18743, size = 39, normalized size = 1.34

$$\frac{2}{\left(-i\,a+\frac{a\sin(dx+c)}{\cos(dx+c)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A] time = 0.456692, size = 35, normalized size = 1.21

$$\frac{i\,e^{(-i\,dx-i\,c)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [A] time = 0.152055, size = 31, normalized size = 1.07

$$\begin{cases} \frac{ie^{-ic}e^{-idx}}{ad} & \text{for } ade^{ic} \neq 0 \\ \frac{xe^{-ic}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*c
)/a, True))
```

Giac [A] time = 1.09313, size = 28, normalized size = 0.97

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

3.156 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

```
[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)
```

Rubi [A] time = 0.0701351, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3092, 3090, 3475}

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}$$
$$= -\frac{i\int (ia+a\tan(c+dx))\,dx}{a^2}$$
$$= \frac{x}{a}-\frac{i\int \tan(c+dx)\,dx}{a}$$
$$= \frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

Mathematica [A] time = 0.0651584, size = 23, normalized size = 1.

$$\frac{i\log(\cos(c+dx))+c+dx}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (c + d*x + I*Log[Cos[c + d*x]])/(a*d)
```

Maple [A] time = 0.121, size = 22, normalized size = 1.

$$\frac{-i\ln\left(i\tan\left(dx+c\right)+1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] -I/d/a*ln(I*tan(d*x+c)+1)
```

Maxima [B] time = 1.1681, size = 136, normalized size = 5.91

$$-\frac{-\frac{i\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{i\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{i\log\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(-I*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - I*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a + I*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 - 1)/a)/d
```

Fricas [A] time = 0.492848, size = 65, normalized size = 2.83

$$\frac{2\,dx + i\,\log\left(e^{(2i\,dx+2i\,c)} + 1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.14518, size = 80, normalized size = 3.48

$$-\frac{\frac{2i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a} - \frac{i\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{i\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*I*log(tan(1/2*d*x + 1/2*c) - I)/a - I*log(abs(tan(1/2*d*x + 1/2*c) + 1)
)/a - I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a)/d
```

3.157 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rubi [A] time = 0.0943114, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 3770, 2606, 8}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int (ia\sec(c+dx)+a\sec(c+dx)\tan(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \sec(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec(c+dx)\,dx}{a}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\,\mathrm{Subst}(\int 1\,dx,x,\sec(c+dx))}{ad}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.193977, size = 35, normalized size = 1.13

$$-\frac{i\left(\sec(c+dx)+2i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I)*((2*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]))/(a*d)
```

Maple [B] time = 0.145, size = 85, normalized size = 2.7

$$\frac{-i}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{1}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{i}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{1}{ad}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] -I/d/a/(tan(1/2*d*x+1/2*c)+1)+1/d/a*ln(tan(1/2*d*x+1/2*c)+1)+I/d/a/(tan(1/2
*d*x+1/2*c)-1)-1/d/a*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.17606, size = 112, normalized size = 3.61

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2}{-ia+\frac{ia\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] time = 0.474333, size = 217, normalized size = 7.

$$\frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
+ 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*I*
c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.15208, size = 81, normalized size = 2.61

$$\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - log(abs(tan(1/2*d*x + 1/2*c) - 1))/
a + 2*I/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

3.158 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)}{ad}-\frac{i\sec^2(c+dx)}{2ad}$$

```
[Out] ((-I/2)*Sec[c + d*x]^2)/(a*d) + Tan[c + d*x]/(a*d)
```

Rubi [A] time = 0.107244, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3767, 8, 2606, 30}

$$\frac{\tan(c+dx)}{ad}-\frac{i\sec^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/2)*Sec[c + d*x]^2)/(a*d) + Tan[c + d*x]/(a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \left(ia\sec^2(c+dx)+a\sec^2(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\
&= -\frac{i\int \sec^2(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^2(c+dx)\,dx}{a}\\
&= -\frac{i\,\mathrm{Subst}(\int x\,dx,x,\sec(c+dx))}{ad}-\frac{\mathrm{Subst}(\int 1\,dx,x,-\tan(c+dx))}{ad}\\
&= -\frac{i\sec^2(c+dx)}{2ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}
$$

Mathematica [A] time = 0.183829, size = 35, normalized size = 1.03

$$
-\frac{i\sec(c+dx)(\sec(c+dx)+2i\sec(c)\sin(dx))}{2ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/2)*Sec[c + d*x]*(Sec[c + d*x] + (2*I)*Sec[c]*Sin[d*x]))/(a*d)
```

Maple [A] time = 0.135, size = 26, normalized size = 0.8

$$\frac{\tan(dx+c) - \frac{i}{2}(\tan(dx+c))^2}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/2*I*tan(d*x+c)^2)
```

Maxima [B] time = 1.18713, size = 146, normalized size = 4.29

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{i\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a - \frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(sin(d*x + c)/(cos(d*x + c) + 1) - I*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a - 2*a*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)
```

Fricas [A] time = 0.438385, size = 88, normalized size = 2.59

$$\frac{2i}{ade^{(4i\,dx+4i\,c)} + 2\,ade^{(2i\,dx+2i\,c)} + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.16536, size = 36, normalized size = 1.06

$$-\frac{i \tan(dx+c)^2 - 2 \tan(dx+c)}{2 ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

3.159 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.117174, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3768, 3770, 2606, 30}

$$-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \left(ia\sec^3(c+dx)+a\sec^3(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\
&= -\frac{i\int \sec^3(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^3(c+dx)\,dx}{a}\\
&= \frac{\sec(c+dx)\tan(c+dx)}{2ad}+\frac{\int \sec(c+dx)\,dx}{2a}-\frac{i\,\mathrm{Subst}\left(\int x^2\,dx,x,\sec(c+dx)\right)}{ad}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.240093, size = 54, normalized size = 0.9

$$-\frac{i\left((4+3i\sin(2(c+dx)))\sec^3(c+dx)+12i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/12)*((12*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(4
+ (3*I)*Sin[2*(c + d*x)])))/(a*d)
```

Maple [B] time = 0.163, size = 258, normalized size = 4.3

$$-\frac{\frac{i}{3}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{1}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{\frac{i}{2}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{2\,ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{\frac{i}{2}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] -1/3*I/d/a/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/a/(tan(1/2*d*x+1/2*c)+1)-1/2*I/d/
a/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^2+1/2*I/d/a/(tan(1/
2*d*x+1/2*c)+1)^2+1/2/d/a*ln(tan(1/2*d*x+1/2*c)+1)+1/3*I/d/a/(tan(1/2*d*x+1
/2*c)-1)^3+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)^2+1/2*I/d/a/(tan(1/2*d*x+1/2*c)-1
)^2+1/2/d/a/(tan(1/2*d*x+1/2*c)-1)+1/2*I/d/a/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a
*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.17412, size = 251, normalized size = 4.18

$$\frac{\frac{4\left(\frac{3i\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{3i\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+2\right)}{6i\,a-\frac{18i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [B] time = 0.481741, size = 505, normalized size = 8.42

$$\frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx}\right.}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*
I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*d*x
 + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I*d*x
 + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.17489, size = 136, normalized size = 2.27

$$\frac{\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a}-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}a}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 3*log(abs(tan(1/2*d*x + 1/2*c
) - 1))/a + 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*ta
n(1/2*d*x + 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

3.160 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^4(c+dx)}{4ad}$$

```
[Out] ((-I/4)*Sec[c + d*x]^4)/(a*d) + Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.115584, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 3767, 2606, 30}

$$\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/4)*Sec[c + d*x]^4)/(a*d) + Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^4(c+dx)+a\sec^4(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^4(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^4(c+dx)\,dx}{a}\\ &= -\frac{i\,\mathrm{Subst}\left(\int x^3\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\mathrm{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{ad}\\ &= -\frac{i\sec^4(c+dx)}{4ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.272148, size = 53, normalized size = 1.02

$$-\frac{i\sec^4(c+dx)(i\sec(c)(4\sin(c+2dx)+\sin(3c+4dx))-3i\tan(c)+3)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/12)*Sec[c + d*x]^4*(3 + I*Sec[c]*(4*Sin[c + 2*d*x] + Sin[3*c + 4*d*x])
 - (3*I)*Tan[c]))/(a*d)
```

Maple [A] time = 0.148, size = 47, normalized size = 0.9

$$\frac{1}{ad}\left(\tan\left(dx+c\right)-\frac{i}{4}\left(\tan\left(dx+c\right)\right)^{4}+\frac{\left(\tan\left(dx+c\right)\right)^{3}}{3}-\frac{i}{2}\left(\tan\left(dx+c\right)\right)^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/4*I*tan(d*x+c)^4+1/3*tan(d*x+c)^3-1/2*I*tan(d*x+c)^2)
```

Maxima [B] time = 1.21298, size = 285, normalized size = 5.48

$$\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{3i\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{5\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{5\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{3i\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{3\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{3\left(a-\frac{4a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{4a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - 3*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/((a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A] time = 0.451651, size = 208, normalized size = 4.

$$\frac{16i\,e^{(2i\,dx+2i\,c)}+4i}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(16*I*e^(2*I*d*x + 2*I*c) + 4*I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*
I*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a*
```

```
d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.17927, size = 63, normalized size = 1.21

$$-\frac{3i\tan\left(dx+c\right)^{4}-4\tan\left(dx+c\right)^{3}+6i\tan\left(dx+c\right)^{2}-12\tan\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan(
d*x + c))/(a*d)
```

3.161 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rubi [A] time = 0.13277, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3768, 3770, 2606, 30}

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2} \\
&= -\frac{i\int \left(ia\sec^5(c+dx)+a\sec^5(c+dx)\tan(c+dx)\right)dx}{a^2} \\
&= -\frac{i\int \sec^5(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^5(c+dx)\,dx}{a} \\
&= \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}+\frac{3\int \sec^3(c+dx)\,dx}{4a}-\frac{i\,\mathrm{Subst}\left(\int x^4\,dx,x,\sec(c+dx)\right)}{ad} \\
&= -\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{4ad}+\frac{3\int \sec
\\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx
\end{aligned}
$$

Mathematica [A] time = 0.477167, size = 66, normalized size = 0.79

$$
-\frac{i\left((70i\sin(2(c+dx))+15i\sin(4(c+dx))+64)\sec^5(c+dx)+240i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{320ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/320)*((240*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5*(
64 + (70*I)*Sin[2*(c + d*x)] + (15*I)*Sin[4*(c + d*x)])))/(a*d)
```

Maple [B] time = 0.169, size = 430, normalized size = 5.1

$$\frac{\frac{5i}{8}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}+\frac{5}{8ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{\frac{3i}{8}}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}+\frac{1}{2ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{\frac{5i}{8}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 5/8*I/d/a/(tan(1/2*d*x+1/2*c)-1)^2+5/8/d/a/(tan(1/2*d*x+1/2*c)+1)+3/8*I/d/a
/(tan(1/2*d*x+1/2*c)-1)+1/2/d/a/(tan(1/2*d*x+1/2*c)+1)^3+5/8*I/d/a/(tan(1/2
*d*x+1/2*c)+1)^2-1/4/d/a/(tan(1/2*d*x+1/2*c)+1)^4+1/5*I/d/a/(tan(1/2*d*x+1/
2*c)-1)^5-7/8/d/a/(tan(1/2*d*x+1/2*c)+1)^2-3/4*I/d/a/(tan(1/2*d*x+1/2*c)+1)
^3+3/8/d/a*ln(tan(1/2*d*x+1/2*c)+1)+1/2*I/d/a/(tan(1/2*d*x+1/2*c)+1)^4+7/8/
d/a/(tan(1/2*d*x+1/2*c)-1)^2-3/8*I/d/a/(tan(1/2*d*x+1/2*c)+1)+5/8/d/a/(tan(
1/2*d*x+1/2*c)-1)+3/4*I/d/a/(tan(1/2*d*x+1/2*c)-1)^3+1/2/d/a/(tan(1/2*d*x+1
/2*c)-1)^3+1/2*I/d/a/(tan(1/2*d*x+1/2*c)-1)^4+1/4/d/a/(tan(1/2*d*x+1/2*c)-1
)^4-1/5*I/d/a/(tan(1/2*d*x+1/2*c)+1)^5-3/8/d/a*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.27656, size = 390, normalized size = 4.64

$$\frac{\frac{16\left(-\frac{75i\sin(dx+c)}{\cos(dx+c)+1}+\frac{30i\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{240\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{30i\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{120\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{75i\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-24\right)}{-120ia+\frac{600ia\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1200ia\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{1200ia\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{600ia\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(16*(-75*I*sin(d*x + c)/(cos(d*x + c) + 1) + 30*I*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 240*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 30*I*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 - 120*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 75*I*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 - 24)/(-120*I*a + 600*I*a*sin(d*x + c)^2
```

```
/(cos(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1200
*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d*x
+ c) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) + 3*log(sin(d*
x + c)/(cos(d*x + c) + 1) + 1)/a - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a)/d
```

Fricas [B] time = 0.496565, size = 810, normalized size = 9.64

$$\frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i}}{40\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8i\,dx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6
*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*
d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c) - 25
6*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))
/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x
 + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.1778, size = 189, normalized size = 2.25

$$\frac{\frac{15 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)}{a} - \frac{15 \log\left(\left|\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)}{a} + \frac{2\left(25 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^9+40i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^8-10 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7+80i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4+10 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-25 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+8i\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-1\right)^5 a}}{40 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 15*log(abs(tan(1/2*d*x + 1/
2*c) - 1))/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 -
 10*tan(1/2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x +
 1/2*c)^3 - 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*
a))/d
```

3.162 $\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{\tan^5(c+dx)}{5ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^6(c+dx)}{6ad}$$

```
[Out] ((-I/6)*Sec[c + d*x]^6)/(a*d) + Tan[c + d*x]/(a*d) + (2*Tan[c + d*x]^3)/(3*
a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.122142, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3092, 3090, 3767, 2606, 30}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/6)*Sec[c + d*x]^6)/(a*d) + Tan[c + d*x]/(a*d) + (2*Tan[c + d*x]^3)/(3*
a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^7(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^6(c+dx)+a\sec^6(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^6(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^6(c+dx)\,dx}{a}\\ &= -\frac{i\,\text{Subst}\left(\int x^5\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\tan(c+dx\right.}{ad}\\ &= -\frac{i\sec^6(c+dx)}{6ad}+\frac{\tan(c+dx)}{ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A] time = 0.349596, size = 67, normalized size = 0.96

$$-\frac{i\sec(c)\sec^6(c+dx)(10\cos(c)-i(-15\sin(c+2dx)-6\sin(3c+4dx)-\sin(5c+6dx)+10\sin(c)))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-I/60)*Sec[c]*Sec[c + d*x]^6*(10*Cos[c] - I*(10*Sin[c] - 15*Sin[c + 2*d*x
] - 6*Sin[3*c + 4*d*x] - Sin[5*c + 6*d*x])))/(a*d)
```

Maple [A] time = 0.153, size = 68, normalized size = 1.

$$\frac{1}{ad}\left(\tan(dx+c)-\frac{i}{6}(\tan(dx+c))^6+\frac{(\tan(dx+c))^5}{5}-\frac{i}{2}(\tan(dx+c))^4+\frac{2\,(\tan(dx+c))^3}{3}-\frac{i}{2}(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-1/6*I*tan(d*x+c)^6+1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4+2/
3*tan(d*x+c)^3-1/2*I*tan(d*x+c)^2)
```

Maxima [B] time = 1.28286, size = 423, normalized size = 6.04

$$\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{15i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{78\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{50i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{78\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15i\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\right.}{15\left(a-\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 15*I*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 78*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - 50*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 78*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*I*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 - 15*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11)/((a - 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [A] time = 0.461256, size = 333, normalized size = 4.76

$$\frac{240i\,e^{(4i\,dx+4i\,c)}+96i\,e^{(2i\,dx+2i\,c)}+16i}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(240*I*e^(4*I*d*x + 4*I*c) + 96*I*e^(2*I*d*x + 2*I*c) + 16*I)/(a*d*e^(
12*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*
c) + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*I
*d*x + 2*I*c) + a*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.1558, size = 90, normalized size = 1.29

$$-\frac{5i\tan\left(dx+c\right)^{6}-6\tan\left(dx+c\right)^{5}+15i\tan\left(dx+c\right)^{4}-20\tan\left(dx+c\right)^{3}+15i\tan\left(dx+c\right)^{2}-30\tan\left(dx+c\right)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

3.163 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$-\frac{2\sin^7(c+dx)}{7a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^7(c+dx)}{7a^2d}$$

```
[Out] (((2*I)/7)*Cos[c + d*x]^7)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (4*Sin[c + d*x]
^3)/(3*a^2*d) + Sin[c + d*x]^5/(a^2*d) - (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.185731, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 270}

$$-\frac{2\sin^7(c+dx)}{7a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^7(c+dx)}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/7)*Cos[c + d*x]^7)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (4*Sin[c + d*x]
^3)/(3*a^2*d) + Sin[c + d*x]^5/(a^2*d) - (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$
$$= -\frac{\int \left(-a^2\cos^7(c+dx)+2ia^2\cos^6(c+dx)\sin(c+dx)+a^2\cos^5(c+dx)\sin^2(c+\right.}{a^4}$$
$$= -\frac{(2i)\int \cos^6(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^7(c+dx)\,dx}{a^2}-\frac{\int \cos^5(c+dx)\sin^2(c}{a^2}$$
$$= \frac{(2i)\operatorname{Subst}\left(\int x^6\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\sin(c+dx)\right)}{a^2d}$$
$$= \frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{3\sin^5(c+dx)}{5a^2d}-\frac{\sin^7(c+dx)}{7a^2d}-$$
$$= \frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{2\sin^7(c+dx)}{7a^2d}$$

Mathematica [A] time = 0.103847, size = 149, normalized size = 1.75

$$\frac{15\sin(c+dx)}{32a^2d}+\frac{11\sin(3(c+dx))}{96a^2d}+\frac{\sin(5(c+dx))}{32a^2d}+\frac{\sin(7(c+dx))}{224a^2d}+\frac{5i\cos(c+dx)}{32a^2d}+\frac{3i\cos(3(c+dx))}{32a^2d}+\frac{i\cos(5(c+}{32a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((5*I)/32)*Cos[c + d*x])/(a^2*d) + (((3*I)/32)*Cos[3*(c + d*x)])/(a^2*d) +
 ((I/32)*Cos[5*(c + d*x)])/(a^2*d) + ((I/224)*Cos[7*(c + d*x)])/(a^2*d) + (
15*Sin[c + d*x])/(32*a^2*d) + (11*Sin[3*(c + d*x)])/(96*a^2*d) + Sin[5*(c +
 d*x)]/(32*a^2*d) + Sin[7*(c + d*x)]/(224*a^2*d)
```

Maple [B] time = 0.138, size = 174, normalized size = 2.1

$$2\frac{1}{da^2}\left(\frac{i}{(\tan(1/2\,dx+c/2)-i)^6}-\frac{5/2\,i}{(\tan(1/2\,dx+c/2)-i)^4}+\frac{\frac{23\,i}{16}}{(\tan(1/2\,dx+c/2)-i)^2}-2/7\,(\tan(1/2\,dx+c/2)-i)^{-7}+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(I/(tan(1/2*d*x+1/2*c)-I)^6-5/2*I/(tan(1/2*d*x+1/2*c)-I)^4+23/16*I/
(tan(1/2*d*x+1/2*c)-I)^2-2/7/(tan(1/2*d*x+1/2*c)-I)^7+2/(tan(1/2*d*x+1/2*c)
-I)^5-55/24/(tan(1/2*d*x+1/2*c)-I)^3+13/16/(tan(1/2*d*x+1/2*c)-I)-1/16*I/(t
an(1/2*d*x+1/2*c)+I)^2-1/24/(tan(1/2*d*x+1/2*c)+I)^3+3/16/(tan(1/2*d*x+1/2*
c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.469126, size = 244, normalized size = 2.87

$$\frac{\left(-7i\,e^{(10i\,dx+10i\,c)} - 105i\,e^{(8i\,dx+8i\,c)} + 210i\,e^{(6i\,dx+6i\,c)} + 70i\,e^{(4i\,dx+4i\,c)} + 21i\,e^{(2i\,dx+2i\,c)} + 3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6*
I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*I)
*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [A] time = 1.81458, size = 233, normalized size = 2.74

$$\begin{cases} \frac{\left(-176160768ia^{10}d^5e^{19ic}e^{3idx} - 2642411520ia^{10}d^5e^{17ic}e^{idx} + 5284823040ia^{10}d^5e^{15ic}e^{-idx} + 1761607680ia^{10}d^5e^{13ic}e^{-3idx} + 528482304ia^{10}d^5e^{11ic}e^{-5idx} + 75497472i}{16911433728a^{12}d^6} \\ \frac{x\left(e^{10ic} + 5e^{8ic} + 10e^{6ic} + 10e^{4ic} + 5e^{2ic} + 1\right)e^{-7ic}}{32a^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*I
*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*ex
p(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 528482304*I
*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c)*ex
p(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(16911433728*a**12*d*
*6*exp(16*I*c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*ex
p(4*I*c) + 5*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

Giac [A] time = 1.11403, size = 196, normalized size = 2.31

$$\frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\right)}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{3}}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-1155i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+2870i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+2037\tan\left(\frac{1}{2}dx+\right.}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{7}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(t
an(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/2*
d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*c)^
3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^7))/d
```

3.164 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$-\frac{1}{16a^2d(-\cot(c+dx)+i)}+\frac{11}{16a^2d(\cot(c+dx)+i)}-\frac{3i}{8a^2d(\cot(c+dx)+i)^2}-\frac{1}{12a^2d(\cot(c+dx)+i)^3}+\frac{x}{4a^2}$$

```
[Out] x/(4*a^2) - 1/(16*a^2*d*(I - Cot[c + d*x])) - 1/(12*a^2*d*(I + Cot[c + d*x]
)^3) - ((3*I)/8)/(a^2*d*(I + Cot[c + d*x])^2) + 11/(16*a^2*d*(I + Cot[c + d
*x]))
```

Rubi [A] time = 0.100526, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3088, 848, 88, 203}

$$-\frac{1}{16a^2d(-\cot(c+dx)+i)}+\frac{11}{16a^2d(\cot(c+dx)+i)}-\frac{3i}{8a^2d(\cot(c+dx)+i)^2}-\frac{1}{12a^2d(\cot(c+dx)+i)^3}+\frac{x}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) - 1/(16*a^2*d*(I - Cot[c + d*x])) - 1/(12*a^2*d*(I + Cot[c + d*x]
)^3) - ((3*I)/8)/(a^2*d*(I + Cot[c + d*x])^2) + 11/(16*a^2*d*(I + Cot[c + d
*x]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(ia+ax)^2\left(1+x^2\right)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{i}{a}+\frac{x}{a}\right)^2(ia+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{16a^2(-i+x)^2}-\frac{1}{4a^2(i+x)^4}-\frac{3i}{4a^2(i+x)^3}+\frac{11}{16a^2(i+x)^2}+\frac{1}{4a^2\left(1+x^2\right)}\right)dx, x, \cot(c$$

$$= -\frac{1}{16a^2d(i-\cot(c+dx))}-\frac{1}{12a^2d(i+\cot(c+dx))^3}-\frac{3i}{8a^2d(i+\cot(c+dx))^2}+$$

$$= \frac{x}{4a^2}-\frac{1}{16a^2d(i-\cot(c+dx))}-\frac{1}{12a^2d(i+\cot(c+dx))^3}-\frac{3i}{8a^2d(i+\cot(c+dx))}$$

Mathematica [A] time = 0.127354, size = 82, normalized size = 0.81

$$\frac{21\sin(2(c+dx))+6\sin(4(c+dx))+\sin(6(c+dx))+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))+24c+2}{96a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (24*c + 24*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[6
*(c + d*x)] + 21*Sin[2*(c + d*x)] + 6*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])/
```

```
(96*a^2*d)
```

Maple [A] time = 0.134, size = 117, normalized size = 1.2

$$-\frac{-\frac{i}{8}\ln\left(\tan\left(dx+c\right)-i\right)}{da^2}-\frac{\frac{i}{8}}{da^2\left(\tan\left(dx+c\right)-i\right)^2}-\frac{1}{12\,da^2\left(\tan\left(dx+c\right)-i\right)^3}+\frac{3}{16\,da^2\left(\tan\left(dx+c\right)-i\right)}+\frac{\frac{i}{8}\ln\left(\tan\left(dx\right.\right.}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] -1/8*I/d/a^2*ln(tan(d*x+c)-I)-1/8*I/d/a^2/(tan(d*x+c)-I)^2-1/12/d/a^2/(tan(
d*x+c)-I)^3+3/16/d/a^2/(tan(d*x+c)-I)+1/8*I/d/a^2*ln(tan(d*x+c)+I)+1/16/d/a
^2/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.470143, size = 198, normalized size = 1.96

$$\frac{\left(24\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(8i\,dx+8i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d*
x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [A] time = 0.58235, size = 190, normalized size = 1.88

$$\begin{cases} \frac{\left(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx}\right)e^{-12ic}}{786432a^8d^4} & \text{for } 786432a^8d^4e^{12ic} \neq 0 \\ x\left(\frac{\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-6ic}}{16a^2} - \frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d**
3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x) +
8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**4),
 Ne(786432*a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*e
xp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) +
 x/(4*a**2)
```

Giac [A] time = 1.1448, size = 139, normalized size = 1.38

$$-\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^2} + \frac{6i\log(\tan(dx+c)-i)}{a^2} + \frac{3(2i\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)} + \frac{-11i\tan(dx+c)^3-42\tan(dx+c)^2+57i\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*(
2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 - 42
*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

3.165 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^5(c+dx)}{5a^2d}$$

```
[Out] (((2*I)/5)*Cos[c + d*x]^5)/(a^2*d) + Sin[c + d*x]/(a^2*d) - Sin[c + d*x]^3/
(a^2*d) + (2*Sin[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.175888, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 14}

$$\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/5)*Cos[c + d*x]^5)/(a^2*d) + Sin[c + d*x]/(a^2*d) - Sin[c + d*x]^3/
(a^2*d) + (2*Sin[c + d*x]^5)/(5*a^2*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}\\
&= -\frac{\int \left(-a^2\cos^5(c+dx)+2ia^2\cos^4(c+dx)\sin(c+dx)+a^2\cos^3(c+dx)\sin^2(c+\right.}{a^4}\\
&= -\frac{(2i)\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^5(c+dx)\,dx}{a^2}-\frac{\int \cos^3(c+dx)\sin^2(c}{a^2}\\
&= \frac{(2i)\,\mathrm{Subst}\left(\int x^4\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\mathrm{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\sin(c+dx)\right)}{a^2d}-\\
&= \frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{5a^2d}-\frac{\mathrm{Subst}\left(\int \left(x^2\right.\right.}{}\\
&= \frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{2\sin^5(c+dx)}{5a^2d}
\end{aligned}$$

Mathematica [A] time = 0.0854094, size = 111, normalized size = 1.63

$$\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{8a^2d}+\frac{\sin(5(c+dx))}{40a^2d}+\frac{i\cos(c+dx)}{4a^2d}+\frac{i\cos(3(c+dx))}{8a^2d}+\frac{i\cos(5(c+dx))}{40a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a^2*d) + ((I/8)*Cos[3*(c + d*x)])/(a^2*d) + ((I/40)*C
os[5*(c + d*x)])/(a^2*d) + Sin[c + d*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(8*a^2
*d) + Sin[5*(c + d*x)]/(40*a^2*d)
```

Maple [A] time = 0.127, size = 108, normalized size = 1.6

$$2\,\frac{1}{da^2}\left(\frac{-i}{(\tan(1/2\,dx+c/2)-i)^4}+\frac{5/4\,i}{(\tan(1/2\,dx+c/2)-i)^2}+2/5\,(\tan(1/2\,dx+c/2)-i)^{-5}-3/2\,(\tan(1/2\,dx+c/2)-i)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(-I/(tan(1/2*d*x+1/2*c)-I)^4+5/4*I/(tan(1/2*d*x+1/2*c)-I)^2+2/5/(ta
n(1/2*d*x+1/2*c)-I)^5-3/2/(tan(1/2*d*x+1/2*c)-I)^3+7/8/(tan(1/2*d*x+1/2*c)-
I)+1/8/(tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.473445, size = 161, normalized size = 2.37

$$\frac{\left(-5i\, e^{(6i\, dx+6i\, c)} + 15i\, e^{(4i\, dx+4i\, c)} + 5i\, e^{(2i\, dx+2i\, c)} + i\right) e^{(-5i\, dx-5i\, c)}}{40\, a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
+ 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [A] time = 0.882052, size = 165, normalized size = 2.43

$$\begin{cases} \frac{\left(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx}\right)e^{-9ic}}{20480a^8d^4} & \text{for}\ 20480a^8d^4e^{9ic} \neq 0 \\ \frac{x\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*exp
(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a**
6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(20480*a*
*8*d**4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*
exp(-5*I*c)/(8*a**2), True))
```

Giac [A] time = 1.16359, size = 126, normalized size = 1.85

$$\frac{\frac{5}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)} + \frac{35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-90i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+70i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}}{20\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90*
I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x +
1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

3.166 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)}+\frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] x/(4*a^2) + ((I/4)*Cos[c + d*x]^2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2
) + ((I/4)*Cos[c + d*x])/(d*(a^2*Cos[c + d*x] + I*a^2*Sin[c + d*x]))
```

Rubi [A] time = 0.0820806, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3082, 8}

$$\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)}+\frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) + ((I/4)*Cos[c + d*x]^2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2
) + ((I/4)*Cos[c + d*x])/(d*(a^2*Cos[c + d*x] + I*a^2*Sin[c + d*x]))
```

Rule 3082

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n)/(2*a*d*n*Cos[c + d*x]^n), x] + Dist[1/(2*a), Int[(a*Cos[c + d*
x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx}{2a}$$
$$= \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)} + $$
$$= \frac{x}{4a^2} + \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+d\right.}$$

Mathematica [A] time = 0.105191, size = 60, normalized size = 0.67

$$\frac{4\sin(2(c+dx))+\sin(4(c+dx))+4i\cos(2(c+dx))+i\cos(4(c+dx))+4c+4dx}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (4*c + 4*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 4*Sin[2*(c + d
*x)] + Sin[4*(c + d*x)])/(16*a^2*d)
```

Maple [A] time = 0.127, size = 79, normalized size = 0.9

$$\frac{-\frac{i}{8}\ln(\tan(dx+c)-i)}{da^2} - \frac{\frac{i}{4}}{da^2(\tan(dx+c)-i)^2} + \frac{1}{4\,da^2(\tan(dx+c)-i)} + \frac{\frac{i}{8}\ln(\tan(dx+c)+i)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] -1/8*I/d/a^2*ln(tan(d*x+c)-I)-1/4*I/d/a^2/(tan(d*x+c)-I)^2+1/4/d/a^2/(tan(d
*x+c)-I)+1/8*I/d/a^2*ln(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.466714, size = 126, normalized size = 1.42

$$\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
- 4*I*c)/(a^2*d)
```

Sympy [A] time = 0.407223, size = 119, normalized size = 1.34

$$\begin{cases} \frac{\left(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx}\right)e^{-6ic}}{64a^4d^2} & \text{for } 64a^4d^2e^{6ic}\neq 0 \\ x\left(\frac{\left(e^{4ic}+2e^{2ic}+1\right)e^{-4ic}}{4a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases}+\frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(64*a**4*d**2*exp(6*I*c), 0)), (
x*((exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True
)) + x/(4*a**2)
```

Giac [A] time = 1.15136, size = 97, normalized size = 1.09

$$-\frac{\frac{2i\log(i\tan(dx+c)+1)}{a^2}-\frac{2i\log(i\tan(dx+c)-1)}{a^2}+\frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*I*log(I*tan(d*x + c) + 1)/a^2 - 2*I*log(I*tan(d*x + c) - 1)/a^2 +
(-3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/
d
```

3.167 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^3(c+dx)}{3a^2d}$$

```
[Out] (((2*I)/3)*Cos[c + d*x]^3)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (2*Sin[c + d*x]
^3)/(3*a^2*d)
```

Rubi [A] time = 0.113771, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.207, Rules used = {3092, 3090, 2633, 2565, 30, 2564}

$$-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/3)*Cos[c + d*x]^3)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (2*Sin[c + d*x]
^3)/(3*a^2*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \cos(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}\\
&= -\frac{\int \left(-a^2\cos^3(c+dx)+2ia^2\cos^2(c+dx)\sin(c+dx)+a^2\cos(c+dx)\sin^2(c+\cdots\right.}{a^4}\\
&= -\frac{(2i)\int \cos^2(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^3(c+dx)\,dx}{a^2}-\frac{\int \cos(c+dx)\sin^2(\cdots}{a^2}\\
&= \frac{(2i)\,\mathrm{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\mathrm{Subst}\left(\int x^2\,dx,x,\sin(c+dx)\right)}{a^2d}-\frac{\mathrm{Subst}\cdots}{}\\
&= \frac{2i\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}
\end{aligned}
$$

Mathematica [A] time = 0.0574055, size = 73, normalized size = 1.4

$$
\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{6a^2d}+\frac{i\cos(c+dx)}{2a^2d}+\frac{i\cos(3(c+dx))}{6a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((I/2)*Cos[c + d*x])/(a^2*d) + ((I/6)*Cos[3*(c + d*x)])/(a^2*d) + Sin[c + d
*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(6*a^2*d)
```

Maple [A] time = 0.112, size = 57, normalized size = 1.1

$$2\,\frac{1}{da^2}\left(\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-1}-2/3\,\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{-3}+\frac{i}{\left(\tan\left(1/2\,dx+c/2\right)-i\right)^{2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*(1/(tan(1/2*d*x+1/2*c)-I)-2/3/(tan(1/2*d*x+1/2*c)-I)^3+I/(tan(1/2*d
*x+1/2*c)-I)^2)
```

Maxima [A] time = 1.04839, size = 61, normalized size = 1.17

$$\frac{i\,\cos\left(3\,dx+3\,c\right)+3i\,\cos\left(dx+c\right)+\sin\left(3\,dx+3\,c\right)+3\,\sin\left(dx+c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x +
 c))/(a^2*d)
```

Fricas [A] time = 0.464128, size = 86, normalized size = 1.65

$$\frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [A] time = 0.416579, size = 94, normalized size = 1.81

$$\begin{cases} \frac{\left(6ia^2de^{3ic}e^{-idx}+2ia^2de^{ic}e^{-3idx}\right)e^{-4ic}}{12a^4d^2} & \text{for } 12a^4d^2e^{4ic} \neq 0 \\ \frac{x\left(e^{2ic}+1\right)e^{-3ic}}{2a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((6*I*a**2*d*exp(3*I*c)*exp(-I*d*x) + 2*I*a**2*d*exp(I*c)*exp(-3*
I*d*x))*exp(-4*I*c)/(12*a**4*d**2), Ne(12*a**4*d**2*exp(4*I*c), 0)), (x*(ex
p(2*I*c) + 1)*exp(-3*I*c)/(2*a**2), True))
```

Giac [A] time = 1.11315, size = 63, normalized size = 1.21

$$\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\right)}{3a^2d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(1
/2*d*x + 1/2*c) - I)^3)
```

3.168 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rubi [A] time = 0.0155068, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx=\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Mathematica [A] time = 0.0432011, size = 31, normalized size = 1.

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Maple [A] time = 0.099, size = 23, normalized size = 0.7

$$\frac{i}{da^2\left(i\tan\left(dx+c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] I/d/a^2/(I*tan(d*x+c)+1)
```

Maxima [A] time = 1.01411, size = 30, normalized size = 0.97

$$\frac{1}{\left(a^2\tan\left(dx+c\right)-i\,a^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/((a^2*tan(d*x + c) - I*a^2)*d)
```

Fricas [A] time = 0.459977, size = 49, normalized size = 1.58

$$\frac{i\,e^{\left(-2i\,dx-2i\,c\right)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [A] time = 0.238383, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for } 2a^2de^{2ic} \neq 0 \\ \frac{xe^{-2ic}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(2*a**2*d*exp(2*I*c),
0)), (x*exp(-2*I*c)/a**2, True))
```

Giac [A] time = 1.14306, size = 41, normalized size = 1.32

$$-\frac{2\,\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{a^2d\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

3.169 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=46

$$\frac{2\sin(c+dx)}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Cos[c + d*x])/(a^2*d) + (2*Sin[c
+ d*x])/(a^2*d)
```

Rubi [A] time = 0.11459, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {3092, 3090, 2637, 2638, 2592, 321, 206}

$$\frac{2\sin(c+dx)}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Cos[c + d*x])/(a^2*d) + (2*Sin[c
+ d*x])/(a^2*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$
$$= -\frac{\int \left(-a^2\cos(c+dx)+2ia^2\sin(c+dx)+a^2\sin(c+dx)\tan(c+dx)\right)\,dx}{a^4}$$
$$= -\frac{(2i)\int \sin(c+dx)\,dx}{a^2}+\frac{\int \cos(c+dx)\,dx}{a^2}-\frac{\int \sin(c+dx)\tan(c+dx)\,dx}{a^2}$$
$$= \frac{2i\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{a^2d}$$
$$= \frac{2i\cos(c+dx)}{a^2d}+\frac{2\sin(c+dx)}{a^2d}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{a^2d}$$
$$= -\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}+\frac{2\sin(c+dx)}{a^2d}$$

Mathematica [B] time = 0.230578, size = 184, normalized size = 4.

$$-\frac{\sec^2(c+dx)\left(\cos\left(\frac{3}{2}(c+dx)\right)+i\sin\left(\frac{3}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*S
in[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*d*(-
I + Tan[c + d*x])^2))
```

Maple [A] time = 0.154, size = 63, normalized size = 1.4

$$4\,\frac{1}{da^2\left(\tan\left(1/2\,dx+c/2\right)-i\right)}-\frac{1}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 4/d/a^2/(tan(1/2*d*x+1/2*c)-I)-1/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)+1/d/a^2*ln(
tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.68299, size = 158, normalized size = 3.43

$$-\frac{-2i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\cos\left(dx+c\right)+\log\left(\cos\left(d\right.\right.}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*
x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

Fricas [A] time = 0.482932, size = 161, normalized size = 3.5

$$-\frac{\left(e^{(idx+ic)}\log\left(e^{(idx+ic)}+i\right)-e^{(idx+ic)}\log\left(e^{(idx+ic)}-i\right)-2i\right)e^{(-idx-ic)}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x +
 I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.17553, size = 80, normalized size = 1.74

$$-\frac{\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1
))/a^2 - 4/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

3.170 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}+\frac{2x}{a^2}$$

```
[Out] (2*x)/a^2 + ((2*I)*Log[Sin[c + d*x]])/(a^2*d) - ((2*I)*Log[Tan[c + d*x]])/(
a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rubi [A] time = 0.0706444, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 77}

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```


```
[Out] (2*x)/a^2 + ((2*I)*Log[Sin[c + d*x]])/(a^2*d) - ((2*I)*Log[Tan[c + d*x]])/(
a^2*d) - Tan[c + d*x]/(a^2*d)
```


Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{1+x^2}{x^2(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \frac{-\frac{i}{a}+\frac{x}{a}}{x^2(ia+ax)}\,dx, x, \cot(c+dx)\right)}{d}\\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2x^2}-\frac{2i}{a^2x}+\frac{2i}{a^2(i+x)}\right)dx, x, \cot(c+dx)\right)}{d}\\
&= \frac{2x}{a^2}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}
\end{aligned}$$

Mathematica [A] time = 0.413786, size = 71, normalized size = 1.29

$$\frac{4\tan^{-1}(\tan(dx))+i\sec(c)\sec(c+dx)\left(\cos(dx)\log\left(\cos^2(c+dx)\right)+\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+2i\sin(dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (4*ArcTan[Tan[d*x]] + I*Sec[c]*Sec[c + d*x]*(Cos[d*x]*Log[Cos[c + d*x]^2] +
 Cos[2*c + d*x]*Log[Cos[c + d*x]^2] + (2*I)*Sin[d*x]))/(2*a^2*d)
```

Maple [A] time = 0.161, size = 35, normalized size = 0.6

$$\frac{-2\,i\ln(\tan(dx+c)-i)}{da^2}-\frac{\tan(dx+c)}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] -2*I/d/a^2*ln(tan(d*x+c)-I)-tan(d*x+c)/a^2/d
```

Maxima [A] time = 1.11533, size = 41, normalized size = 0.75

$$\frac{-\frac{2i\log(\tan(dx+c)-i)}{a^2}-\frac{\tan(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (-2*I*log(tan(d*x + c) - I)/a^2 - tan(d*x + c)/a^2)/d
```

Fricas [A] time = 0.485937, size = 192, normalized size = 3.49

$$\frac{4\,dxe^{(2i\,dx+2i\,c)}+4\,dx+\left(2i\,e^{(2i\,dx+2i\,c)}+2i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-2i}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] (4*d*x*e^(2*I*d*x + 2*I*c) + 4*d*x + (2*I*e^(2*I*d*x + 2*I*c) + 2*I)*log(e^
(2*I*d*x + 2*I*c) + 1) - 2*I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.20462, size = 138, normalized size = 2.51

$$\frac{2\left(-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^2}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{-i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(-2*I*log(tan(1/2*d*x + 1/2*c) - I)/a^2 + I*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/a^2 + I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + (-I*tan(1/2*d*x + 1/
2*c)^2 + tan(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

3.171 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$-\frac{2i\sec(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{2a^2d}$$

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - ((2*I)*Sec[c + d*x])/(a^2*d) - (Sec[c
 + d*x]*Tan[c + d*x])/(2*a^2*d)
```

Rubi [A] time = 0.145636, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3770, 2606, 8, 2611}

$$-\frac{2i\sec(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - ((2*I)*Sec[c + d*x])/(a^2*d) - (Sec[c
 + d*x]*Tan[c + d*x])/(2*a^2*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}\\ &= -\frac{\int \left(-a^2\sec(c+dx)+2ia^2\sec(c+dx)\tan(c+dx)+a^2\sec(c+dx)\tan^2(c+\right.}{a^4}\\ &= -\frac{(2i)\int \sec(c+dx)\tan(c+dx)\,dx}{a^2}+\frac{\int \sec(c+dx)\,dx}{a^2}-\frac{\int \sec(c+dx)\tan^2(c}{a^2}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{2a^2d}+\frac{\int \sec(c+dx)\,dx}{2a^2}-\frac{(2i)\,\mathrm{Sub}}{}\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{2a^2d}\end{aligned}$$

Mathematica [B] time = 0.394743, size = 146, normalized size = 2.61

$$-\frac{\sec^2(c+dx)\left(2\sin(c+dx)+8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}\right.\right.\right.\right.}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 2*Sin[c + d*x]))/(4*a^2*d)
```

Maple [B] time = 0.184, size = 170, normalized size = 3.

$$-\frac{1}{2da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{2i}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{1}{2da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{3}{2da^2}\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] -1/2/d/a^2/(tan(1/2*d*x+1/2*c)+1)-2*I/d/a^2/(tan(1/2*d*x+1/2*c)+1)+1/2/d/a^
2/(tan(1/2*d*x+1/2*c)+1)^2+3/2/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(ta
n(1/2*d*x+1/2*c)-1)+2*I/d/a^2/(tan(1/2*d*x+1/2*c)-1)-1/2/d/a^2/(tan(1/2*d*x
+1/2*c)-1)^2-3/2/d/a^2*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.05135, size = 225, normalized size = 4.02

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+4i\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^2)/d
```

Fricas [B] time = 0.480987, size = 375, normalized size = 6.7

$$\frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(3i\,dx+3i\,c)}-10i}{2\left(a^2de^{(4i\,dx+4i\,c)}+2\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c
) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
 I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4*I*
d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.19903, size = 131, normalized size = 2.34

$$\frac{\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*log(abs(tan(1/2*d*x + 1/2
*c) - 1))/a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

3.172 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{i\tan^3(c+dx)(-\cot(c+dx)+i)^3}{3a^2d}$$

```
[Out] ((-I/3)*(I - Cot[c + d*x])^3*Tan[c + d*x]^3)/(a^2*d)
```

Rubi [A] time = 0.0636908, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 37}

$$-\frac{i\tan^3(c+dx)(-\cot(c+dx)+i)^3}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-I/3)*(I - Cot[c + d*x])^3*Tan[c + d*x]^3)/(a^2*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^2}{x^4}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{i(i-\cot(c+dx))^3\tan^3(c+dx)}{3a^2d}$$

Mathematica [A] time = 0.251387, size = 68, normalized size = 2.

$$-\frac{\sec(c)\sec^3(c+dx)(3\sin(2c+dx)-2\sin(2c+3dx)+3i\cos(2c+dx)-3\sin(dx)+3i\cos(dx))}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c]*Sec[c + d*x]^3*((3*I)*Cos[d*x] + (3*I)*Cos[2*c + d*x] - 3*Sin[d*x]
 + 3*Sin[2*c + d*x] - 2*Sin[2*c + 3*d*x]))/(6*a^2*d)
```

Maple [A] time = 0.172, size = 36, normalized size = 1.1

$$\frac{1}{da^2}\left(\tan(dx+c)-\frac{(\tan(dx+c))^3}{3}-i(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/3*tan(d*x+c)^3-I*tan(d*x+c)^2)
```

Maxima [A] time = 1.00289, size = 47, normalized size = 1.38

$$-\frac{\tan(dx+c)^3 + 3i\,\tan(dx+c)^2 - 3\,\tan(dx+c)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Fricas [B] time = 0.447857, size = 139, normalized size = 4.09

$$\frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)} + 3\,a^2de^{(4i\,dx+4i\,c)} + 3\,a^2de^{(2i\,dx+2i\,c)} + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.10297, size = 47, normalized size = 1.38

$$-\frac{\tan\left(dx+c\right)^{3}+3i\,\tan\left(dx+c\right)^{2}-3\,\tan\left(dx+c\right)}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

3.173 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{4a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*
a^2*d)
```

Rubi [A] time = 0.192213, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 3768, 3770, 2606, 30, 2611}

$$-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{4a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*
a^2*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}$$
$$= -\frac{\int \left(-a^2\sec^3(c+dx)+2ia^2\sec^3(c+dx)\tan(c+dx)+a^2\sec^3(c+dx)\tan^2(c\right.}{a^4}$$
$$= -\frac{(2i)\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2}+\frac{\int \sec^3(c+dx)\,dx}{a^2}-\frac{\int \sec^3(c+dx)\tan}{a^2}$$
$$= \frac{\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{\sec^3(c+dx)\tan(c+dx)}{4a^2d}+\frac{\int \sec^3(c+dx)\,dx}{4a^2}+\frac{\int \mathrm{s}}{}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}-\frac{\sec^3(c+}{}$$
$$= \frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}-\frac{\sec^3(c}{}$$

Mathematica [B] time = 0.94825, size = 215, normalized size = 2.56

$$-\frac{\sec^4(c+dx)\left(18\sin(c+dx)-30\sin(3(c+dx))+128i\cos(c+dx)+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\,\mathrm{c}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
 - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x] - 30*Sin[3
*(c + d*x)]))/(192*a^2*d)
```

Maple [B] time = 0.197, size = 342, normalized size = 4.1

$$\frac{3}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{\frac{2\,i}{3}}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{1}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{i}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 3/8/d/a^2/(tan(1/2*d*x+1/2*c)+1)-2/3*I/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3-1/8/d
/a^2/(tan(1/2*d*x+1/2*c)+1)^2+I/d/a^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2/d/a^2/(t
an(1/2*d*x+1/2*c)+1)^3+I/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+1/4/d/a^2/(tan(1/2*
d*x+1/2*c)+1)^4+5/8/d/a^2*ln(tan(1/2*d*x+1/2*c)+1)+3/8/d/a^2/(tan(1/2*d*x+1
/2*c)-1)+2/3*I/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3+1/8/d/a^2/(tan(1/2*d*x+1/2*c)
-1)^2-I/d/a^2/(tan(1/2*d*x+1/2*c)+1)-1/2/d/a^2/(tan(1/2*d*x+1/2*c)-1)^3+I/d
/a^2/(tan(1/2*d*x+1/2*c)-1)-1/4/d/a^2/(tan(1/2*d*x+1/2*c)-1)^4-5/8/d/a^2*ln
(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.09991, size = 398, normalized size = 4.74

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{33\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{48i\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{33\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{48i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{9\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-16i\right)}{a^2-\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}-\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 16*I
)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [B] time = 0.487309, size = 667, normalized size = 7.94

$$\frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}\cdots\right.}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*
c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x +
 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*
I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e^(5*
I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^2*d*e
^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x + 4*I
*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.20561, size = 207, normalized size = 2.46

$$\frac{\frac{15\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{15\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-33\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-33\tan\cdots\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4a^2}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 15*log(abs(tan(1/2*d*x +
1/2*c) - 1))/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^
6 - 33*tan(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*
x + 1/2*c)^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)
/((tan(1/2*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

3.174 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$-\frac{\tan^5(c+dx)}{5a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{i\tan^2(c+dx)}{a^2d}+\frac{\tan(c+dx)}{a^2d}$$

```
[Out] Tan[c + d*x]/(a^2*d) - (I*Tan[c + d*x]^2)/(a^2*d) - ((I/2)*Tan[c + d*x]^4)/
(a^2*d) - Tan[c + d*x]^5/(5*a^2*d)
```

Rubi [A] time = 0.0784553, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 75}

$$-\frac{\tan^5(c+dx)}{5a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{i\tan^2(c+dx)}{a^2d}+\frac{\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] Tan[c + d*x]/(a^2*d) - (I*Tan[c + d*x]^2)/(a^2*d) - ((I/2)*Tan[c + d*x]^4)/
(a^2*d) - Tan[c + d*x]^5/(5*a^2*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^3(ia+ax)}{x^6}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2x^6}-\frac{2i}{a^2x^5}-\frac{2i}{a^2x^3}+\frac{1}{a^2x^2}\right)dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{a^2d}-\frac{i\tan^2(c+dx)}{a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{\tan^5(c+dx)}{5a^2d}\end{aligned}$$

Mathematica [A] time = 0.407311, size = 77, normalized size = 1.1

$$\frac{\sec(c)\sec^5(c+dx)(-5\sin(2c+dx)+5\sin(2c+3dx)+\sin(4c+5dx)-5i\cos(2c+dx)+5\sin(dx)-5i\cos(dx))}{20a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c]*Sec[c + d*x]^5*((-5*I)*Cos[d*x] - (5*I)*Cos[2*c + d*x] + 5*Sin[d*x]
 - 5*Sin[2*c + d*x] + 5*Sin[2*c + 3*d*x] + Sin[4*c + 5*d*x]))/(20*a^2*d)
```

Maple [A] time = 0.175, size = 47, normalized size = 0.7

$$\frac{1}{da^2}\left(\tan(dx+c)-\frac{(\tan(dx+c))^5}{5}-\frac{i}{2}(\tan(dx+c))^4-i(\tan(dx+c))^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(tan(d*x+c)-1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4-I*tan(d*x+c)^2)
```

Maxima [A] time = 0.998652, size = 63, normalized size = 0.9

$$-\frac{6\,\tan\left(dx+c\right)^{5}+15i\,\tan\left(dx+c\right)^{4}+30i\,\tan\left(dx+c\right)^{2}-30\,\tan\left(dx+c\right)}{30\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/30*(6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 + 30*I*tan(d*x + c)^2 - 30*ta
n(d*x + c))/(a^2*d)
```

Fricas [A] time = 0.456679, size = 267, normalized size = 3.81

$$\frac{40i\,e^{(2i\,dx+2i\,c)}+8i}{5\left(a^{2}de^{(10i\,dx+10i\,c)}+5\,a^{2}de^{(8i\,dx+8i\,c)}+10\,a^{2}de^{(6i\,dx+6i\,c)}+10\,a^{2}de^{(4i\,dx+4i\,c)}+5\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/5*(40*I*e^(2*I*d*x + 2*I*c) + 8*I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d
*e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x +
 4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.13239, size = 63, normalized size = 0.9

$$-\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

3.175 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=125

$$-\frac{1}{32a^3d(-\cot(c+dx)+i)}+\frac{13}{16a^3d(\cot(c+dx)+i)}-\frac{23i}{32a^3d(\cot(c+dx)+i)^2}-\frac{1}{3a^3d(\cot(c+dx)+i)^3}+\frac{i}{16a^3d(\cot(c+$$

```
[Out] (5*x)/(32*a^3) - 1/(32*a^3*d*(I - Cot[c + d*x])) + (I/16)/(a^3*d*(I + Cot[c
 + d*x])^4) - 1/(3*a^3*d*(I + Cot[c + d*x])^3) - ((23*I)/32)/(a^3*d*(I + Co
t[c + d*x])^2) + 13/(16*a^3*d*(I + Cot[c + d*x]))
```

Rubi [A] time = 0.111441, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {3088, 848, 88, 203}

$$-\frac{1}{32a^3d(-\cot(c+dx)+i)}+\frac{13}{16a^3d(\cot(c+dx)+i)}-\frac{23i}{32a^3d(\cot(c+dx)+i)^2}-\frac{1}{3a^3d(\cot(c+dx)+i)^3}+\frac{i}{16a^3d(\cot(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(32*a^3) - 1/(32*a^3*d*(I - Cot[c + d*x])) + (I/16)/(a^3*d*(I + Cot[c
 + d*x])^4) - 1/(3*a^3*d*(I + Cot[c + d*x])^3) - ((23*I)/32)/(a^3*d*(I + Co
t[c + d*x])^2) + 13/(16*a^3*d*(I + Cot[c + d*x]))
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{x^5}{(ia+ax)^3\left(1+x^2\right)^2}\,dx,x,\cot(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{x^5}{\left(-\frac{i}{a}+\frac{x}{a}\right)^2(ia+ax)^5}\,dx,x,\cot(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{32a^3(-i+x)^2}+\frac{i}{4a^3(i+x)^5}-\frac{1}{a^3(i+x)^4}-\frac{23i}{16a^3(i+x)^3}+\frac{13}{16a^3(i+x)^2}+\frac{5}{32a^3\left(1+x^2\right)}\right.\right.}{d}$$

$$= -\frac{1}{32a^3d(i-\cot(c+dx))}+\frac{i}{16a^3d(i+\cot(c+dx))^4}-\frac{1}{3a^3d(i+\cot(c+dx))^3}-$$

$$= \frac{5x}{32a^3}-\frac{1}{32a^3d(i-\cot(c+dx))}+\frac{i}{16a^3d(i+\cot(c+dx))^4}-\frac{1}{3a^3d(i+\cot(c+a}$$

Mathematica [A] time = 0.189515, size = 106, normalized size = 0.85

$$\frac{132\sin(2(c+dx))+60\sin(4(c+dx))+20\sin(6(c+dx))+3\sin(8(c+dx))+108i\cos(2(c+dx))+60i\cos(4(c+dx))}{768a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (120*c + 120*d*x + (108*I)*Cos[2*(c + d*x)] + (60*I)*Cos[4*(c + d*x)] + (20
*I)*Cos[6*(c + d*x)] + (3*I)*Cos[8*(c + d*x)] + 132*Sin[2*(c + d*x)] + 60*S
```

```
in[4*(c + d*x)] + 20*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)])/(768*a^3*d)
```

Maple [A] time = 0.202, size = 137, normalized size = 1.1

$$\frac{-\frac{5i}{64}\ln\left(\tan\left(dx+c\right)-i\right)}{da^{3}}+\frac{\frac{i}{16}}{da^{3}\left(\tan\left(dx+c\right)-i\right)^{4}}-\frac{\frac{3i}{32}}{da^{3}\left(\tan\left(dx+c\right)-i\right)^{2}}-\frac{1}{12\,da^{3}\left(\tan\left(dx+c\right)-i\right)^{3}}+\frac{1}{8\,da^{3}\left(\tan\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] -5/64*I/d/a^3*ln(tan(d*x+c)-I)+1/16*I/d/a^3/(tan(d*x+c)-I)^4-3/32*I/d/a^3/(
tan(d*x+c)-I)^2-1/12/d/a^3/(tan(d*x+c)-I)^3+1/8/d/a^3/(tan(d*x+c)-I)+5/64*I
/d/a^3*ln(tan(d*x+c)+I)+1/32/d/a^3/(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.472251, size = 247, normalized size = 1.98

$$\frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^(
6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [A] time = 0.92502, size = 226, normalized size = 1.81

$$\begin{cases} \frac{\left(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d^4e^{12ic}e^{-8idx}\right)e^{-20ic}}{6442450944a^{15}d^5} & \text{fo} \\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) & \text{o} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*I
*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c)*
exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165824
*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15*d**
5), Ne(6442450944*a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I
*c) + 10*exp(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**
3) - 5/(32*a**3)), True)) + 5*x/(32*a**3)
```

Giac [A] time = 1.15703, size = 161, normalized size = 1.29

$$-\frac{-\frac{60i\log(-i\tan(dx+c)+1)}{a^3}+\frac{60i\log(-i\tan(dx+c)-1)}{a^3}-\frac{12(5\tan(dx+c)+7i)}{a^3(i\tan(dx+c)-1)}+\frac{-125i\tan(dx+c)^4-596\tan(dx+c)^3+1110i\tan(dx+c)^2+996\tan(dx+c}{a^3(\tan(dx+c)-i)^4}}{768d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(-60*I*log(-I*tan(d*x + c) + 1)/a^3 + 60*I*log(-I*tan(d*x + c) - 1)/
a^3 - 12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*
x + c)^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) -
405*I)/(a^3*(tan(d*x + c) - I)^4))/d
```

3.176 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=106

$$-\frac{4\sin^7(c+dx)}{7a^3d}+\frac{9\sin^5(c+dx)}{5a^3d}-\frac{2\sin^3(c+dx)}{a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^7(c+dx)}{7a^3d}-\frac{i\cos^5(c+dx)}{5a^3d}$$

```
[Out] ((-I/5)*Cos[c + d*x]^5)/(a^3*d) + (((4*I)/7)*Cos[c + d*x]^7)/(a^3*d) + Sin[
c + d*x]/(a^3*d) - (2*Sin[c + d*x]^3)/(a^3*d) + (9*Sin[c + d*x]^5)/(5*a^3*d
) - (4*Sin[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.234448, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 270, 14}

$$-\frac{4\sin^7(c+dx)}{7a^3d}+\frac{9\sin^5(c+dx)}{5a^3d}-\frac{2\sin^3(c+dx)}{a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^7(c+dx)}{7a^3d}-\frac{i\cos^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/5)*Cos[c + d*x]^5)/(a^3*d) + (((4*I)/7)*Cos[c + d*x]^7)/(a^3*d) + Sin[
c + d*x]/(a^3*d) - (2*Sin[c + d*x]^3)/(a^3*d) + (9*Sin[c + d*x]^5)/(5*a^3*d
) - (4*Sin[c + d*x]^7)/(7*a^3*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\cos^7(c+dx)-3a^3\cos^6(c+dx)\sin(c+dx)+3ia^3\cos^5(c+dx)\sin^2(c\right.}{a^6}$$
$$= \frac{i\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \cos^6(c+dx)\sin(c+dx)\,dx}{a^3} + \frac{\int \cos^7(c}{a}$$
$$= -\frac{i\,\mathrm{Subst}\left(\int x^4\left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{a^3d} + \frac{(3i)\,\mathrm{Subst}\left(\int x^6\,dx,x,\cos(c+dx)\right)}{a^3d}$$
$$= \frac{3i\cos^7(c+dx)}{7a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{\sin^3(c+dx)}{a^3d} + \frac{3\sin^5(c+dx)}{5a^3d} - \frac{\sin^7(c+dx)}{7a^3d} -$$
$$= -\frac{i\cos^5(c+dx)}{5a^3d} + \frac{4i\cos^7(c+dx)}{7a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{2\sin^3(c+dx)}{a^3d} + \frac{9\sin^5(c+a}{5a^3d}$$

Mathematica [A] time = 0.0888874, size = 149, normalized size = 1.41

$$\frac{5\sin(c+dx)}{16a^3d} + \frac{\sin(3(c+dx))}{8a^3d} + \frac{\sin(5(c+dx))}{20a^3d} + \frac{\sin(7(c+dx))}{112a^3d} + \frac{3i\cos(c+dx)}{16a^3d} + \frac{i\cos(3(c+dx))}{8a^3d} + \frac{i\cos(5(c+dx))}{20a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (((3*I)/16)*Cos[c + d*x])/(a^3*d) + ((I/8)*Cos[3*(c + d*x)])/(a^3*d) + ((I/
20)*Cos[5*(c + d*x)])/(a^3*d) + ((I/112)*Cos[7*(c + d*x)])/(a^3*d) + (5*Sin
[c + d*x])/(16*a^3*d) + Sin[3*(c + d*x)]/(8*a^3*d) + Sin[5*(c + d*x)]/(20*a
^3*d) + Sin[7*(c + d*x)]/(112*a^3*d)
```

Maple [A] time = 0.149, size = 141, normalized size = 1.3

$$2\frac{1}{da^3}\left(\frac{2i}{(\tan(1/2\,dx+c/2)-i)^6} - \frac{9/2\,i}{(\tan(1/2\,dx+c/2)-i)^4} + \frac{\frac{17\,i}{8}}{(\tan(1/2\,dx+c/2)-i)^2} - 4/7\,(\tan(1/2\,dx+c/2)-i)^{-7} + \frac{}{5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(2*I/(tan(1/2*d*x+1/2*c)-I)^6-9/2*I/(tan(1/2*d*x+1/2*c)-I)^4+17/8*I
/(tan(1/2*d*x+1/2*c)-I)^2-4/7/(tan(1/2*d*x+1/2*c)-I)^7+19/5/(tan(1/2*d*x+1/
2*c)-I)^5-15/4/(tan(1/2*d*x+1/2*c)-I)^3+15/16/(tan(1/2*d*x+1/2*c)-I)+1/16/(
tan(1/2*d*x+1/2*c)+I))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.470762, size = 205, normalized size = 1.93

$$\frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*
d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [A] time = 1.1694, size = 199, normalized size = 1.88

$$\begin{cases} \frac{\left(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx}\right)e^{-16ic}}{1146880a^{15}d^5} & \text{for } 1146880a^{15}d^5e^{16 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-7ic}}{16a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d**
4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x) +
 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9*I*
c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(1146880*a**15*d**5*
exp(16*I*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*
c) + 1)*exp(-7*I*c)/(16*a**3), True))
```

Giac [A] time = 1.19541, size = 161, normalized size = 1.52

$$\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)}+\frac{525\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1960i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-4025\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+4480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3143\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1176i\tan\left(\frac{1}{2}dx+\right.}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(1/
2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/2*c
) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

3.177 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{i\cos(c+dx)}{8d\left(a^3\cos(c+dx)+ia^3\sin(c+dx)\right)}+\frac{x}{8a^3}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}+\frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] x/(8*a^3) + ((I/6)*Cos[c + d*x]^3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3
) + ((I/8)*Cos[c + d*x]^2)/(a*d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2) + ((
I/8)*Cos[c + d*x])/(d*(a^3*Cos[c + d*x] + I*a^3*Sin[c + d*x]))
```

Rubi [A] time = 0.140537, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3082, 8}

$$\frac{i\cos(c+dx)}{8d\left(a^3\cos(c+dx)+ia^3\sin(c+dx)\right)}+\frac{x}{8a^3}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}+\frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] x/(8*a^3) + ((I/6)*Cos[c + d*x]^3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3
) + ((I/8)*Cos[c + d*x]^2)/(a*d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2) + ((
I/8)*Cos[c + d*x])/(d*(a^3*Cos[c + d*x] + I*a^3*Sin[c + d*x]))
```

Rule 3082

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n)/(2*a*d*n*Cos[c + d*x]^n), x] + Dist[1/(2*a), Int[(a*Cos[c + d*
x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b, c
, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx}{2a}$$

$$= \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{\int}{}$$

$$= \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))^2} + \frac{}{8d}$$

$$= \frac{x}{8a^3} + \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))}$$

Mathematica [A] time = 0.117344, size = 84, normalized size = 0.64

$$\frac{18\sin(2(c+dx))+9\sin(4(c+dx))+2\sin(6(c+dx))+18i\cos(2(c+dx))+9i\cos(4(c+dx))+2i\cos(6(c+dx))+12c}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (12*c + 12*d*x + (18*I)*Cos[2*(c + d*x)] + (9*I)*Cos[4*(c + d*x)] + (2*I)*C
os[6*(c + d*x)] + 18*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + 2*Sin[6*(c + d
*x)])/(96*a^3*d)
```

Maple [A] time = 0.142, size = 98, normalized size = 0.8

$$\frac{-\frac{i}{16}\ln(\tan(dx+c)-i)}{da^3} - \frac{\frac{i}{8}}{da^3(\tan(dx+c)-i)^2} - \frac{1}{6\,da^3(\tan(dx+c)-i)^3} + \frac{1}{8\,da^3(\tan(dx+c)-i)} + \frac{\frac{i}{16}\ln(\tan(dx+}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] -1/16*I/d/a^3*ln(tan(d*x+c)-I)-1/8*I/d/a^3/(tan(d*x+c)-I)^2-1/6/d/a^3/(tan(
d*x+c)-I)^3+1/8/d/a^3/(tan(d*x+c)-I)+1/16*I/d/a^3*ln(tan(d*x+c)+I)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError
```

Fricas [A] time = 0.463655, size = 166, normalized size = 1.27

$$\frac{\left(12\,dxe^{(6i\,dx+6i\,c)} + 18i\,e^{(4i\,dx+4i\,c)} + 9i\,e^{(2i\,dx+2i\,c)} + 2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A] time = 0.639258, size = 156, normalized size = 1.19

$$\begin{cases} \frac{\left(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } 24576a^9d^3e^{12ic} \neq 0 \\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-6ic}}{8a^3} - \frac{1}{8a^3}\right) & \text{otherwise} \end{cases} + \frac{x}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*e
xp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12
*I*c)/(24576*a**9*d**3), Ne(24576*a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*
c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), T
```

```
rue)) + x/(8*a**3)
```

Giac [A] time = 1.15852, size = 108, normalized size = 0.82

$$-\frac{\frac{6i\log(\tan(dx+c)-i)}{a^3}-\frac{6i\log(i\tan(dx+c)-1)}{a^3}+\frac{-11i\tan(dx+c)^3-45\tan(dx+c)^2+69i\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*I*log(tan(d*x + c) - I)/a^3 - 6*I*log(I*tan(d*x + c) - 1)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(tan
(d*x + c) - I)^3))/d
```

3.178 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=90

$$\frac{4\sin^5(c+dx)}{5a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}-\frac{i\cos^3(c+dx)}{3a^3d}$$

```
[Out] ((-I/3)*Cos[c + d*x]^3)/(a^3*d) + (((4*I)/5)*Cos[c + d*x]^5)/(a^3*d) + Sin[
c + d*x]/(a^3*d) - (5*Sin[c + d*x]^3)/(3*a^3*d) + (4*Sin[c + d*x]^5)/(5*a^3
*d)
```

Rubi [A] time = 0.220501, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {3092, 3090, 2633, 2565, 30, 2564, 14}

$$\frac{4\sin^5(c+dx)}{5a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}-\frac{i\cos^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/3)*Cos[c + d*x]^3)/(a^3*d) + (((4*I)/5)*Cos[c + d*x]^5)/(a^3*d) + Sin[
c + d*x]/(a^3*d) - (5*Sin[c + d*x]^3)/(3*a^3*d) + (4*Sin[c + d*x]^5)/(5*a^3
*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\cos^5(c+dx)-3a^3\cos^4(c+dx)\sin(c+dx)+3ia^3\cos^3(c+dx)\sin^2(\right.}{a^6}$$
$$= \frac{i\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^3} + \frac{\int \cos^5}{}$$
$$= -\frac{i\,\mathrm{Subst}\left(\int x^2\left(1-x^2\right)dx, x, \cos(c+dx)\right)}{a^3d} + \frac{(3i)\,\mathrm{Subst}\left(\int x^4\,dx, x, \cos(c+d\right.}{a^3d}$$
$$= \frac{3i\cos^5(c+dx)}{5a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{2\sin^3(c+dx)}{3a^3d} + \frac{\sin^5(c+dx)}{5a^3d} - \frac{i\,\mathrm{Subst}\left(\int\left(\right.\right.}{}$$
$$= -\frac{i\cos^3(c+dx)}{3a^3d} + \frac{4i\cos^5(c+dx)}{5a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{5\sin^3(c+dx)}{3a^3d} + \frac{4\sin^5(c+}{5a^3d}$$

Mathematica [A] time = 0.0787693, size = 111, normalized size = 1.23

$$\frac{\sin(c+dx)}{4a^3d} + \frac{\sin(3(c+dx))}{6a^3d} + \frac{\sin(5(c+dx))}{20a^3d} + \frac{i\cos(c+dx)}{4a^3d} + \frac{i\cos(3(c+dx))}{6a^3d} + \frac{i\cos(5(c+dx))}{20a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a^3*d) + ((I/6)*Cos[3*(c + d*x)])/(a^3*d) + ((I/20)*C
os[5*(c + d*x)])/(a^3*d) + Sin[c + d*x]/(4*a^3*d) + Sin[3*(c + d*x)]/(6*a^3
*d) + Sin[5*(c + d*x)]/(20*a^3*d)
```

Maple [A] time = 0.137, size = 90, normalized size = 1.

$$2\,\frac{1}{da^3}\left(\frac{-2\,i}{(\tan(1/2\,dx+c/2)-i)^4} + (\tan(1/2\,dx+c/2)-i)^{-1} + 4/5\,(\tan(1/2\,dx+c/2)-i)^{-5} + \frac{2\,i}{(\tan(1/2\,dx+c/2)-i)^2} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(-2*I/(tan(1/2*d*x+1/2*c)-I)^4+1/(tan(1/2*d*x+1/2*c)-I)+4/5/(tan(1/
2*d*x+1/2*c)-I)^5+2*I/(tan(1/2*d*x+1/2*c)-I)^2-8/3/(tan(1/2*d*x+1/2*c)-I)^3
```

)

Maxima [A] time = 1.13629, size = 93, normalized size = 1.03

$$\frac{3i\cos(5\,dx+5\,c)+10i\cos(3\,dx+3\,c)+15i\cos(dx+c)+3\sin(5\,dx+5\,c)+10\sin(3\,dx+3\,c)+15\sin(dx+c)}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3*
sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

Fricas [A] time = 0.461456, size = 128, normalized size = 1.42

$$\frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

Sympy [A] time = 0.64325, size = 133, normalized size = 1.48

$$\begin{cases} \frac{\left(120ia^6d^2e^{8ic}e^{-idx}+80ia^6d^2e^{6ic}e^{-3idx}+24ia^6d^2e^{4ic}e^{-5idx}\right)e^{-9ic}}{480a^9d^3} & \text{for } 480a^9d^3e^{9ic}\neq 0 \\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-5ic}}{4a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((120*I*a**6*d**2*exp(8*I*c)*exp(-I*d*x) + 80*I*a**6*d**2*exp(6*I
*c)*exp(-3*I*d*x) + 24*I*a**6*d**2*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(4
80*a**9*d**3), Ne(480*a**9*d**3*exp(9*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I
*c) + 1)*exp(-5*I*c)/(4*a**3), True))
```

Giac [A] time = 1.19289, size = 99, normalized size = 1.1

$$\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+20i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\right)}{15\,a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*
d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2*c
) - I)^5)
```

3.179 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=32

$$\frac{i\cot^2(c+dx)}{2a^3d(\cot(c+dx)+i)^2}$$

```
[Out] ((I/2)*Cot[c + d*x]^2)/(a^3*d*(I + Cot[c + d*x])^2)
```

Rubi [A] time = 0.0318248, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.069, Rules used = {3088, 37}

$$\frac{i\cot^2(c+dx)}{2a^3d(\cot(c+dx)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/2)*Cot[c + d*x]^2)/(a^3*d*(I + Cot[c + d*x])^2)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x}{(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{i\cot^2(c+dx)}{2a^3d(i+\cot(c+dx))^2}$$

Mathematica [B] time = 0.0593853, size = 77, normalized size = 2.41

$$\frac{\sin(2(c+dx))}{4a^3d} + \frac{\sin(4(c+dx))}{8a^3d} + \frac{i\cos(2(c+dx))}{4a^3d} + \frac{i\cos(4(c+dx))}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*Cos[2*(c + d*x)])/(a^3*d) + ((I/8)*Cos[4*(c + d*x)])/(a^3*d) + Sin[2
*(c + d*x)]/(4*a^3*d) + Sin[4*(c + d*x)]/(8*a^3*d)
```

Maple [A] time = 0.115, size = 23, normalized size = 0.7

$$\frac{\frac{i}{2}}{da^3\,(i\tan(dx+c)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/2*I/d/a^3/(I*tan(d*x+c)+1)^2
```

Maxima [A] time = 1.15423, size = 69, normalized size = 2.16

$$\frac{i\,\cos(4\,dx+4\,c) + 2i\,\cos(2\,dx+2\,c) + \sin(4\,dx+4\,c) + 2\,\sin(2\,dx+2\,c)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) + sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))/(a^3*d)
```

Fricas [A] time = 0.462933, size = 86, normalized size = 2.69

$$\frac{\left(2i\,e^{(2i\,dx+2i\,c)} + i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [A] time = 0.419022, size = 97, normalized size = 3.03

$$\begin{cases} \frac{\left(8ia^3de^{4ic}e^{-2idx}+4ia^3de^{2ic}e^{-4idx}\right)e^{-6ic}}{32a^6d^2} & \text{for } 32a^6d^2e^{6ic} \neq 0 \\ \frac{x\left(e^{2ic}+1\right)e^{-4ic}}{2a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((8*I*a**3*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**3*d*exp(2*I*c)*exp
(-4*I*d*x))*exp(-6*I*c)/(32*a**6*d**2), Ne(32*a**6*d**2*exp(6*I*c), 0)), (x
*(exp(2*I*c) + 1)*exp(-4*I*c)/(2*a**3), True))
```

Giac [B] time = 1.12068, size = 77, normalized size = 2.41

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - i\,\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{a^3d\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c
))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

3.180 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=31

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rubi [A] time = 0.0164085, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {3071}

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rule 3071

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(a*(a*Cos[c + d*x] + b*Sin[c + d*x])^n)/(b*d*n), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx=\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Mathematica [A] time = 0.0404673, size = 31, normalized size = 1.

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Maple [B] time = 0.128, size = 57, normalized size = 1.8

$$2\,{\frac {1}{d{a}^{3}}}\left(\left(\tan \left(1/2\,dx+c/2 \right) -i \right) ^{-1}+{\frac {2\,i}{ \left(\tan \left(1/2\,dx+c/2 \right) -i \right) ^{2}}}-4/3\, \left(\tan \left(1/2\,dx+c/2 \right) -i \right) ^{-3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 2/d/a^3*(1/(tan(1/2*d*x+1/2*c)-I)+2*I/(tan(1/2*d*x+1/2*c)-I)^2-4/3/(tan(1/2
*d*x+1/2*c)-I)^3)
```

Maxima [A] time = 1.01792, size = 39, normalized size = 1.26

$$\frac{i\,\cos\left(3\,dx + 3\,c\right) + \sin\left(3\,dx + 3\,c\right)}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Fricas [A] time = 0.457797, size = 49, normalized size = 1.58

$$\frac{i\,e^{\left(-3i\,dx - 3i\,c\right)}}{3\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [A] time = 0.19882, size = 46, normalized size = 1.48

$$\begin{cases} \frac{ie^{-3ic}e^{-3idx}}{3a^3d} & \text{for } 3a^3de^{3ic} \neq 0 \\ \frac{xe^{-3ic}}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(3*a**3*d*exp(3*I*c),
0)), (x*exp(-3*I*c)/a**3, True))
```

Giac [A] time = 1.10577, size = 49, normalized size = 1.58

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - 1\right)}{3\,a^3d\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - i\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

3.181 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=61

$$\frac{2}{a^3d(\cot(c+dx)+i)}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}-\frac{x}{a^3}$$

```
[Out] -(x/a^3) + 2/(a^3*d*(I + Cot[c + d*x])) - (I*Log[Sin[c + d*x]])/(a^3*d) + (
I*Log[Tan[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.0643177, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3088, 848, 77}

$$\frac{2}{a^3d(\cot(c+dx)+i)}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + 2/(a^3*d*(I + Cot[c + d*x])) - (I*Log[Sin[c + d*x]])/(a^3*d) + (
I*Log[Tan[c + d*x]])/(a^3*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{1+x^2}{x(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{-\frac{i}{a}+\frac{x}{a}}{x(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{i}{a^3x}+\frac{2}{a^3(i+x)^2}-\frac{i}{a^3(i+x)}\right)dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{x}{a^3}+\frac{2}{a^3d(i+\cot(c+dx))}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}\end{aligned}$$

Mathematica [A] time = 0.261845, size = 91, normalized size = 1.49

$$\frac{i\sec^2(c+dx)(\sin(2(c+dx))-i\cos(2(c+dx)))(\log(\cos(c+dx))+\tan(c+dx)(i\log(\cos(c+dx))+dx+i)-idx-1)}{a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (I*Sec[c + d*x]^2*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-1 - I*d*x +
Log[Cos[c + d*x]] + (I + d*x + I*Log[Cos[c + d*x]])*Tan[c + d*x]))/(a^3*d*(
-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.185, size = 40, normalized size = 0.7

$$\frac{i\ln(\tan(dx+c)-i)}{da^3}+2\,\frac{1}{da^3(\tan(dx+c)-i)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] I/d/a^3*ln(tan(d*x+c)-I)+2/d/a^3/(tan(d*x+c)-I)
```

Maxima [A] time = 1.61716, size = 134, normalized size = 2.2

$$-\frac{4\,dx + 4\,c - 2\,\arctan\left(\sin\left(2\,dx + 2\,c\right), \cos\left(2\,dx + 2\,c\right) + 1\right) - 2i\,\cos\left(2\,dx + 2\,c\right) + i\,\log\left(\cos\left(2\,dx + 2\,c\right)^2 + \sin\left(2\,dx\right.\right.}{2\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*d*x + 4*c - 2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) - 2*I
*cos(2*d*x + 2*c) + I*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - 2*sin(2*d*x + 2*c))/(a^3*d)
```

Fricas [A] time = 0.480306, size = 157, normalized size = 2.57

$$-\frac{\left(2\,dxe^{(2i\,dx+2i\,c)} + i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)} + 1\right) - i\right)e^{(-2i\,dx-2i\,c)}}{a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.14531, size = 138, normalized size = 2.26

$$-\frac{-\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3i}{a^3\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(-2*I*log(tan(1/2*d*x + 1/2*c) - I)/a^3 + I*log(abs(tan(1/2*d*x + 1/2*c) +
 1))/a^3 + I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + (3*I*tan(1/2*d*x + 1/
2*c)^2 + 10*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))
/d
```

3.182 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=62

$$\frac{4\sin(c+dx)}{a^3d}+\frac{4i\cos(c+dx)}{a^3d}+\frac{i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}$$

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((4*I)*Cos[c + d*x])/(a^3*d) + (I*Sec[
c + d*x])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d)
```

Rubi [A] time = 0.161956, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.29, Rules used = {3092, 3090, 2637, 2638, 2592, 321, 206, 2590, 14}

$$\frac{4\sin(c+dx)}{a^3d}+\frac{4i\cos(c+dx)}{a^3d}+\frac{i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((4*I)*Cos[c + d*x])/(a^3*d) + (I*Sec[
c + d*x])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\cos(c+dx)-3a^3\sin(c+dx)+3ia^3\sin(c+dx)\tan(c+dx)+a^3\sin\right.}{a^6}$$
$$= \frac{i\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{(3i)\int \sin(c+dx)\,dx}{a^3} + \frac{\int \cos(c+dx)\,dx}{a^3} - \frac{3}{}$$
$$= \frac{3i\cos(c+dx)}{a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{i\,\mathrm{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{a^3d} - \frac{3\,\mathrm{Subst}\,(}{}$$
$$= \frac{3i\cos(c+dx)}{a^3d} + \frac{4\sin(c+dx)}{a^3d} - \frac{i\,\mathrm{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos(c+dx)\right)}{a^3d} - \frac{3}{}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{4i\cos(c+dx)}{a^3d} + \frac{i\sec(c+dx)}{a^3d} + \frac{4\sin(c+dx)}{a^3d}$$

Mathematica [A] time = 0.314773, size = 109, normalized size = 1.76

$$-\frac{i\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3\left((\tan(c+dx)-5i)(\cos(2c-dx)+i\sin(2c-dx))+6(\cos(3c)+i\sin(3c))\tanh^{-1}\right.}{a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I)*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*T
an[(d*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*
(-5*I + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A] time = 0.196, size = 108, normalized size = 1.7

$$8\,\frac{1}{da^3\,(\tan(1/2\,dx+c/2)-i)} + \frac{i}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - 3\,\frac{\ln(\tan(1/2\,dx+c/2)+1)}{da^3} - \frac{i}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 8/d/a^3/(tan(1/2*d*x+1/2*c)-I)+I/d/a^3/(tan(1/2*d*x+1/2*c)+1)-3/d/a^3*ln(ta
n(1/2*d*x+1/2*c)+1)-I/d/a^3/(tan(1/2*d*x+1/2*c)-1)+3/d/a^3*ln(tan(1/2*d*x+1
/2*c)-1)
```

Maxima [B] time = 1.6332, size = 444, normalized size = 7.16

$$\frac{(6\cos(3\,dx+3\,c)+6\cos(dx+c)+6i\sin(3\,dx+3\,c)+6i\sin(dx+c))\arctan(\cos(dx+c),\sin(dx+c)+1)+(6\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] ((6*cos(3*d*x + 3*c) + 6*cos(d*x + c) + 6*I*sin(3*d*x + 3*c) + 6*I*sin(d*x
+ c))*arctan2(cos(d*x + c), sin(d*x + c) + 1) + (6*cos(3*d*x + 3*c) + 6*cos
(d*x + c) + 6*I*sin(3*d*x + 3*c) + 6*I*sin(d*x + c))*arctan2(cos(d*x + c),
-sin(d*x + c) + 1) - (-3*I*cos(3*d*x + 3*c) - 3*I*cos(d*x + c) + 3*sin(3*d*
x + 3*c) + 3*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x
+ c) + 1) - (3*I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) - 3*sin(3*d*x + 3*c) -
 3*sin(d*x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)
+ 12*cos(2*d*x + 2*c) + 12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3
*c) - 2*I*a^3*cos(d*x + c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d
)
```

Fricas [A] time = 0.493061, size = 302, normalized size = 4.87

$$-\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}-4i}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(e
^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I
*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.20389, size = 151, normalized size = 2.44

$$-\frac{\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{3\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{2\left(4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 3*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/a^3 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((ta
n(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)
*a^3))/d
```

3.183 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}+\frac{4x}{a^3}$$

```
[Out] (4*x)/a^3 + ((4*I)*Log[Sin[c + d*x]])/(a^3*d) - ((4*I)*Log[Tan[c + d*x]])/(
a^3*d) - (3*Tan[c + d*x])/(a^3*d) + ((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rubi [A] time = 0.0805796, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 88}

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}+\frac{4x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (4*x)/a^3 + ((4*I)*Log[Sin[c + d*x]])/(a^3*d) - ((4*I)*Log[Tan[c + d*x]])/(
a^3*d) - (3*Tan[c + d*x])/(a^3*d) + ((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^2}{x^3(ia+ax)}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{i}{a^3x^3}-\frac{3}{a^3x^2}-\frac{4i}{a^3x}+\frac{4i}{a^3(i+x)}\right)dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{4x}{a^3}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+d}{2a^3d}\end{aligned}$$

Mathematica [A] time = 0.599273, size = 110, normalized size = 1.47

$$\frac{i\sec(c)\sec^2(c+dx)(\cos(c)(4\log(\cos(c+dx))-4idx+1)-i(2\cos(c+2dx)(dx+i\log(\cos(c+dx)))+2\cos(3c+2dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/2)*Sec[c]*Sec[c + d*x]^2*(Cos[c]*(1 - (4*I)*d*x + 4*Log[Cos[c + d*x]])
- I*(2*Cos[c + 2*d*x]*(d*x + I*Log[Cos[c + d*x]]) + 2*Cos[3*c + 2*d*x]*(d*x
 + I*Log[Cos[c + d*x]]) - 6*Cos[c + d*x]*Sin[d*x])))/(a^3*d)
```

Maple [A] time = 0.196, size = 52, normalized size = 0.7

$$-3\,\frac{\tan(dx+c)}{da^3}+\frac{\frac{i}{2}(\tan(dx+c))^2}{da^3}-\frac{4\,i\ln(\tan(dx+c)-i)}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] -3*tan(d*x+c)/a^3/d+1/2*I*tan(d*x+c)^2/a^3/d-4*I/d/a^3*ln(tan(d*x+c)-I)
```

Maxima [B] time = 1.64775, size = 404, normalized size = 5.39

$$-8i\,dx + (4i\,\cos(4\,dx + 4\,c) + 8i\,\cos(2\,dx + 2\,c) - 4\,\sin(4\,dx + 4\,c) - 8\,\sin(2\,dx + 2\,c) + 4i)\arctan(\sin(2\,dx + 2\,c)\,,\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (-8*I*d*x + (4*I*cos(4*d*x + 4*c) + 8*I*cos(2*d*x + 2*c) - 4*sin(4*d*x + 4*
c) - 8*sin(2*d*x + 2*c) + 4*I)*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1) + (-8*I*d*x - 8*I*c)*cos(4*d*x + 4*c) + (-16*I*d*x - 16*I*c - 4)*cos(2*
d*x + 2*c) + (2*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 2*I*sin(4*d*x + 4*c
) + 4*I*sin(2*d*x + 2*c) + 2)*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1) + 8*(d*x + c)*sin(4*d*x + 4*c) + (16*d*x + 16*c -
4*I)*sin(2*d*x + 2*c) - 8*I*c - 6)/((-I*a^3*cos(4*d*x + 4*c) - 2*I*a^3*cos(
2*d*x + 2*c) + a^3*sin(4*d*x + 4*c) + 2*a^3*sin(2*d*x + 2*c) - I*a^3)*d)
```

Fricas [A] time = 0.484404, size = 317, normalized size = 4.23

$$\frac{8\,dxe^{(4i\,dx+4i\,c)} + 8\,dx + (16\,dx - 4i)e^{(2i\,dx+2i\,c)} + \left(4i\,e^{(4i\,dx+4i\,c)} + 8i\,e^{(2i\,dx+2i\,c)} + 4i\right)\log\left(e^{(2i\,dx+2i\,c)} + 1\right) - 6i}{a^3de^{(4i\,dx+4i\,c)} + 2\,a^3de^{(2i\,dx+2i\,c)} + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] (8*d*x*e^(4*I*d*x + 4*I*c) + 8*d*x + (16*d*x - 4*I)*e^(2*I*d*x + 2*I*c) + (
4*I*e^(4*I*d*x + 4*I*c) + 8*I*e^(2*I*d*x + 2*I*c) + 4*I)*log(e^(2*I*d*x + 2
*I*c) + 1) - 6*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.18891, size = 176, normalized size = 2.35

$$\frac{2\left(-\frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3}+\frac{2i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}+\frac{2i\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}+\frac{-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(-4*I*log(tan(1/2*d*x + 1/2*c) - I)/a^3 + 2*I*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/a^3 + 2*I*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + (-3*I*tan(1/2*d*
x + 1/2*c)^4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*ta
n(1/2*d*x + 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

3.184 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{i\sec^3(c+dx)}{3a^3d}-\frac{4i\sec(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{3\tan(c+dx)\sec(c+dx)}{2a^3d}$$

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - ((4*I)*Sec[c + d*x])/(a^3*d) + ((I/3)
*Sec[c + d*x]^3)/(a^3*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d)
```

Rubi [A] time = 0.181894, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3092, 3090, 3770, 2606, 8, 2611}

$$\frac{i\sec^3(c+dx)}{3a^3d}-\frac{4i\sec(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{3\tan(c+dx)\sec(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - ((4*I)*Sec[c + d*x])/(a^3*d) + ((I/3)
*Sec[c + d*x]^3)/(a^3*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$

$$= \frac{i\int \left(-ia^3\sec(c+dx)-3a^3\sec(c+dx)\tan(c+dx)+3ia^3\sec(c+dx)\tan^2(c+\right.}{a^6}$$

$$= \frac{i\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \sec(c+dx)\tan(c+dx)\,dx}{a^3} + \frac{\int \sec(c}{a}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{3\sec(c+dx)\tan(c+dx)}{2a^3d} + \frac{3\int \sec(c+dx)\,dx}{2a^3} + \frac{i\,\mathrm{Sul}}{}$$

$$= \frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{4i\sec(c+dx)}{a^3d} + \frac{i\sec^3(c+dx)}{3a^3d} - \frac{3\sec(c+dx)\tan(c}{2a^3d}$$

Mathematica [A] time = 0.471151, size = 64, normalized size = 0.84

$$\frac{i\left(\sec^3(c+dx)(9i\sin(2(c+dx))-24\cos(2(c+dx))-20)-60i\tanh^{-1}\left(\cos(c)\tan\left(\frac{dx}{2}\right)+\sin(c)\right)\right)}{12a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/12)*((-60*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-2
0 - 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/(a^3*d)
```

Maple [B] time = 0.215, size = 258, normalized size = 3.4

$$\frac{\frac{i}{3}}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{3}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{\frac{i}{2}}{da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{3}{2\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/3*I/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3+3/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2-1/2
*I/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2-3/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)-7/2*I/d/
a^3/(tan(1/2*d*x+1/2*c)+1)+5/2/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)-1/3*I/d/a^3/(
tan(1/2*d*x+1/2*c)-1)^3-3/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2-1/2*I/d/a^3/(tan
(1/2*d*x+1/2*c)-1)^2-3/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)+7/2*I/d/a^3/(tan(1/2*
d*x+1/2*c)-1)-5/2/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.08496, size = 290, normalized size = 3.82

$$\frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{9i\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+22\right)}{6i\,a^3-\frac{18i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{18i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{6i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 -
```

```
5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [B] time = 0.486938, size = 521, normalized size = 6.86

$$\frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i}\right.}{6\left(a^3de^{(6i\,dx+6i\,c)}+3\,a^3de^{(4i\,dx+4i\,c)}+3\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5*I*
d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*e^(6
*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c)
 + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.17367, size = 154, normalized size = 2.03

$$\frac{\frac{15\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{15\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}-\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-22i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 15*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4
 + 48*I*tan(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d
*x + 1/2*c)^2 - 1)^3*a^3))/d
```

3.185 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=34

$$\frac{i\tan^4(c+dx)(-\cot(c+dx)+i)^4}{4a^3d}$$

```
[Out] ((I/4)*(I - Cot[c + d*x])^4*Tan[c + d*x]^4)/(a^3*d)
```

Rubi [A] time = 0.062903, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {3088, 848, 37}

$$\frac{i\tan^4(c+dx)(-\cot(c+dx)+i)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*(I - Cot[c + d*x])^4*Tan[c + d*x]^4)/(a^3*d)
```

Rule 3088

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[(x^m*(b +
 a*x)^n)/(1 + x^2)^((m + n + 2)/2), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rule 848

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c*x)/e)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{
```

```
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^3}{x^5}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{i(i-\cot(c+dx))^4\tan^4(c+dx)}{4a^3d}$$

Mathematica [B] time = 0.465381, size = 90, normalized size = 2.65

$$-\frac{i\sec(c)\sec^4(c+dx)(2i\sin(c+2dx)-2i\sin(3c+2dx)+i\sin(3c+4dx)+2\cos(c+2dx)+2\cos(3c+2dx)-3i\sin(c)\cdots}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I/4)*Sec[c]*Sec[c + d*x]^4*(3*Cos[c] + 2*Cos[c + 2*d*x] + 2*Cos[3*c + 2*
d*x] - (3*I)*Sin[c] + (2*I)*Sin[c + 2*d*x] - (2*I)*Sin[3*c + 2*d*x] + I*Sin
[3*c + 4*d*x]))/(a^3*d)
```

Maple [A] time = 0.197, size = 47, normalized size = 1.4

$$\frac{\tan(dx+c)+\frac{i}{4}(\tan(dx+c))^4-(\tan(dx+c))^3-\frac{3i}{2}(\tan(dx+c))^2}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(tan(d*x+c)+1/4*I*tan(d*x+c)^4-tan(d*x+c)^3-3/2*I*tan(d*x+c)^2)
```

Maxima [B] time = 1.08891, size = 324, normalized size = 9.53

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{3i\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{8i\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{7\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{3i\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{\left(a^3 - \frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 8*I*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 7*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*I*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 - sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/((a^3 - 4*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8)*d)
```

Fricas [B] time = 0.448625, size = 177, normalized size = 5.21

$$\frac{4i}{a^3de^{(8i\,dx+8i\,c)} + 4\,a^3de^{(6i\,dx+6i\,c)} + 6\,a^3de^{(4i\,dx+4i\,c)} + 4\,a^3de^{(2i\,dx+2i\,c)} + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(4
*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.21989, size = 63, normalized size = 1.85

$$-\frac{-i\tan\left(dx+c\right)^{4}+4\tan\left(dx+c\right)^{3}+6i\tan\left(dx+c\right)^{2}-4\tan\left(dx+c\right)}{4a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

3.186 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=104

$$\frac{i\sec^5(c+dx)}{5a^3d}-\frac{4i\sec^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{3\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}$$

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((4*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
((I/5)*Sec[c + d*x]^5)/(a^3*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) -
(3*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*d)
```

Rubi [A] time = 0.229947, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3092, 3090, 3768, 3770, 2606, 30, 2611, 14}

$$\frac{i\sec^5(c+dx)}{5a^3d}-\frac{4i\sec^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{3\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((4*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
((I/5)*Sec[c + d*x]^5)/(a^3*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) -
(3*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*d)
```

Rule 3092

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3090

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\sec^3(c+dx) - 3a^3\sec^3(c+dx)\tan(c+dx) + 3ia^3\sec^3(c+dx)\tan^2(\right.}{a^6}$$
$$= \frac{i\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^3} + \frac{\int \sec^3}{}$$
$$= \frac{\sec(c+dx)\tan(c+dx)}{2a^3d} - \frac{3\sec^3(c+dx)\tan(c+dx)}{4a^3d} + \frac{\int \sec(c+dx)\,dx}{2a^3} + \frac{3}{}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{2a^3d} - \frac{i\sec^3(c+dx)}{a^3d} + \frac{7\sec(c+dx)\tan(c+dx)}{8a^3d} - \frac{3\sec^3(c+}{}$$
$$= \frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d} - \frac{4i\sec^3(c+dx)}{3a^3d} + \frac{i\sec^5(c+dx)}{5a^3d} + \frac{7\sec(c+dx)\tan(}{8a^3d}$$

Mathematica [A] time = 0.443505, size = 115, normalized size = 1.11

$$\frac{i\sec^8(c+dx)(\sin(3(c+dx)) - i\cos(3(c+dx)))\left(-150i\sin(2(c+dx)) + 105i\sin(4(c+dx)) + 640\cos(2(c+dx)) + 1680\right.}{960a^3d(\tan(c+dx)-i)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/960)*Sec[c + d*x]^8*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(448 + (
1680*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c
 + d*x)] - (150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(a^3*d*(-I
 + Tan[c + d*x])^3)
```

Maple [B] time = 0.224, size = 430, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x)
```

```
[Out] -1/5*I/d/a^3/(tan(1/2*d*x+1/2*c)-1)^5+5/8/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2-13
/8*I/d/a^3/(tan(1/2*d*x+1/2*c)+1)+3/4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4+11/8*I
```

```
/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+1/8/d/a^3/(tan(1/2*d*x+1/2*c)+1)-1/2*I/d/a^
3/(tan(1/2*d*x+1/2*c)-1)^4-3/2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3+7/12*I/d/a^3/
(tan(1/2*d*x+1/2*c)-1)^3+7/8/d/a^3*ln(tan(1/2*d*x+1/2*c)+1)+11/8*I/d/a^3/(t
an(1/2*d*x+1/2*c)-1)^2+1/8/d/a^3/(tan(1/2*d*x+1/2*c)-1)+13/8*I/d/a^3/(tan(1
/2*d*x+1/2*c)-1)-3/4/d/a^3/(tan(1/2*d*x+1/2*c)-1)^4+1/5*I/d/a^3/(tan(1/2*d*
x+1/2*c)+1)^5-5/8/d/a^3/(tan(1/2*d*x+1/2*c)-1)^2-7/12*I/d/a^3/(tan(1/2*d*x+
1/2*c)+1)^3-3/2/d/a^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2*I/d/a^3/(tan(1/2*d*x+1/2
*c)+1)^4-7/8/d/a^3*ln(tan(1/2*d*x+1/2*c)-1)
```

Maxima [B] time = 1.10627, size = 460, normalized size = 4.42

$$\frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136\right)}{-120ia^3+\frac{600ia^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200ia^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200ia^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600ia^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120ia^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right.}{a^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600*I*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [B] time = 0.501209, size = 836, normalized size = 8.04

$$\frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(10i\,dx+10i\,c)}+5\,e\right.}{120\left(a^3de^{(10i\,dx+10i\,c)}+5\,a^3de^{(8}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^
(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*I*c)
- 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(I*d*x
+ I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.21218, size = 224, normalized size = 2.15

$$\frac{\frac{105 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{105 \log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{2\left(15 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360i \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-390 \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-960i \tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx\cdots\right.\right.}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 105*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/
2*c)^8 - 390*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*
tan(1/2*d*x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1
/2*c)^2 - 15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*
```

```
a^3))/d
```

3.187 $\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=66

$$-\frac{i\cos^{-n}(c+dx)\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(1+i\tan(c+dx))\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

```
[Out] ((-I/2)*Hypergeometric2F1[1, n, 1 + n, (1 + I*Tan[c + d*x])/2]*(a*Cos[c + d
*x] + I*a*Sin[c + d*x])^n)/(d*n*Cos[c + d*x]^n)
```

Rubi [A] time = 0.0611622, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.03, Rules used = {3084}

$$-\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(i\tan(c+dx)+1)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

```
[Out] ((-I/2)*Hypergeometric2F1[1, n, 1 + n, (1 + I*Tan[c + d*x])/2]*(a*Cos[c + d
*x] + I*a*Sin[c + d*x])^n)/(d*n*Cos[c + d*x]^n)
```

Rule 3084

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*(a*Cos[c + d*x] + b*Sin[
c + d*x])^n*Hypergeometric2F1[1, n, n + 1, (a + b*Tan[c + d*x])/(2*a)])/(2*
a*d*n*Cos[c + d*x]^n), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] &&
EqQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a\cos(c+$$

Mathematica [A] time = 2.07308, size = 90, normalized size = 1.36

$$\frac{\cos^{-n}(c+dx)\left(n(\tan(c+dx)-i)\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{1}{2}(1+i\tan(c+dx))\right)-2i(n+1)\right)(a(\cos(c+dx)}{4dn(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

```
[Out] ((a*(Cos[c + d*x] + I*Sin[c + d*x]))^n*((-2*I)*(1 + n) + n*Hypergeometric2F
1[1, 1 + n, 2 + n, (1 + I*Tan[c + d*x])/2]*(-I + Tan[c + d*x])))/(4*d*n*(1
+ n)*Cos[c + d*x]^n)
```

Maple [F] time = 0.586, size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+ia\sin(dx+c))^n}{(\cos(dx+c))^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

```
[Out] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\cos(dx+c)+i\,a\sin(dx+c))^n\cos(dx+c)^{-n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*cos(d*x + c)^(-n), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(ae^{(i\,dx+i\,c)}\right)^n}{\left(\frac{1}{2}\left(e^{(2i\,dx+2i\,c)}+1\right)e^{(-i\,dx-i\,c)}\right)^n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="fric
as")
```

```
[Out] integral((a*e^(I*d*x + I*c))^n/(1/2*(e^(2*I*d*x + 2*I*c) + 1)*e^(-I*d*x - I
*c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(cos(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\cos(dx+c)+i\,a\sin(dx+c))^n}{\cos(dx+c)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="giac
")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/cos(d*x + c)^n, x)
```

3.188 $\int \frac{1}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=5

$$\log(\sin(x)+1)$$

```
[Out] Log[1 + Sin[x]]
```

Rubi [A] time = 0.0231376, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3159, 2667, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] Log[1 + Sin[x]]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos(x)}{1+\sin(x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0178865, size = 16, normalized size = 3.2

$$2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] 2*Log[Cos[x/2] + Sin[x/2]]
```

Maple [A] time = 0.068, size = 6, normalized size = 1.2

$$\ln(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x)),x)
```

```
[Out] ln(sin(x)+1)
```

Maxima [B] time = 0.996776, size = 42, normalized size = 8.4

$$2\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.479156, size = 23, normalized size = 4.6

$$\log(\sin(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [B] time = 0.173068, size = 17, normalized size = 3.4

$$\log(\tan(x) + \sec(x)) - \frac{\log\left(\tan^2(x) + 1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x)
```

```
[Out] log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

Giac [B] time = 1.13293, size = 30, normalized size = 6.

$$-\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 2\log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

3.189 $\int \frac{\sin(x)}{\sec(x)+\tan(x)} \, dx$

Optimal. Leaf size=10

$$\sin(x) - \log(\sin(x) + 1)$$

```
[Out] -Log[1 + Sin[x]] + Sin[x]
```

Rubi [A] time = 0.0678942, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4391, 2833, 43}

$$\sin(x) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -Log[1 + Sin[x]] + Sin[x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\cos(x)\sin(x)}{1+\sin(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, \sin(x)\right)$$
$$= -\log(1+\sin(x))+\sin(x)$$

Mathematica [A] time = 0.0206266, size = 19, normalized size = 1.9

$$\sin(x)-2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -2*Log[Cos[x/2] + Sin[x/2]] + Sin[x]
```

Maple [A] time = 0.089, size = 11, normalized size = 1.1

$$-\ln(\sin(x)+1)+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sec(x)+tan(x)),x)
```

```
[Out] -ln(sin(x)+1)+sin(x)
```

Maxima [B] time = 1.57702, size = 73, normalized size = 7.3

$$\frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}-2\,\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(x
) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.47127, size = 36, normalized size = 3.6

$$-\log\left(\sin\left(x\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -log(sin(x) + 1) + sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(x\right)}{\tan\left(x\right)+\sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(sin(x)/(tan(x) + sec(x)), x)
```

Giac [A] time = 1.09507, size = 14, normalized size = 1.4

$$-\log\left(\sin\left(x\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(sin(x) + 1) + sin(x)
```

3.190 $\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=4

$$x + \cos(x)$$

```
[Out] x + Cos[x]
```

Rubi [A] time = 0.0601193, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4391, 2682, 8}

$$x + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x + Cos[x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\cos^2(x)}{1+\sin(x)}\,dx$$
$$= \cos(x) + \int 1\,dx$$
$$= x + \cos(x)$$

Mathematica [A] time = 0.0184668, size = 4, normalized size = 1.

$$x + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x + Cos[x]
```

Maple [B] time = 0.091, size = 15, normalized size = 3.8

$$2\left(\left(\tan\left(x/2\right)\right)^{2}+1\right)^{-1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sec(x)+tan(x)),x)
```

```
[Out] 2/(tan(1/2*x)^2+1)+x
```

Maxima [B] time = 1.64385, size = 41, normalized size = 10.25

$$\frac{2}{\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+1}+2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.467904, size = 16, normalized size = 4.

$$x + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] x + cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\tan(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(cos(x)/(tan(x) + sec(x)), x)
```

Giac [B] time = 1.12482, size = 19, normalized size = 4.75

$$x + \frac{2}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] x + 2/(tan(1/2*x)^2 + 1)
```

3.191 $\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=11

$$x+\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] x + Cos[x]/(1 + Sin[x])
```

Rubi [A] time = 0.0528004, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4391, 2735, 2648}

$$x+\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x + Cos[x]/(1 + Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sin(x)}{1+\sin(x)}\,dx$$
$$= x - \int \frac{1}{1+\sin(x)}\,dx$$
$$= x + \frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] time = 0.0336051, size = 25, normalized size = 2.27

$$x-\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.065, size = 13, normalized size = 1.2

$$2\,(\tan(x/2)+1)^{-1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(sec(x)+tan(x)),x)
```

```
[Out] 2/(tan(1/2*x)+1)+x
```

Maxima [B] time = 1.66223, size = 38, normalized size = 3.45

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}+2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 0.464028, size = 88, normalized size = 8.

$$\frac{(x+1)\cos(x)+(x-1)\sin(x)+x+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(tan(x)/(tan(x) + sec(x)), x)
```

Giac [A] time = 1.12496, size = 16, normalized size = 1.45

$$x+\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] x + 2/(tan(1/2*x) + 1)
```

3.192 $\int \frac{\cot(x)}{\sec(x)+\tan(x)} dx$

Optimal. Leaf size=9

$$-x - \tanh^{-1}(\cos(x))$$

```
[Out] -x - ArcTanh[Cos[x]]
```

Rubi [A] time = 0.0784214, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4391, 2839, 3770, 8}

$$-x - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -x - ArcTanh[Cos[x]]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos(x)\cot(x)}{1+\sin(x)}\,dx\\ &= -\int 1\,dx + \int \csc(x)\,dx\\ &= -x - \tanh^{-1}(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.023098, size = 20, normalized size = 2.22

$$-x + \log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.099, size = 10, normalized size = 1.1

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sec(x)+tan(x)),x)
```

```
[Out] ln(tan(1/2*x))-x
```

Maxima [B] time = 1.62816, size = 31, normalized size = 3.44

$$-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 0.491269, size = 82, normalized size = 9.11

$$-x - \frac{1}{2}\,\log\left(\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right) + \frac{1}{2}\,\log\left(-\frac{1}{2}\,\cos\left(x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot\left(x\right)}{\tan\left(x\right) + \sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(cot(x)/(tan(x) + sec(x)), x)
```

Giac [A] time = 1.13378, size = 14, normalized size = 1.56

$$-x + \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -x + log(abs(tan(1/2*x)))
```

3.193 $\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{\sin(x)+1}$$

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rubi [A] time = 0.0244206, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3165, 2648}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Sec[x] + Tan[x]),x]

[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{1}{1+\sin(x)}\,dx$$
$$= -\frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] time = 0.0171352, size = 23, normalized size = 2.3

$$\frac{2\sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A] time = 0.061, size = 11, normalized size = 1.1

$$-2\,\left(\tan\left(x/2\right)+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(sec(x)+tan(x)),x)
```

```
[Out] -2/(tan(1/2*x)+1)
```

Maxima [A] time = 1.06971, size = 20, normalized size = 2.

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

Fricas [A] time = 0.461033, size = 62, normalized size = 6.2

$$-\frac{\cos(x)-\sin(x)+1}{\cos(x)+\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(sec(x)/(tan(x) + sec(x)), x)
```

Giac [A] time = 1.12133, size = 14, normalized size = 1.4

$$-\frac{2}{\tan\left(\frac{1}{2}x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) + 1)
```

3.194 $\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rubi [A] time = 0.0546386, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4391, 2707, 36, 29, 31}

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\sin(x)}\,dx \\
&= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right) \\
&= \log(\sin(x)) - \log(1+\sin(x))
\end{aligned}
$$

Mathematica [A] time = 0.0216489, size = 20, normalized size = 1.82

$$\log(\sin(x)) - 2\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -2*Log[Cos[x/2] + Sin[x/2]] + Log[Sin[x]]
```

Maple [A] time = 0.089, size = 8, normalized size = 0.7

$$-\ln(1+\csc(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(sec(x)+tan(x)),x)
```

```
[Out] -ln(1+csc(x))
```

Maxima [B] time = 1.11594, size = 34, normalized size = 3.09

$$-2\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.485571, size = 47, normalized size = 4.27

$$\log\left(\frac{1}{2}\sin(x)\right)-\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + sec(x)), x)
```

Giac [A] time = 1.11951, size = 16, normalized size = 1.45

$$-\log(\sin(x)+1)+\log(|\sin(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

3.195 $\int \frac{1}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=9

$$-\log(1-\sin(x))$$

```
[Out] -Log[1 - Sin[x]]
```

Rubi [A] time = 0.0270939, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3159, 2667, 31}

$$-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] - Tan[x])^(-1),x]
```

```
[Out] -Log[1 - Sin[x]]
```

Rule 3159

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rule 2667

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\int \frac{1}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos(x)}{1-\sin(x)}\,dx$$
$$= -\operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\sin(x)\right)$$
$$= -\log(1-\sin(x))$$

Mathematica [A] time = 0.0195591, size = 18, normalized size = 2.

$$-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] - Tan[x])^(-1),x]
```

```
[Out] -2*Log[Cos[x/2] - Sin[x/2]]
```

Maple [A] time = 0.067, size = 8, normalized size = 0.9

$$-\ln(\sin(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)-tan(x)),x)
```

```
[Out] -ln(sin(x)-1)
```

Maxima [B] time = 1.09662, size = 39, normalized size = 4.33

$$-2\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.482665, size = 26, normalized size = 2.89

$$-\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] -log(-sin(x) + 1)
```

Sympy [B] time = 0.233267, size = 17, normalized size = 1.89

$$-\log\left(-\tan\left(x\right)+\sec\left(x\right)\right)+\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x)
```

```
[Out] -log(-tan(x) + sec(x)) + log(tan(x)**2 + 1)/2
```

Giac [B] time = 1.11513, size = 27, normalized size = 3.

$$\log\left(\tan\left(\frac{1}{2}x\right)^{2}+1\right)-2\log\left(\left|\tan\left(\frac{1}{2}x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) - 1))
```

3.196 $\int \frac{\sin(x)}{\sec(x)-\tan(x)} dx$

Optimal. Leaf size=14

$$-\sin(x) - \log(1 - \sin(x))$$

```
[Out] -Log[1 - Sin[x]] - Sin[x]
```

Rubi [A] time = 0.0819025, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4391, 2833, 43}

$$-\sin(x) - \log(1 - \sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -Log[1 - Sin[x]] - Sin[x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos(x)\sin(x)}{1-\sin(x)}\,dx$$
$$= \mathrm{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\sin(x)\right)$$
$$= \mathrm{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, -\sin(x)\right)$$
$$= -\log(1-\sin(x)) - \sin(x)$$

Mathematica [A] time = 0.0220718, size = 23, normalized size = 1.64

$$-\sin(x) - 2\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -2*Log[Cos[x/2] - Sin[x/2]] - Sin[x]
```

Maple [A] time = 0.083, size = 13, normalized size = 0.9

$$-\sin(x) - \ln(\sin(x) - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sec(x)-tan(x)),x)
```

```
[Out] -sin(x)-ln(sin(x)-1)
```

Maxima [B] time = 1.66421, size = 73, normalized size = 5.21

$$-\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} - 2\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right) + \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(
x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.48163, size = 38, normalized size = 2.71

$$-\log\left(-\sin\left(x\right)+1\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] -log(-sin(x) + 1) - sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(x\right)}{-\tan\left(x\right)+\sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(sin(x)/(-tan(x) + sec(x)), x)
```

Giac [A] time = 1.10918, size = 19, normalized size = 1.36

$$-\log\left(-\sin\left(x\right)+1\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] -log(-sin(x) + 1) - sin(x)
```

3.197 $\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=6

$$x - \cos(x)$$

```
[Out] x - Cos[x]
```

Rubi [A] time = 0.0600477, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4391, 2682, 8}

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sec[x] - Tan[x]),x]

[Out] x - Cos[x]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos^2(x)}{1-\sin(x)}\,dx$$
$$= -\cos(x) + \int 1\,dx$$
$$= x - \cos(x)$$

Mathematica [A] time = 0.0214677, size = 6, normalized size = 1.

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Maple [B] time = 0.099, size = 15, normalized size = 2.5

$$-2\left((\tan(x/2))^2+1\right)^{-1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sec(x)-tan(x)),x)
```

```
[Out] -2/(tan(1/2*x)^2+1)+x
```

Maxima [B] time = 1.48269, size = 41, normalized size = 6.83

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.470495, size = 16, normalized size = 2.67

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] x - cos(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{-\tan(x) + \sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(cos(x)/(-tan(x) + sec(x)), x)
```

Giac [B] time = 1.12075, size = 19, normalized size = 3.17

$$x - \frac{2}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] x - 2/(tan(1/2*x)^2 + 1)
```

3.198 $\int \frac{\tan(x)}{\sec(x)-\tan(x)} dx$

Optimal. Leaf size=15

$$\frac{\cos(x)}{1-\sin(x)} - x$$

```
[Out] -x + Cos[x]/(1 - Sin[x])
```

Rubi [A] time = 0.0604004, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4391, 2735, 2648}

$$\frac{\cos(x)}{1-\sin(x)} - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -x + Cos[x]/(1 - Sin[x])
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{\sin(x)}{1-\sin(x)}\,dx \\
&= -x + \int \frac{1}{1-\sin(x)}\,dx \\
&= -x + \frac{\cos(x)}{1-\sin(x)}
\end{aligned}
$$

Mathematica [A] time = 0.0360226, size = 29, normalized size = 1.93

$$
\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}-x
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -x + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0.067, size = 15, normalized size = 1.

$$
-2\,\left(\tan\left(x/2\right)-1\right)^{-1}-x
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(sec(x)-tan(x)),x)
```

```
[Out] -2/(tan(1/2*x)-1)-x
```

Maxima [A] time = 1.49618, size = 38, normalized size = 2.53

$$
-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}-2\,\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.468083, size = 89, normalized size = 5.93

$$-\frac{(x-1)\cos(x)-(x+1)\sin(x)+x-1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] -((x - 1)*cos(x) - (x + 1)*sin(x) + x - 1)/(cos(x) - sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan(x)}{-\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(tan(x)/(-tan(x) + sec(x)), x)
```

Giac [A] time = 1.14186, size = 19, normalized size = 1.27

$$-x-\frac{2}{\tan\left(\frac{1}{2}\,x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] -x - 2/(tan(1/2*x) - 1)
```

3.199 $\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=7

$$x - \tanh^{-1}(\cos(x))$$

```
[Out] x - ArcTanh[Cos[x]]
```

Rubi [A] time = 0.0888162, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4391, 2839, 3770, 8}

$$x - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - ArcTanh[Cos[x]]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{\cos(x)\cot(x)}{1-\sin(x)}\,dx\\ &= \int 1\,dx+\int \csc(x)\,dx\\ &= x-\tanh^{-1}(\cos(x))\end{aligned}$$

Mathematica [B] time = 0.0221246, size = 18, normalized size = 2.57

$$x+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A] time = 0.105, size = 8, normalized size = 1.1

$$\ln\left(\tan\left(\frac{x}{2}\right)\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sec(x)-tan(x)),x)
```

```
[Out] ln(tan(1/2*x))+x
```

Maxima [B] time = 1.50547, size = 31, normalized size = 4.43

$$2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

Fricas [B] time = 0.490267, size = 81, normalized size = 11.57

$$x - \frac{1}{2} \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + \frac{1}{2} \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{-\tan(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(cot(x)/(-tan(x) + sec(x)), x)
```

Giac [A] time = 1.1556, size = 11, normalized size = 1.57

$$x + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] x + log(abs(tan(1/2*x)))
```

3.200 $\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] Cos[x]/(1 - Sin[x])
```

Rubi [A] time = 0.0287537, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3165, 2648}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 3165

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sec(x)}{\sec(x) - \tan(x)}\,dx = \int \frac{1}{1 - \sin(x)}\,dx$$
$$= \frac{\cos(x)}{1 - \sin(x)}$$

Mathematica [B] time = 0.0163485, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A] time = 0.059, size = 11, normalized size = 1.

$$-2\,(\tan(x/2) - 1)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(sec(x)-tan(x)),x)
```

```
[Out] -2/(tan(1/2*x)-1)
```

Maxima [A] time = 0.982344, size = 20, normalized size = 1.82

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A] time = 0.457423, size = 61, normalized size = 5.55

$$\frac{\cos(x) + \sin(x) + 1}{\cos(x) - \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(x)}{-\tan(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(sec(x)/(-tan(x) + sec(x)), x)
```

Giac [A] time = 1.15903, size = 14, normalized size = 1.27

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) - 1)
```

3.201 $\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=13

$$\log(\sin(x)) - \log(1 - \sin(x))$$

```
[Out] -Log[1 - Sin[x]] + Log[Sin[x]]
```

Rubi [A] time = 0.0620936, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4391, 2707, 36, 29, 31}

$$\log(\sin(x)) - \log(1 - \sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -Log[1 - Sin[x]] + Log[Sin[x]]
```

Rule 4391

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{\cot(x)}{1-\sin(x)}\,dx \\
&= \mathrm{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, -\sin(x)\right) \\
&= \mathrm{Subst}\left(\int \frac{1}{x}\,dx, x, -\sin(x)\right) - \mathrm{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\sin(x)\right) \\
&= -\log(1-\sin(x)) + \log(\sin(x))
\end{aligned}
$$

Mathematica [A] time = 0.0220258, size = 22, normalized size = 1.69

$$\log(\sin(x)) - 2\log\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -2*Log[Cos[x/2] - Sin[x/2]] + Log[Sin[x]]
```

Maple [A] time = 0.09, size = 8, normalized size = 0.6

$$-\ln\left(-1+\csc(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(sec(x)-tan(x)),x)
```

```
[Out] -ln(-1+csc(x))
```

Maxima [A] time = 1.01512, size = 34, normalized size = 2.62

$$-2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)+\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.487989, size = 49, normalized size = 3.77

$$\log\left(\frac{1}{2}\,\sin\left(x\right)\right)-\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(-sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(x\right)}{-\tan\left(x\right)+\sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(csc(x)/(-tan(x) + sec(x)), x)
```

Giac [A] time = 1.12615, size = 19, normalized size = 1.46

$$-\log\left(-\sin\left(x\right)+1\right)+\log\left(\left|\sin\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] -log(-sin(x) + 1) + log(abs(sin(x)))
```

3.202 $\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx$

Optimal. Leaf size=23

$$-\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

```
[Out] -(Cot[c + d*x]/d) - Csc[c + d*x]/d
```

Rubi [A] time = 0.0941638, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4397, 2669, 3767, 8}

$$-\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]/d) - Csc[c + d*x]/d
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx &= \int (1+\cos(c+dx))\csc^2(c+dx)\,dx\\ &= -\frac{\csc(c+dx)}{d}+\int \csc^2(c+dx)\,dx\\ &= -\frac{\csc(c+dx)}{d}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}\\ &= -\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0142455, size = 15, normalized size = 0.65

$$-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
```

```
[Out] -(Cot[(c + d*x)/2]/d)
```

Maple [A] time = 0.027, size = 24, normalized size = 1.

$$\frac{1}{d}\left(-(\sin(dx+c))^{-1}-\cot(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x)
```

```
[Out] 1/d*(-1/sin(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 1.04373, size = 30, normalized size = 1.3

$$-\frac{\frac{1}{\sin(dx+c)}+\frac{1}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(1/sin(d*x + c) + 1/tan(d*x + c))/d
```

Fricas [A] time = 0.457527, size = 51, normalized size = 2.22

$$-\frac{\cos(dx+c)+1}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c) + 1)/(d*sin(d*x + c))
```

Sympy [A] time = 6.24435, size = 27, normalized size = 1.17

$$\begin{cases} \frac{-\cot(c+dx)-\csc(c+dx)}{d} & \text{for } d \neq 0 \\ x(\cot(c)+\csc(c))\csc(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x)
```

```
[Out] Piecewise(((-cot(c + d*x) - csc(c + d*x))/d, Ne(d, 0)), (x*(cot(c) + csc(c)
)*csc(c), True))
```

Giac [A] time = 1.12489, size = 22, normalized size = 0.96

$$-\frac{1}{d\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/(d*tan(1/2*d*x + 1/2*c))
```

3.203 $\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=6

$$x - \sin(x)$$

```
[Out] x - Sin[x]
```

Rubi [A] time = 0.070243, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$, Rules used = {4392, 2682, 8}

$$x - \sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] x - Sin[x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\sin^2(x)}{1+\cos(x)}\,dx$$
$$= -\sin(x) + \int 1\,dx$$
$$= x - \sin(x)$$

Mathematica [B] time = 0.0074676, size = 14, normalized size = 2.33

$$2\left(\frac{x}{2} - \frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] 2*(x/2 - Sin[x]/2)
```

Maple [B] time = 0.083, size = 19, normalized size = 3.2

$$-2\,\frac{\tan(x/2)}{(\tan(x/2))^2+1} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cot(x)+csc(x)),x)
```

```
[Out] -2*tan(1/2*x)/(tan(1/2*x)^2+1)+x
```

Maxima [B] time = 1.66621, size = 51, normalized size = 8.5

$$-\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(c
os(x) + 1))
```

Fricas [A] time = 0.465335, size = 16, normalized size = 2.67

$$x - \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] x - sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(x)}{\cot(x) + \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(sin(x)/(cot(x) + csc(x)), x)
```

Giac [B] time = 1.0958, size = 24, normalized size = 4.

$$x - \frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] x - 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

3.204 $\int \frac{\cos(x)}{\cot(x)+\csc(x)} \, dx$

Optimal. Leaf size=10

$$\log(\cos(x) + 1) - \cos(x)$$

```
[Out] -Cos[x] + Log[1 + Cos[x]]
```

Rubi [A] time = 0.0673322, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4392, 2833, 43}

$$\log(\cos(x) + 1) - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -Cos[x] + Log[1 + Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{1+\cos(x)}\,dx \\ &= -\operatorname{Subst}\left(\int \frac{x}{1+x}\,dx, x, \cos(x)\right) \\ &= -\operatorname{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, \cos(x)\right) \\ &= -\cos(x)+\log(1+\cos(x))\end{aligned}$$

Mathematica [A] time = 0.0067374, size = 20, normalized size = 2.

$$2\log\left(\cos\left(\frac{x}{2}\right)\right)-2\cos^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -2*Cos[x/2]^2 + 2*Log[Cos[x/2]]
```

Maple [A] time = 0.082, size = 11, normalized size = 1.1

$$-\cos(x)+\ln(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(cot(x)+csc(x)),x)
```

```
[Out] -cos(x)+ln(cos(x)+1)
```

Maxima [B] time = 1.60049, size = 46, normalized size = 4.6

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.485423, size = 45, normalized size = 4.5

$$-\cos(x) + \log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -cos(x) + log(1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(x)}{\cot(x) + \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(cos(x)/(cot(x) + csc(x)), x)
```

Giac [A] time = 1.12215, size = 14, normalized size = 1.4

$$-\cos(x) + \log(\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] -cos(x) + log(cos(x) + 1)
```

3.205 $\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=7

$$\tanh^{-1}(\sin(x)) - x$$

```
[Out] -x + ArcTanh[Sin[x]]
```

Rubi [A] time = 0.0845227, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {4392, 2839, 3770, 8}

$$\tanh^{-1}(\sin(x)) - x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -x + ArcTanh[Sin[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\sin(x)\tan(x)}{1+\cos(x)}\,dx\\ &= -\int 1\,dx+\int \sec(x)\,dx\\ &= -x+\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0265616, size = 36, normalized size = 5.14

$$-x-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Cot[x] + Csc[x]),x]

[Out] -x - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [B] time = 0.086, size = 21, normalized size = 3.

$$\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cot(x)+csc(x)),x)

[Out] ln(tan(1/2*x)+1)-ln(tan(1/2*x)-1)-x
```

Maxima [B] time = 1.64086, size = 53, normalized size = 7.57

$$-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/
(cos(x) + 1) - 1)
```

Fricas [B] time = 0.490319, size = 66, normalized size = 9.43

$$-x + \frac{1}{2} \log\left(\sin\left(x\right) + 1\right) - \frac{1}{2} \log\left(-\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(x\right)}{\cot\left(x\right) + \csc\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(tan(x)/(cot(x) + csc(x)), x)
```

Giac [B] time = 1.13856, size = 30, normalized size = 4.29

$$-x + \log\left(\left|\tan\left(\frac{1}{2}\, x\right) + 1\right|\right) - \log\left(\left|\tan\left(\frac{1}{2}\, x\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] -x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

3.206 $\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=12

$$x-\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] x - Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0535888, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {4392, 2735, 2648}

$$x-\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] x - Sin[x]/(1 + Cos[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\cos(x)}{1+\cos(x)}\,dx \\
&= x-\int \frac{1}{1+\cos(x)}\,dx \\
&= x-\frac{\sin(x)}{1+\cos(x)}
\end{aligned}
$$

Mathematica [A] time = 0.017545, size = 10, normalized size = 0.83

$$x-\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] x - Tan[x/2]
```

Maple [A] time = 0.057, size = 9, normalized size = 0.8

$$-\tan\left(\frac{x}{2}\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(cot(x)+csc(x)),x)
```

```
[Out] -tan(1/2*x)+x
```

Maxima [A] time = 1.66572, size = 31, normalized size = 2.58

$$-\frac{\sin(x)}{\cos(x)+1}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -sin(x)/(cos(x) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.48709, size = 51, normalized size = 4.25

$$\frac{x\cos(x) + x - \sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] (x*cos(x) + x - sin(x))/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{\cot(x) + \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(cot(x)/(cot(x) + csc(x)), x)
```

Giac [A] time = 1.13233, size = 11, normalized size = 0.92

$$x - \tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] x - tan(1/2*x)
```

3.207 $\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=11

$$\log(\cos(x)+1)-\log(\cos(x))$$

```
[Out] -Log[Cos[x]] + Log[1 + Cos[x]]
```

Rubi [A] time = 0.0595973, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {4392, 2707, 36, 29, 31}

$$\log(\cos(x)+1)-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -Log[Cos[x]] + Log[1 + Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\tan(x)}{1+\cos(x)}\,dx \\
&= -\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \cos(x)\right) \\
&= -\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \cos(x)\right) + \operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right) \\
&= -\log(\cos(x)) + \log(1+\cos(x))
\end{aligned}$$

Mathematica [B] time = 0.0085433, size = 25, normalized size = 2.27

$$2\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(1 - 2\cos^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] 2*Log[Cos[x/2]] - Log[1 - 2*Cos[x/2]^2]
```

Maple [A] time = 0.078, size = 6, normalized size = 0.6

$$\ln(1 + \sec(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(cot(x)+csc(x)),x)
```

```
[Out] ln(1+sec(x))
```

Maxima [B] time = 1.10753, size = 39, normalized size = 3.55

$$-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A] time = 0.485049, size = 53, normalized size = 4.82

$$-\log(-\cos(x))+\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -log(-cos(x)) + log(1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(sec(x)/(cot(x) + csc(x)), x)
```

Giac [A] time = 1.14099, size = 16, normalized size = 1.45

$$\log(\cos(x)+1)-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] log(cos(x) + 1) - log(abs(cos(x)))
```

3.208 $\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{\cos(x)+1}$$

```
[Out] Sin[x]/(1 + Cos[x])
```

Rubi [A] time = 0.0255203, antiderivative size = 9, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3166, 2648}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{1}{1+\cos(x)}\,dx$$
$$= \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A] time = 0.0193816, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] Tan[x/2]
```

Maple [A] time = 0.05, size = 5, normalized size = 0.6

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(cot(x)+csc(x)),x)
```

```
[Out] tan(1/2*x)
```

Maxima [A] time = 1.0759, size = 12, normalized size = 1.33

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A] time = 0.458424, size = 28, normalized size = 3.11

$$\frac{\sin(x)}{\cos(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(csc(x)/(cot(x) + csc(x)), x)
```

Giac [A] time = 1.1713, size = 5, normalized size = 0.56

$$\tan\left(\frac{1}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] tan(1/2*x)
```

3.209 $\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=4

$$x+\sin(x)$$

```
[Out] x + Sin[x]
```

Rubi [A] time = 0.0697868, antiderivative size = 4, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4392, 2682, 8}

$$x+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] x + Sin[x]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2682

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1))/(b*f*(p - 1)), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\sin^2(x)}{1-\cos(x)}\,dx$$
$$= \sin(x) + \int 1\,dx$$
$$= x + \sin(x)$$

Mathematica [B] time = 0.006656, size = 14, normalized size = 3.5

$$2\left(\frac{x}{2}+\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] 2*(x/2 + Sin[x]/2)
```

Maple [B] time = 0.079, size = 19, normalized size = 4.8

$$2\,\frac{\tan(x/2)}{(\tan(x/2))^2+1}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(-cot(x)+csc(x)),x)
```

```
[Out] 2*tan(1/2*x)/(tan(1/2*x)^2+1)+x
```

Maxima [B] time = 1.53232, size = 51, normalized size = 12.75

$$\frac{2\,\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}+2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(co
s(x) + 1))
```

Fricas [A] time = 0.469711, size = 16, normalized size = 4.

$$x + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] x + sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sin(x)}{\cot(x) - \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(sin(x)/(cot(x) - csc(x)), x)
```

Giac [B] time = 1.12148, size = 24, normalized size = 6.

$$x + \frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] x + 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

3.210 $\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=10

$$\cos(x) + \log(1 - \cos(x))$$

```
[Out] Cos[x] + Log[1 - Cos[x]]
```

Rubi [A] time = 0.0684733, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4392, 2833, 43}

$$\cos(x) + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] Cos[x] + Log[1 - Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx$$
$$= -\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)dx, x, -\cos(x)\right)$$
$$= \cos(x)+\log(1-\cos(x))$$

Mathematica [A] time = 0.0100583, size = 20, normalized size = 2.

$$2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\sin^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] 2*Log[Sin[x/2]] - 2*Sin[x/2]^2
```

Maple [A] time = 0.078, size = 9, normalized size = 0.9

$$\cos(x)+\ln(-1+\cos(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(-cot(x)+csc(x)),x)
```

```
[Out] cos(x)+ln(-1+cos(x))
```

Maxima [B] time = 1.66173, size = 62, normalized size = 6.2

$$\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}+2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*log(sin(x)/(cos(x) + 1)) - log(sin(x)^2
/(cos(x) + 1)^2 + 1)
```

Fricas [A] time = 0.480792, size = 45, normalized size = 4.5

$$\cos(x) + \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] cos(x) + log(-1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\cos(x)}{\cot(x) - \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(cos(x)/(cot(x) - csc(x)), x)
```

Giac [A] time = 1.09205, size = 14, normalized size = 1.4

$$\cos(x) + \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] cos(x) + log(-cos(x) + 1)
```

3.211 $\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=5

$$x + \tanh^{-1}(\sin(x))$$

```
[Out] x + ArcTanh[Sin[x]]
```

Rubi [A] time = 0.0944992, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4392, 2839, 3770, 8}

$$x + \tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] x + ArcTanh[Sin[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{\sin(x)\tan(x)}{1-\cos(x)}\,dx\\ &= \int 1\,dx + \int \sec(x)\,dx\\ &= x + \tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] time = 0.0163073, size = 46, normalized size = 9.2

$$2\left(\frac{x}{2}-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(-Cot[x] + Csc[x]),x]

[Out] 2*(x/2 - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Cos[x/2] + Sin[x/2]]/2)
```

Maple [B] time = 0.095, size = 19, normalized size = 3.8

$$\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(-cot(x)+csc(x)),x)

[Out] ln(tan(1/2*x)+1)-ln(tan(1/2*x)-1)+x
```

Maxima [B] time = 1.67212, size = 53, normalized size = 10.6

$$2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(
cos(x) + 1) - 1)
```

Fricas [B] time = 0.487646, size = 65, normalized size = 13.

$$x + \frac{1}{2}\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\log\left(-\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\tan\left(x\right)}{\cot\left(x\right) - \csc\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(tan(x)/(cot(x) - csc(x)), x)
```

Giac [B] time = 1.13415, size = 27, normalized size = 5.4

$$x + \log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

3.212 $\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=16

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rubi [A] time = 0.0598725, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4392, 2735, 2648}

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)}{1-\cos(x)}\,dx$$
$$= -x + \int \frac{1}{1-\cos(x)}\,dx$$
$$= -x - \frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.0152774, size = 16, normalized size = 1.

$$\frac{1}{2}\left(-2x - 2\cot\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] (-2*x - 2*Cot[x/2])/2
```

Maple [A] time = 0.063, size = 13, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1} - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -1/tan(1/2*x)-x
```

Maxima [A] time = 1.64224, size = 31, normalized size = 1.94

$$-\frac{\cos(x)+1}{\sin(x)} - 2\,\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A] time = 0.464927, size = 45, normalized size = 2.81

$$-\frac{x\sin(x)+\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -(x*sin(x) + cos(x) + 1)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\cot(x)}{\cot(x)-\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(cot(x)/(cot(x) - csc(x)), x)
```

Giac [A] time = 1.09296, size = 16, normalized size = 1.

$$-x-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] -x - 1/tan(1/2*x)
```

3.213 $\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=13

$$\log(1-\cos(x))-\log(\cos(x))$$

```
[Out] Log[1 - Cos[x]] - Log[Cos[x]]
```

Rubi [A] time = 0.0605882, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4392, 2707, 36, 29, 31}

$$\log(1-\cos(x))-\log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] Log[1 - Cos[x]] - Log[Cos[x]]
```

Rule 4392

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{\tan(x)}{1-\cos(x)}\,dx\\
&= -\operatorname{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, -\cos(x)\right)\\
&= -\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, -\cos(x)\right) + \operatorname{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\cos(x)\right)\\
&= \log(1-\cos(x)) - \log(\cos(x))
\end{aligned}$$

Mathematica [A] time = 0.0098065, size = 25, normalized size = 1.92

$$2\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(1-2\sin^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] 2*Log[Sin[x/2]] - Log[1 - 2*Sin[x/2]^2]
```

Maple [A] time = 0.081, size = 6, normalized size = 0.5

$$\ln(\sec(x)-1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(-cot(x)+csc(x)),x)
```

```
[Out] ln(sec(x)-1)
```

Maxima [B] time = 1.09332, size = 55, normalized size = 4.23

$$-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)+2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1) + 2*log(sin(x)
/(cos(x) + 1))
```

Fricas [A] time = 0.479766, size = 54, normalized size = 4.15

$$-\log(-\cos(x))+\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -log(-cos(x)) + log(-1/2*cos(x) + 1/2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\sec(x)}{\cot(x)-\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(sec(x)/(cot(x) - csc(x)), x)
```

Giac [A] time = 1.10371, size = 19, normalized size = 1.46

$$\log(-\cos(x)+1)-\log(|\cos(x)|)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] log(-cos(x) + 1) - log(abs(cos(x)))
```

3.214 $\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -(Sin[x]/(1 - Cos[x]))
```

Rubi [A] time = 0.029823, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3166, 2648}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(-Cot[x] + Csc[x]),x]

[Out] -(Sin[x]/(1 - Cos[x]))
```

Rule 3166

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\csc(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{1}{1-\cos(x)}\,dx$$
$$= -\frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A] time = 0.0067818, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] -Cot[x/2]
```

Maple [A] time = 0.054, size = 9, normalized size = 0.8

$$-\left(\tan\left(\frac{x}{2}\right)\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -1/tan(1/2*x)
```

Maxima [A] time = 1.11778, size = 14, normalized size = 1.17

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A] time = 0.454914, size = 30, normalized size = 2.5

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\csc(x)}{\cot(x)-\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(csc(x)/(cot(x) - csc(x)), x)
```

Giac [A] time = 1.13066, size = 11, normalized size = 0.92

$$-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] -1/tan(1/2*x)
```

3.215 $\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}d}$$

```
[Out] -(ArcTanh[Cos[c + d*x]/Sqrt[2]]/(Sqrt[2]*d))
```

Rubi [A] time = 0.0386754, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {4397, 3186, 206}

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]/Sqrt[2]]/(Sqrt[2]*d))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\sin(c+dx)}{1+\sin^2(c+dx)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}d}$$

Mathematica [C] time = 0.184958, size = 61, normalized size = 2.65

$$-\frac{\tanh^{-1}\left(\frac{\cos(c)-(\sin(c)-i)\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{\cos(c)-(\sin(c)+i)\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
```

```
[Out] -((ArcTanh[(Cos[c] - (-I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]] + ArcTanh[(Cos[c]
 - (I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]])/(Sqrt[2]*d))
```

Maple [A] time = 0.077, size = 21, normalized size = 0.9

$$-\frac{\sqrt{2}}{2\,d}\text{Artanh}\left(\frac{\cos(dx+c)\sqrt{2}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] -1/2*arctanh(1/2*cos(d*x+c)*2^(1/2))/d*2^(1/2)
```

Maxima [B] time = 1.66338, size = 238, normalized size = 10.35

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}+1\right)\cos(dx+c)-\cos(dx+c)^2-\sin(dx+c)^2-2\sqrt{2}-3}{2\left(\sqrt{2}-1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2-2\sqrt{2}+3}\right)+\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-1\right)\cos(dx+c)-\cos(dx+c)^2-\sin(dx+c)^2+2\sqrt{2}-3}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*log(-(2*(sqrt(2) + 1)*cos(d*x + c) - cos(d*x + c)^2 - sin(d*x
+ c)^2 - 2*sqrt(2) - 3)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + si
n(d*x + c)^2 - 2*sqrt(2) + 3)) + sqrt(2)*log(-(2*(sqrt(2) - 1)*cos(d*x + c)
 - cos(d*x + c)^2 - sin(d*x + c)^2 + 2*sqrt(2) - 3)/(2*(sqrt(2) + 1)*cos(d*
x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)))/d
```

Fricas [B] time = 0.485088, size = 119, normalized size = 5.17

$$\frac{\sqrt{2}\log\left(-\frac{\cos(dx+c)^2-2\sqrt{2}\cos(dx+c)+2}{\cos(dx+c)^2-2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - 2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(1/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] time = 1.24742, size = 92, normalized size = 4.

$$\frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+6\right|}{\left|4\sqrt{2}-\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+6\right|}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
6)/abs(4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6))/d
```

3.216 $\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$-\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}d}-\frac{x}{\sqrt{2}}+x$$

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d
*x]^2)]/(Sqrt[2]*d)
```

Rubi [A] time = 0.172122, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1130, 203}

$$-\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}d}-\frac{x}{\sqrt{2}}+x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d
*x]^2)]/(Sqrt[2]*d)
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1+3x^2+2x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= x - \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}d}$$

Mathematica [A] time = 0.0722669, size = 30, normalized size = 0.59

$$-\frac{\tan^{-1}\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}d} + \frac{c}{d} + x$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] c/d + x - ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

Maple [A] time = 0.087, size = 30, normalized size = 0.6

$$-\frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(dx+c)\right)}{2\,d} + \frac{dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] -1/2/d*2^(1/2)*arctan(2^(1/2)*tan(d*x+c))+1/d*(d*x+c)
```

Maxima [B] time = 1.69157, size = 340, normalized size = 6.67

$$\frac{4\,dx - \sqrt{2}\arctan\left(\frac{2\sqrt{2}\sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}, \frac{\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right) + \sqrt{2}\arctan\left(\cdots\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*d*x - sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d*
x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^2
+ sin(d*x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + cos(
d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) + sqrt(2)*arctan2(2*sqrt(2)*s
in(d*x + c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2
 - 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) - 1)/(
2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqrt(2)
+ 3)) + 4*c)/d
```

Fricas [A] time = 0.49443, size = 140, normalized size = 2.75

$$\frac{4\,dx + \sqrt{2}\arctan\left(\frac{3\,\sqrt{2}\cos(dx+c)^2-2\,\sqrt{2}}{4\,\cos(dx+c)\sin(dx+c)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x + sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(cos
(d*x + c)*sin(d*x + c))))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin{(c + dx)}}{\sin{(c + dx)} + \csc{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.19908, size = 111, normalized size = 2.18

$$\frac{2\,dx - \sqrt{2}\left(dx + c + \arctan\left(-\frac{\sqrt{2}\sin\left(2\,dx+2\,c\right)-2\,\sin\left(2\,dx+2\,c\right)}{\sqrt{2}\cos\left(2\,dx+2\,c\right)+\sqrt{2}-2\,\cos\left(2\,dx+2\,c\right)+2}\right)\right) + 2\,c}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*d*x - sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(2
*d*x + 2*c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2))
) + 2*c)/d
```

3.217 $\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(\sin^2(c+dx)+1\right)}{2d}$$

```
[Out] Log[1 + Sin[c + d*x]^2]/(2*d)
```

Rubi [A] time = 0.0311779, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4334, 260}

$$\frac{\log\left(\sin^2(c+dx)+1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]^2]/(2*d)
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sin(c+dx)\right)}{d} = \frac{\log\left(1+\sin^2(c+dx)\right)}{2d}$$

Mathematica [A] time = 0.107006, size = 20, normalized size = 1.11

$$\frac{\log(3-\cos(2(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] Log[3 - Cos[2*(c + d*x)]]/(2*d)
```

Maple [A] time = 0.042, size = 17, normalized size = 0.9

$$\frac{\ln\left((\cos(dx+c))^2-2\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] 1/2/d*ln(cos(d*x+c)^2-2)
```

Maxima [A] time = 1.67404, size = 22, normalized size = 1.22

$$\frac{\log\left(\sin(dx+c)^2+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(d*x + c)^2 + 1)/d
```

Fricas [A] time = 0.492568, size = 43, normalized size = 2.39

$$\frac{\log\left(-\cos\left(dx+c\right)^2+2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-cos(d*x + c)^2 + 2)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.15153, size = 22, normalized size = 1.22

$$\frac{\log\left(\sin\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(d*x + c)^2 + 1)/d
```

3.218 $\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\tanh^{-1}(\sin(c+dx))}{2d}-\frac{\tan^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -ArcTan[Sin[c + d*x]]/(2*d) + ArcTanh[Sin[c + d*x]]/(2*d)
```

Rubi [A] time = 0.0375861, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {298, 203, 206}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2d}-\frac{\tan^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] -ArcTan[Sin[c + d*x]]/(2*d) + ArcTanh[Sin[c + d*x]]/(2*d)
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(c+dx)\right)}{2d}$$
$$= -\frac{\tan^{-1}(\sin(c+dx))}{2d} + \frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Mathematica [A] time = 0.0352153, size = 24, normalized size = 0.83

$$\frac{\tanh^{-1}(\sin(c+dx)) - \tan^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] (-ArcTan[Sin[c + d*x]] + ArcTanh[Sin[c + d*x]])/(2*d)
```

Maple [A] time = 0.102, size = 42, normalized size = 1.5

$$-\frac{\arctan(\sin(dx+c))}{2d} + \frac{\ln(1+\sin(dx+c))}{4d} - \frac{\ln(\sin(dx+c)-1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] -1/2*arctan(sin(d*x+c))/d+1/4/d*ln(1+sin(d*x+c))-1/4/d*ln(sin(d*x+c)-1)
```

Maxima [A] time = 1.61021, size = 47, normalized size = 1.62

$$-\frac{2\arctan(\sin(dx+c)) - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
))/d
```

Fricas [A] time = 0.519186, size = 111, normalized size = 3.83

$$-\frac{2\arctan\left(\sin\left(dx+c\right)\right)-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(-\sin\left(dx+c\right)+1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(-sin(d*x + c) +
1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.19726, size = 50, normalized size = 1.72

$$-\frac{2\arctan\left(\sin\left(dx+c\right)\right)-\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*x
 + c) - 1)))/d
```

3.219 $\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{\tan^{-1}(\sin(c+dx))}{d}$$

```
[Out] ArcTan[Sin[c + d*x]]/d
```

Rubi [A] time = 0.0253427, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4338, 203}

$$\frac{\tan^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sin[c + d*x]]/d
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\tan^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.0362957, size = 11, normalized size = 1.

$$\frac{\tan^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sin[c + d*x]]/d
```

Maple [A] time = 0.045, size = 13, normalized size = 1.2

$$-\frac{\arctan(\csc(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] -1/d*arctan(csc(d*x+c))
```

Maxima [A] time = 1.66518, size = 15, normalized size = 1.36

$$\frac{\arctan(\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

[Out] arctan(sin(d*x + c))/d

Fricas [A] time = 0.486142, size = 32, normalized size = 2.91

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] arctan(sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cot\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.13988, size = 15, normalized size = 1.36

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] arctan(sin(d*x + c))/d
```

3.220 $\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}$$

```
[Out] ArcTanh[Sin[c + d*x]^2]/(2*d)
```

Rubi [A] time = 0.0317371, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {275, 206}

$$\frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]^2]/(2*d)
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x}{1-x^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}$$

Mathematica [A] time = 0.0475111, size = 30, normalized size = 1.88

$$\frac{\log\left(2-\cos^2(c+dx)\right)-2\log(\cos(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] (-2*Log[Cos[c + d*x]] + Log[2 - Cos[c + d*x]^2])/(4*d)
```

Maple [A] time = 0.05, size = 19, normalized size = 1.2

$$\frac{\ln\left(2\,(\sec(dx+c))^2-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] 1/4/d*ln(2*sec(d*x+c)^2-1)
```

Maxima [B] time = 1.57702, size = 53, normalized size = 3.31

$$\frac{\log\left(\sin(dx+c)^2+1\right)-\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(log(sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
))/d
```

Fricas [B] time = 0.496821, size = 77, normalized size = 4.81

$$\frac{\log\left(-\cos\left(dx+c\right)^2+2\right)-2\,\log\left(-\cos\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(-cos(d*x + c)^2 + 2) - 2*log(-cos(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sec\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] time = 1.23052, size = 107, normalized size = 6.69

$$-\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\log\left(\left|-\frac{6\,(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right|\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - log(abs(-6*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1)))/d
```

3.221 $\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}d}+\frac{x}{\sqrt{2}}$$

```
[Out] x/Sqrt[2] + ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d*x]^
2)]/(Sqrt[2]*d)
```

Rubi [A] time = 0.10813, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {203}

$$\frac{\tan^{-1}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}d}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d*x]^
2)]/(Sqrt[2]*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}d}\end{aligned}$$

Mathematica [A] time = 0.0249839, size = 22, normalized size = 0.46

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

Maple [A] time = 0.07, size = 20, normalized size = 0.4

$$\frac{\sqrt{2}\arctan\left(\sqrt{2}\tan(dx+c)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] 1/2/d*2^(1/2)*arctan(2^(1/2)*tan(d*x+c))
```

Maxima [B] time = 1.83484, size = 331, normalized size = 6.9

$$\frac{\sqrt{2}\arctan\left(\frac{2\sqrt{2}\sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}, \frac{\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right)-\sqrt{2}\arctan\left(\frac{}{2\left(\sqrt{2}-1\right.}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d*x + c) +
 cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + cos(d*x + c)
^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) - sqrt(2)*arctan2(2*sqrt(2)*sin(d*x +
 c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqr
t(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) - 1)/(2*(sqrt(
```

```
2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqrt(2) + 3)))/d
```

Fricas [A] time = 0.485324, size = 128, normalized size = 2.67

$$-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(dx+c)^2-2\sqrt{2}}{4\cos(dx+c)\sin(dx+c)}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(cos(d*x + c
)*sin(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.18044, size = 97, normalized size = 2.02

$$\frac{\sqrt{2}\left(dx+c+\arctan\left(-\frac{\sqrt{2}\sin(2dx+2c)-2\sin(2dx+2c)}{\sqrt{2}\cos(2dx+2c)+\sqrt{2}-2\cos(2dx+2c)+2}\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(2*d*x + 2*
c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2)))/d
```

3.222 $\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=10

$$\frac{\sec(c+dx)}{d}$$

```
[Out] Sec[c + d*x]/d
```

Rubi [A] time = 0.0250854, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4397, 2606, 8}

$$\frac{\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]
```

```
[Out] Sec[c + d*x]/d
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx = \int \sec(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int 1\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{\sec(c+dx)}{d}$$

Mathematica [A] time = 0.0091558, size = 10, normalized size = 1.

$$\frac{\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]
```

```
[Out] Sec[c + d*x]/d
```

Maple [A] time = 0.076, size = 13, normalized size = 1.3

$$\frac{1}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] 1/d/cos(d*x+c)
```

Maxima [B] time = 1.04778, size = 38, normalized size = 3.8

$$-\frac{2}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A] time = 0.449018, size = 27, normalized size = 2.7

$$\frac{1}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(1/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] time = 1.10124, size = 38, normalized size = 3.8

$$\frac{2}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))
```

3.223 $\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=14

$$\frac{\tan(c+dx)}{d}-x$$

```
[Out] -x + Tan[c + d*x]/d
```

Rubi [A] time = 0.146682, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {321, 203}

$$\frac{\tan(c+dx)}{d}-x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -x + Tan[c + d*x]/d
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{d} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -x + \frac{\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0079127, size = 23, normalized size = 1.64

$$\frac{\tan(c+dx)}{d} - \frac{\tan^{-1}(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

Maple [A] time = 0.079, size = 24, normalized size = 1.7

$$\frac{\tan(dx+c)}{d} - \frac{\arctan(\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] tan(d*x+c)/d-1/d*arctan(tan(d*x+c))
```

Maxima [B] time = 1.62272, size = 86, normalized size = 6.14

$$-\frac{2\left(\frac{\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)} + \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*(sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) +
 1)) + arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [B] time = 0.470194, size = 72, normalized size = 5.14

$$-\frac{dx\cos(dx+c)-\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) - sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.12026, size = 24, normalized size = 1.71

$$-\frac{dx+c-\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(d*x + c - tan(d*x + c))/d
```

3.224 $\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(c+dx))}{d}$$

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rubi [A] time = 0.0306763, antiderivative size = 12, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4334, 260}

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rule 4334

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= -\frac{\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.007376, size = 12, normalized size = 1.

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Maple [A] time = 0.041, size = 13, normalized size = 1.1

$$-\frac{\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] -ln(cos(d*x+c))/d
```

Maxima [A] time = 1.11038, size = 32, normalized size = 2.67

$$-\frac{\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
```

Fricas [A] time = 0.487465, size = 31, normalized size = 2.58

$$-\frac{\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -log(-cos(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] time = 1.15391, size = 35, normalized size = 2.92

$$-\frac{\log(|\sin(dx+c)+1|)+\log(|\sin(dx+c)-1|)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*x + c) - 1)))/d
```

3.225 $\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.0391738, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {288, 206}

$$\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sec(c+dx)\tan(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{2d}$$
$$= -\frac{\tanh^{-1}(\sin(c+dx))}{2d} + \frac{\sec(c+dx)\tan(c+dx)}{2d}$$

Mathematica [A] time = 0.0156429, size = 34, normalized size = 1.

$$\frac{\tan(c+dx)\sec(c+dx)}{2d} - \frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -ArcTanh[Sin[c + d*x]]/(2*d) + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.103, size = 60, normalized size = 1.8

$$-\frac{1}{4\,d\,(1+\sin(dx+c))} - \frac{\ln(1+\sin(dx+c))}{4\,d} - \frac{1}{4\,d\,(\sin(dx+c)-1)} + \frac{\ln(\sin(dx+c)-1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] -1/4/d/(1+sin(d*x+c))-1/4/d*ln(1+sin(d*x+c))-1/4/d/(sin(d*x+c)-1)+1/4/d*ln(
sin(d*x+c)-1)
```

Maxima [A] time = 1.1188, size = 62, normalized size = 1.82

$$-\frac{\frac{2\sin(dx+c)}{\sin(dx+c)^2-1} + \log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1))/d
```

Fricas [B] time = 0.497837, size = 163, normalized size = 4.79

$$-\frac{\cos(dx+c)^2\log(\sin(dx+c)+1)-\cos(dx+c)^2\log(-\sin(dx+c)+1)-2\sin(dx+c)}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(d*x + c)^2*log(sin(d*x + c) + 1) - cos(d*x + c)^2*log(-sin(d*x +
c) + 1) - 2*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tan(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.19485, size = 65, normalized size = 1.91

$$-\frac{\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(|\sin(dx+c)+1|)-\log(|\sin(dx+c)-1|)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(abs(sin(d*x + c) + 1)) - lo
g(abs(sin(d*x + c) - 1)))/d
```

3.226 $\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] ArcTanh[Sin[c + d*x]]/d
```

Rubi [A] time = 0.0264199, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {4338, 206}

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/d
```

Rule 4338

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A] time = 0.0016969, size = 11, normalized size = 1.

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/d
```

Maple [A] time = 0.075, size = 12, normalized size = 1.1

$$\frac{\text{Artanh}\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] arctanh(sin(d*x+c))/d
```

Maxima [B] time = 1.07318, size = 35, normalized size = 3.18

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))/d
```

Fricas [B] time = 0.489127, size = 76, normalized size = 6.91

$$\frac{\log(\sin(dx+c)+1) - \log(-\sin(dx+c)+1)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] time = 1.18535, size = 38, normalized size = 3.45

$$\frac{\log(|\sin(dx+c)+1|) - \log(|\sin(dx+c)-1|)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)))/d
```

3.227 $\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=15

$$\frac{\sec^2(c+dx)}{2d}$$

```
[Out] Sec[c + d*x]^2/(2*d)
```

Rubi [A] time = 0.0317387, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {261}

$$\frac{\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^2/(2*d)
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{\left(1-x^2\right)^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\sec^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0117939, size = 15, normalized size = 1.

$$\frac{\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^2/(2*d)
```

Maple [A] time = 0.046, size = 14, normalized size = 0.9

$$\frac{(\sec(dx+c))^2}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] 1/2*sec(d*x+c)^2/d
```

Maxima [A] time = 1.09814, size = 23, normalized size = 1.53

$$-\frac{1}{2\left(\sin(dx+c)^2-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2/((sin(d*x + c)^2 - 1)*d)
```

Fricas [A] time = 0.480102, size = 32, normalized size = 2.13

$$\frac{1}{2d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] time = 1.14365, size = 62, normalized size = 4.13

$$-\frac{2(\cos(dx+c)-1)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(cos(d*x + c) - 1)/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2*(cos
(d*x + c) + 1))
```

3.228 $\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=10

$$\frac{\tan(c+dx)}{d}$$

```
[Out] Tan[c + d*x]/d
```

Rubi [A] time = 0.0862574, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {8}

$$\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Tan[c + d*x]/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int 1\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\tan(c+dx)}{d}$$

Mathematica [A] time = 0.0039757, size = 10, normalized size = 1.

$$\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Tan[c + d*x]/d
```

Maple [A] time = 0.067, size = 11, normalized size = 1.1

$$\frac{\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] tan(d*x+c)/d
```

Maxima [B] time = 1.05784, size = 59, normalized size = 5.9

$$-\frac{2\sin(dx+c)}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*sin(d*x + c)/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c)
+ 1))
```

Fricas [A] time = 0.447982, size = 42, normalized size = 4.2

$$\frac{\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A] time = 1.14395, size = 14, normalized size = 1.4

$$\frac{\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] tan(d*x + c)/d
```

3.229 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\cos^4(c+dx)}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.0619245, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4377, 12, 2565, 30}

$$-\frac{a\cos^4(c+dx)}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]^4)/(4*d)
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \cos^3(c+dx)\sin(c+dx)\,dx+\int b\cos^2(c+dx)\sin(c+dx)\,dx\\ &= b\int \cos^2(c+dx)\sin(c+dx)\,dx-\frac{a\,\text{Subst}\left(\int x^3\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{a\cos^4(c+dx)}{4d}-\frac{b\,\text{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{b\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.0120894, size = 33, normalized size = 1.

$$-\frac{a\cos^4(c+dx)}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -(b*Cos[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.032, size = 29, normalized size = 0.9

$$-\frac{1}{d}\left(\frac{(\cos(dx+c))^4a}{4}+\frac{b(\cos(dx+c))^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*(1/4*cos(d*x+c)^4*a+1/3*b*cos(d*x+c)^3)
```

Maxima [A] time = 1.10022, size = 38, normalized size = 1.15

$$-\frac{3\,a\cos\left(dx+c\right)^{4}+4\,b\cos\left(dx+c\right)^{3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

Fricas [A] time = 0.487187, size = 69, normalized size = 2.09

$$-\frac{3\,a\cos\left(dx+c\right)^{4}+4\,b\cos\left(dx+c\right)^{3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.230 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{b\sin^2(c+dx)}{2d}-\frac{a\cos^3(c+dx)}{3d}$$

```
[Out] -(a*Cos[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0538762, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4377, 12, 2564, 30, 2565}

$$\frac{b\sin^2(c+dx)}{2d}-\frac{a\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -(a*Cos[c + d*x]^3)/(3*d) + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \cos^2(c+dx)\sin(c+dx)\,dx+\int b\cos(c+dx)\sin(c+dx)\,dx\\
&= b\int \cos(c+dx)\sin(c+dx)\,dx-\frac{a\,\mathrm{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{d}\\
&= -\frac{a\cos^3(c+dx)}{3d}+\frac{b\,\mathrm{Subst}(\int x\,dx,x,\sin(c+dx))}{d}\\
&= -\frac{a\cos^3(c+dx)}{3d}+\frac{b\sin^2(c+dx)}{2d}
\end{aligned}
$$

Mathematica [A] time = 0.11502, size = 38, normalized size = 1.15

$$
-\frac{3a\cos(c+dx)+a\cos(3(c+dx))+3b\cos(2(c+dx))}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -(3*a*Cos[c + d*x] + 3*b*Cos[2*(c + d*x)] + a*Cos[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.03, size = 29, normalized size = 0.9

$$
-\frac{1}{d}\left(\frac{(\cos(dx+c))^3 a}{3}+\frac{b(\cos(dx+c))^2}{2}\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*(1/3*cos(d*x+c)^3*a+1/2*b*cos(d*x+c)^2)
```

Maxima [A] time = 1.06575, size = 38, normalized size = 1.15

$$-\frac{2\,a\cos\left(dx+c\right)^{3}-3\,b\sin\left(dx+c\right)^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^3 - 3*b*sin(d*x + c)^2)/d
```

Fricas [A] time = 0.486196, size = 68, normalized size = 2.06

$$-\frac{2\,a\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] time = 1.21127, size = 134, normalized size = 4.06

$$-\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d}-\frac{b\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-b\tan\left(dx\right)^{2}-4\,b\tan\left(dx\right)\tan\left(c\right)-b\tan\left(c\right)^{2}+b}{4\left(d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+d\tan\left(dx\right)^{2}+d\tan\left(c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/4*(b*tan(d*x)^2*tan(c
)^2 - b*tan(d*x)^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 + b)/(d*tan(d*x)^2*ta
n(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

3.231 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=22

$$-\frac{(a\cos(c+dx)+b)^2}{2ad}$$

```
[Out] -(b + a*Cos[c + d*x])^2/(2*a*d)
```

Rubi [A] time = 0.0302323, antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4377, 12, 2638, 2564, 30}

$$\frac{a\sin^2(c+dx)}{2d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \cos(c+dx)\sin(c+dx)\,dx+\int b\sin(c+dx)\,dx\\ &= b\int \sin(c+dx)\,dx+\frac{a\,\text{Subst}\left(\int x\,dx,x,\sin(c+dx)\right)}{d}\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0107171, size = 40, normalized size = 1.82

$$-\frac{a\cos^2(c+dx)}{2d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) - (a*Cos[c + d*x]^2)/(2*d) + (b*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.023, size = 26, normalized size = 1.2

$$-\frac{1}{d}\left(\frac{(\cos(dx+c))^2 a}{2}+b\cos(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d*(1/2*cos(d*x+c)^2*a+b*cos(d*x+c))
```

Maxima [A] time = 1.10742, size = 34, normalized size = 1.55

$$-\frac{a\cos\left(dx+c\right)^{2}+2\,b\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

Fricas [A] time = 0.477275, size = 62, normalized size = 2.82

$$-\frac{a\cos\left(dx+c\right)^{2}+2\,b\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x), x)
```

Giac [B] time = 1.19092, size = 138, normalized size = 6.27

$$-\frac{a\cos(2\,dx+2\,c)}{4\,d}-\frac{b\tan\left(\frac{1}{2}\,dx\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,c\right)^{2}+b}{d\tan\left(\frac{1}{2}\,dx\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+d\tan\left(\frac{1}{2}\,dx\right)^{2}+d\tan\left(\frac{1}{2}\,c\right)^{2}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*a*cos(2*d*x + 2*c)/d - (b*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*tan(1/2*d*x)
^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b*tan(1/2*c)^2 + b)/(d*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

3.232 $\int (a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0130983, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {2638, 3475}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Sin[c + d*x] + b*Tan[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \sin(c+dx)\,dx+b\int \tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A] time = 0.01777, size = 37, normalized size = 1.42

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sin[c + d*x] + b*Tan[c + d*x],x]
```

```
[Out] -((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.014, size = 31, normalized size = 1.2

$$-\frac{a\cos(dx+c)}{d}+\frac{b\ln\left((\tan(dx+c))^{2}+1\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sin(d*x+c)+b*tan(d*x+c),x)
```

```
[Out] -a*cos(d*x+c)/d+1/2*b/d*ln(tan(d*x+c)^2+1)
```

Maxima [A] time = 1.06457, size = 34, normalized size = 1.31

$$-\frac{a\cos(dx+c)}{d}+\frac{b\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] -a*cos(d*x + c)/d + b*log(sec(d*x + c))/d
```

Fricas [A] time = 0.501073, size = 59, normalized size = 2.27

$$-\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

Sympy [A] time = 0.560458, size = 37, normalized size = 1.42

$$a\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for } d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}\frac{\log\left(\tan^2(c+dx)+1\right)}{2d} & \text{for } d\neq 0\\ x\tan(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x)
```

```
[Out] a*Piecewise((-cos(c + d*x)/d, Ne(d, 0)), (x*sin(c), True)) + b*Piecewise((l
og(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

Giac [A] time = 1.12648, size = 36, normalized size = 1.38

$$-\frac{a\cos(dx+c)}{d}-\frac{b\log(|\cos(dx+c)|)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -a*cos(d*x + c)/d - b*log(abs(cos(d*x + c)))/d
```

3.233 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=25

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rubi [A] time = 0.0280599, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4377, 12, 2606, 8, 3475}

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \tan(c+dx)\,dx+\int b\sec(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\log(\cos(c+dx))}{d}+b\int \sec(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\,\text{Subst}(\int 1\,dx,x,\sec(c+dx))}{d}\\ &= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0130665, size = 25, normalized size = 1.

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Maple [A] time = 0.028, size = 25, normalized size = 1.

$$\frac{b\sec(dx+c)}{d}+\frac{a\ln(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] b*sec(d*x+c)/d+1/d*a*ln(sec(d*x+c))
```

Maxima [A] time = 1.11643, size = 43, normalized size = 1.72

$$-\frac{a \log\left(-\sin\left(dx+c\right)^2+1\right)-\frac{2\,b}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(-sin(d*x + c)^2 + 1) - 2*b/cos(d*x + c))/d
```

Fricas [A] time = 0.50081, size = 80, normalized size = 3.2

$$-\frac{a \cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x), x)
```

Giac [B] time = 1.16685, size = 144, normalized size = 5.76

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a+2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.234 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

```
[Out] (a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0501234, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4377, 12, 2606, 30, 8}

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \sec(c+dx)\tan(c+dx)\,dx+\int b\sec^2(c+dx)\tan(c+dx)\,dx\\ &= b\int \sec^2(c+dx)\tan(c+dx)\,dx+\frac{a\,\mathrm{Subst}\left(\int 1\,dx,x,\sec(c+dx)\right)}{d}\\ &= \frac{a\sec(c+dx)}{d}+\frac{b\,\mathrm{Subst}\left(\int x\,dx,x,\sec(c+dx)\right)}{d}\\ &= \frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0197068, size = 28, normalized size = 1.

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

Maple [A] time = 0.03, size = 25, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b\left(\sec(dx+c)\right)^2}{2}+a\sec(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/2*b*sec(d*x+c)^2+a*sec(d*x+c))
```

Maxima [A] time = 1.02535, size = 36, normalized size = 1.29

$$\frac{b\tan\left(dx+c\right)^2+\frac{2a}{\cos(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a/cos(d*x + c))/d
```

Fricas [A] time = 0.468432, size = 63, normalized size = 2.25

$$\frac{2a\cos\left(dx+c\right)+b}{2d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*cos(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] time = 1.1582, size = 96, normalized size = 3.43

$$\frac{2\left(a+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*(a + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)
```

3.235 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] (a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0569483, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4377, 12, 2606, 30}

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \sec^2(c+dx)\tan(c+dx)\,dx+\int b\sec^3(c+dx)\tan(c+dx)\,dx\\ &= b\int \sec^3(c+dx)\tan(c+dx)\,dx+\frac{a\,\mathrm{Subst}(\int x\,dx,x,\sec(c+dx))}{d}\\ &= \frac{a\sec^2(c+dx)}{2d}+\frac{b\,\mathrm{Subst}\left(\int x^2\,dx,x,\sec(c+dx)\right)}{d}\\ &= \frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0225567, size = 33, normalized size = 1.

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

Maple [A] time = 0.033, size = 28, normalized size = 0.9

$$\frac{1}{d}\left(\frac{(\sec(dx+c))^3 b}{3}+\frac{a(\sec(dx+c))^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d*(1/3*sec(d*x+c)^3*b+1/2*a*sec(d*x+c)^2)
```

Maxima [A] time = 1.05449, size = 43, normalized size = 1.3

$$-\frac{\frac{3\,a}{\sin(dx+c)^2-1}-\frac{2\,b}{\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*a/(sin(d*x + c)^2 - 1) - 2*b/cos(d*x + c)^3)/d
```

Fricas [A] time = 0.471809, size = 66, normalized size = 2.

$$\frac{3\,a\cos\left(dx+c\right)+2\,b}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] time = 1.16724, size = 131, normalized size = 3.97

$$\frac{2\left(b-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{3\,d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(b - 3*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(d
*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)
```

3.236 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=106

$$\frac{(4a^2+b^2)\sin^3(c+dx)}{30d}+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}+\frac{b\sin^3(c+dx)(a\cos(c+dx)+b)}{10d}-\frac{ab\sin(c+dx)\cos(c+d}{4d}$$

```
[Out] (a*b*x)/4 - (a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((4*a^2 + b^2)*Sin[c +
d*x]^3)/(30*d) + (b*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(10*d) + ((b + a*C
os[c + d*x])^2*Sin[c + d*x]^3)/(5*d)
```

Rubi [A] time = 0.348789, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2862, 2669, 2635, 8}

$$\frac{(4a^2+b^2)\sin^3(c+dx)}{30d}+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}+\frac{b\sin^3(c+dx)(a\cos(c+dx)+b)}{10d}-\frac{ab\sin(c+dx)\cos(c+d}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*x)/4 - (a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((4*a^2 + b^2)*Sin[c +
d*x]^3)/(30*d) + (b*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(10*d) + ((b + a*C
os[c + d*x])^2*Sin[c + d*x]^3)/(5*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2862

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*(
g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^m)/(f*g*(m + p + 1)), x] + Dis
t[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp[a
*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] &&
  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Simp
lerQ[c + d*x, a + b*x])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int \cos(c+dx)(b+a\cos(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sin^3(c+dx)}{5d} + \frac{1}{5}\int (b+a\cos(c+dx))(2a+2$$
$$= \frac{b(b+a\cos(c+dx))\sin^3(c+dx)}{10d} + \frac{(b+a\cos(c+dx))^2\sin^3(c+d$$
$$= \frac{\left(4a^2+b^2\right)\sin^3(c+dx)}{30d} + \frac{b(b+a\cos(c+dx))\sin^3(c+dx)}{10d} + \frac{(b+$$
$$= -\frac{ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{\left(4a^2+b^2\right)\sin^3(c+dx)}{30d} + \frac{b(b+a\cos$$
$$= \frac{abx}{4} - \frac{ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{\left(4a^2+b^2\right)\sin^3(c+dx)}{30d} + \frac{b(b$$

Mathematica [A] time = 0.396168, size = 77, normalized size = 0.73

$$\frac{30\left(a^2+2b^2\right)\sin(c+dx)-5\left(a^2+4b^2\right)\sin(3(c+dx))-3a(a\sin(5(c+dx))-20b(c+dx)+5b\sin(4(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (30*(a^2 + 2*b^2)*Sin[c + d*x] - 5*(a^2 + 4*b^2)*Sin[3*(c + d*x)] - 3*a*(-2
0*b*(c + d*x) + 5*b*Sin[4*(c + d*x)] + a*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.063, size = 100, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^4}{5}+\frac{\left(2+(\cos(dx+c))^2\right)\sin(dx+c)}{15}\right)+2\,ab\left(-1/4\,\sin(dx+c)(\cos(dx+c))^3+1/8\,\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+2*
a*b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+
1/3*b^2*sin(d*x+c)^3)
```

Maxima [A] time = 1.11882, size = 92, normalized size = 0.87

$$\frac{80\,b^2\sin(dx+c)^3-16\left(3\,\sin(dx+c)^5-5\,\sin(dx+c)^3\right)a^2+15\,(4\,dx+4\,c-\sin(4\,dx+4\,c))ab}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(80*b^2*sin(d*x + c)^3 - 16*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2
 + 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b)/d
```

Fricas [A] time = 0.4998, size = 211, normalized size = 1.99

$$\frac{15\,abdx-\left(12\,a^2\cos(dx+c)^4+30\,ab\cos(dx+c)^3-15\,ab\cos(dx+c)-4\left(a^2-5\,b^2\right)\cos(dx+c)^2-8\,a^2-20\,b^2\right)\sin(d}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*b*d*x - (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 - 15*a*b*
cos(d*x + c) - 4*(a^2 - 5*b^2)*cos(d*x + c)^2 - 8*a^2 - 20*b^2)*sin(d*x + c
))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\cos^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

3.237 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=86

$$-\frac{(a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(a^2+4b^2\right)+\frac{5ab\sin^3(c+dx)}{12d}+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

```
[Out] ((a^2 + 4*b^2)*x)/8 - ((a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*
a*b*Sin[c + d*x]^3)/(12*d) + (a*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.189586, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2692, 2669, 2635, 8}

$$-\frac{(a^2+4b^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(a^2+4b^2\right)+\frac{5ab\sin^3(c+dx)}{12d}+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```



```
[Out] ((a^2 + 4*b^2)*x)/8 - ((a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*
a*b*Sin[c + d*x]^3)/(12*d) + (a*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(4*d)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2692

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m - 1))/(f*g*(m + p)), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(a
+ b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*Si
n[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] &&
 GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
```

```
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx &= \int (b+a\cos(c+dx))^2\sin^2(c+dx)\,dx\\ &= \frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d}+\frac{1}{4}\int \left(a^2+4b^2+5ab\cos(c+d\right.\\ &= \frac{5ab\sin^3(c+dx)}{12d}+\frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d}+\frac{1}{4}\left(a^2+4b^2\right.\\ &= -\frac{\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{5ab\sin^3(c+dx)}{12d}+\frac{a(b+a\,c}{}\\ &= \frac{1}{8}\left(a^2+4b^2\right)x-\frac{\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{5ab\sin^3(c+}{12d}\end{aligned}$$

Mathematica [A] time = 0.222586, size = 82, normalized size = 0.95

$$\frac{-3a^2\sin(4(c+dx))+12a^2c+12a^2dx+48ab\sin(c+dx)-16ab\sin(3(c+dx))-24b^2\sin(2(c+dx))+48b^2c+48b^2dx}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (12*a^2*c + 48*b^2*c + 12*a^2*d*x + 48*b^2*d*x + 48*a*b*Sin[c + d*x] - 24*b
^2*Sin[2*(c + d*x)] - 16*a*b*Sin[3*(c + d*x)] - 3*a^2*Sin[4*(c + d*x)])/(96
*d)
```

Maple [A] time = 0.059, size = 86, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\frac{\sin(dx+c)(\cos(dx+c))^3}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+\frac{2ab(\sin(dx+c))^3}{3}+b^2\left(-\frac{\cos(dx+c)\sin(}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)+2/3*a*b*sin(d*x+c)^3+b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.0166, size = 89, normalized size = 1.03

$$\frac{64\,ab\sin(dx+c)^3+3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2+24\,(2\,dx+2\,c-\sin(2\,dx+2\,c))b^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(64*a*b*sin(d*x + c)^3 + 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 + 24*(
2*d*x + 2*c - sin(2*d*x + 2*c))*b^2)/d
```

Fricas [A] time = 0.498968, size = 178, normalized size = 2.07

$$\frac{3\left(a^2+4\,b^2\right)dx-\left(6\,a^2\cos(dx+c)^3+16\,ab\cos(dx+c)^2-16\,ab-3\left(a^2-4\,b^2\right)\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 + 4*b^2)*d*x - (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2 -
 16*a*b - 3*(a^2 - 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

Giac [B] time = 7.74578, size = 6967, normalized size = 81.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*a^2*x - 1/32*a^2*sin(4*d*x + 4*c)/d + 1/6*(3*b^2*d*x*tan(d*x)^2*tan(1/2
*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2
*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d
*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(1/2*d*x)
^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan
(c) + 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(d
*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan
(1/2*c)^6 + 3*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*ta
n(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*t
an(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 9*b
^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(1/2*
d*x)^4*tan(1/2*c)^6*tan(c)^2 - 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6 +
 9*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c) + 9*b^2*tan(d*x)^2*tan
(1/2*d*x)^4*tan(1/2*c)^6*tan(c) - 3*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)
- 32*a*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^3*tan(c)^2 - 96*a*b*tan(d*x)^
2*tan(1/2*d*x)^5*tan(1/2*c)^4*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(
1/2*c)^4*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^5*tan(c)^2
- 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^6*tan(c)^2 + 9*b^2*tan(d*x)*t
an(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*t
an(1/2*c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 9*b^2*d*x
*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*
c)^6 + 9*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*b^2*d*x*tan(d*x)^2*tan(1/2
*d*x)^6*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)
```

^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2
*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c
)^4*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(1
/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*
c)^3 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^4 - 9*b^2*tan(d*x)*tan(1
/2*d*x)^6*tan(1/2*c)^4 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^5 - 32
*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^6 - 9*b^2*tan(d*x)*tan(1/2*d*x)^4
*tan(1/2*c)^6 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c) + 27*b^
2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 9*b^2*tan(1/2*d*x)^6*tan(
1/2*c)^4*tan(c) + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c) - 9*b
^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c) + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*ta
n(1/2*c)^2*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 +
 288*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^3*tan(c)^2 - 32*a*b*tan(1/2*d
*x)^6*tan(1/2*c)^3*tan(c)^2 + 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^
4*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 96*a*b*
tan(1/2*d*x)^5*tan(1/2*c)^4*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan
(1/2*c)^5*tan(c)^2 - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c)^5*tan(c)^2 + 9*b^2*ta
n(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 - 32*a*b*tan(1/2*d*x)^3*tan(1/2
*c)^6*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6 + 27*b^2*d*x*tan(d*x)^
2*tan(1/2*d*x)^4*tan(1/2*c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*
b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4 + 27*b^2*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^6 + 9*b^2*d*x*tan(1/2*d*x)^2
*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 3*b^2*d*x*ta
n(1/2*d*x)^6*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 9*b^2*d*x*tan(d
*x)^2*tan(1/2*c)^4*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)
^2 + 3*b^2*d*x*tan(1/2*c)^6*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan
(1/2*c)^2 - 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 288*a*b*tan(d*x)^2
*tan(1/2*d*x)^4*tan(1/2*c)^3 - 32*a*b*tan(1/2*d*x)^6*tan(1/2*c)^3 + 288*a*b
*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 27*b^2*tan(d*x)*tan(1/2*d*x)^4*ta
n(1/2*c)^4 - 96*a*b*tan(1/2*d*x)^5*tan(1/2*c)^4 + 96*a*b*tan(d*x)^2*tan(1/2
*d*x)^2*tan(1/2*c)^5 - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c)^5 - 9*b^2*tan(d*x)*
tan(1/2*d*x)^2*tan(1/2*c)^6 - 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)^6 + 3*b^2*ta
n(d*x)^2*tan(1/2*d*x)^6*tan(c) + 27*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c
)^2*tan(c) - 9*b^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c) + 27*b^2*tan(d*x)^2*t
an(1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 27*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(
c) + 3*b^2*tan(d*x)^2*tan(1/2*c)^6*tan(c) - 9*b^2*tan(1/2*d*x)^2*tan(1/2*c)
^6*tan(c) + 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(
1/2*d*x)^4*tan(1/2*c)*tan(c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*
c)^2*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 96*a
*b*tan(1/2*d*x)^5*tan(1/2*c)^2*tan(c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^3*tan(c)^2 + 288*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3*tan(c)^2 - 96*
a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2
*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 288*a*b*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^
2 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 + 3*b^2*tan(d*x)*tan(1/2*c)

^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4 + 3*b^2*d*x*tan(1/2*d*x)^
6 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 27*b^2*d*x*tan(1/2*
d*x)^4*tan(1/2*c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*c)^4 + 27*b^2*d*x*tan(1/
2*d*x)^2*tan(1/2*c)^4 + 3*b^2*d*x*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^2*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^4*tan(c)^2 + 9*b^2*d*x*tan(d*x)
^2*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2
+ 9*b^2*d*x*tan(1/2*c)^4*tan(c)^2 - 3*b^2*tan(d*x)*tan(1/2*d*x)^6 - 96*a*b*
tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c) - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 - 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^2 + 96*a*b*tan(1/2*d
*x)^5*tan(1/2*c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^3 + 288*a
*b*tan(1/2*d*x)^4*tan(1/2*c)^3 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^
4 - 27*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^4 + 288*a*b*tan(1/2*d*x)^3*ta
n(1/2*c)^4 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)^5 - 3*b^2*tan(d*x)*tan(1/2*c)
^6 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(c) - 3*b^2*tan(1/2*d*x)^6*tan(c) +
27*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 27*b^2*tan(1/2*d*x)
^4*tan(1/2*c)^2*tan(c) + 9*b^2*tan(d*x)^2*tan(1/2*c)^4*tan(c) - 27*b^2*tan(
1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 3*b^2*tan(1/2*c)^6*tan(c) + 32*a*b*tan(d*x
)^2*tan(1/2*d*x)^3*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(c)^2 + 96*a
*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 - 96*a*b*tan(1/2*d*x)^4*ta
n(1/2*c)*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 +
27*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - 288*a*b*tan(1/2*d*x)
^3*tan(1/2*c)^2*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*c)^3*tan(c)^2 - 288*a*
b*tan(1/2*d*x)^2*tan(1/2*c)^3*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*c)^4*tan(c)
^2 - 96*a*b*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1
/2*d*x)^2 + 9*b^2*d*x*tan(1/2*d*x)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*c)^2 +
27*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 9*b^2*d*x*tan(1/2*c)^4 + 3*b^2*d*x
*tan(d*x)^2*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^2*tan(c)^2 + 9*b^2*d*x*tan(1/
2*c)^2*tan(c)^2 + 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3 - 9*b^2*tan(d*x)*tan(1/2
*d*x)^4 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c) - 96*a*b*tan(1/2*d*x)
^4*tan(1/2*c) + 96*a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2 - 27*b^2*tan(d*
x)*tan(1/2*d*x)^2*tan(1/2*c)^2 - 288*a*b*tan(1/2*d*x)^3*tan(1/2*c)^2 + 32*a
*b*tan(d*x)^2*tan(1/2*c)^3 - 288*a*b*tan(1/2*d*x)^2*tan(1/2*c)^3 - 9*b^2*ta
n(d*x)*tan(1/2*c)^4 - 96*a*b*tan(1/2*d*x)*tan(1/2*c)^4 + 9*b^2*tan(d*x)^2*t
an(1/2*d*x)^2*tan(c) - 9*b^2*tan(1/2*d*x)^4*tan(c) + 9*b^2*tan(d*x)^2*tan(1
/2*c)^2*tan(c) - 27*b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - 9*b^2*tan(1/2*
c)^4*tan(c) + 9*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(c)^2 + 32*a*b*tan(1/2*d*x)^
3*tan(c)^2 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 + 9*b^2*tan(d*x)*tan
(1/2*c)^2*tan(c)^2 + 96*a*b*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 + 32*a*b*tan
(1/2*c)^3*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2 + 9*b^2*d*x*tan(1/2*d*x)^2 + 9*b^
2*d*x*tan(1/2*c)^2 + 3*b^2*d*x*tan(c)^2 - 9*b^2*tan(d*x)*tan(1/2*d*x)^2 + 3
2*a*b*tan(1/2*d*x)^3 + 96*a*b*tan(1/2*d*x)^2*tan(1/2*c) - 9*b^2*tan(d*x)*ta
n(1/2*c)^2 + 96*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + 32*a*b*tan(1/2*c)^3 + 3*b^2
*tan(d*x)^2*tan(c) - 9*b^2*tan(1/2*d*x)^2*tan(c) - 9*b^2*tan(1/2*c)^2*tan(c
) + 3*b^2*tan(d*x)*tan(c)^2 + 3*b^2*d*x - 3*b^2*tan(d*x) - 3*b^2*tan(c))/(d
*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x

)^6*tan(1/2*c)^6 + 3*d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 3*
d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + d*tan(1/2*d*x)^6*tan(1/
2*c)^6*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(d*x)
^2*tan(1/2*d*x)^4*tan(1/2*c)^6 + d*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*d*tan(d*
x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^4*t
an(1/2*c)^4*tan(c)^2 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 3*d*tan(d
*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x)^4*tan(1/2*c)^
6*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*tan(d*x)^2*ta
n(1/2*d*x)^4*tan(1/2*c)^4 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^4 + 3*d*tan(d*x)^
2*tan(1/2*d*x)^2*tan(1/2*c)^6 + 3*d*tan(1/2*d*x)^4*tan(1/2*c)^6 + d*tan(d*x
)^2*tan(1/2*d*x)^6*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^2*ta
n(c)^2 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*
d*x)^2*tan(1/2*c)^4*tan(c)^2 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + d
*tan(d*x)^2*tan(1/2*c)^6*tan(c)^2 + 3*d*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^
2 + d*tan(d*x)^2*tan(1/2*d*x)^6 + 9*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^
2 + 3*d*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2
*c)^4 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^4 + d*tan(d*x)^2*tan(1/2*c)^6 + 3*d*t
an(1/2*d*x)^2*tan(1/2*c)^6 + 3*d*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + d*tan
(1/2*d*x)^6*tan(c)^2 + 9*d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2
+ 9*d*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*c)^4*ta
n(c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + d*tan(1/2*c)^6*tan(c)^2
 + 3*d*tan(d*x)^2*tan(1/2*d*x)^4 + d*tan(1/2*d*x)^6 + 9*d*tan(d*x)^2*tan(1/
2*d*x)^2*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^4*tan(1/2*c)^2 + 3*d*tan(d*x)^2*ta
n(1/2*c)^4 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^4 + d*tan(1/2*c)^6 + 3*d*tan(d*x
)^2*tan(1/2*d*x)^2*tan(c)^2 + 3*d*tan(1/2*d*x)^4*tan(c)^2 + 3*d*tan(d*x)^2*
tan(1/2*c)^2*tan(c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 3*d*tan(
1/2*c)^4*tan(c)^2 + 3*d*tan(d*x)^2*tan(1/2*d*x)^2 + 3*d*tan(1/2*d*x)^4 + 3*
d*tan(d*x)^2*tan(1/2*c)^2 + 9*d*tan(1/2*d*x)^2*tan(1/2*c)^2 + 3*d*tan(1/2*c
)^4 + d*tan(d*x)^2*tan(c)^2 + 3*d*tan(1/2*d*x)^2*tan(c)^2 + 3*d*tan(1/2*c)^
2*tan(c)^2 + d*tan(d*x)^2 + 3*d*tan(1/2*d*x)^2 + 3*d*tan(1/2*c)^2 + d*tan(c
)^2 + d)

3.238 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\sin(c+dx)\cos(c+dx)}{3d}-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}+abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/(3*d)
- (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) - ((b + a*Cos[c + d*x])^2*Sin[c + d
*x])/(3*d)
```

Rubi [A] time = 0.321384, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4397, 2889, 3050, 3033, 3023, 2735, 3770}

$$\frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\sin(c+dx)\cos(c+dx)}{3d}-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}+abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/(3*d)
- (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) - ((b + a*Cos[c + d*x])^2*Sin[c + d
*x])/(3*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
```

```
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sin(c+dx)\tan(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= -\frac{(b+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (b+a\cos(c+dx))\left(3b+a\right.$$
$$= -\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-\frac{(b+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{6}$$
$$= \frac{\left(a^2-2b^2\right)\sin(c+dx)}{3d}-\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-\frac{(b+a\cos(c+}{}$$
$$= abx+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{3d}-\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-\frac{(b+a\,c}{}$$
$$= abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{\left(a^2-2b^2\right)\sin(c+dx)}{3d}-\frac{ab\cos(c+}{}$$

Mathematica [A] time = 0.152097, size = 117, normalized size = 1.34

$$\frac{3\left(a^2-4b^2\right)\sin(c+dx)+a^2(-\sin(3(c+dx)))-6ab\sin(2(c+dx))+12abc+12abdx-12b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (12*a*b*c + 12*a*b*d*x - 12*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(a^2 - 4*b^2)*Sin[c + d
*x] - 6*a*b*Sin[2*(c + d*x)] - a^2*Sin[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.052, size = 83, normalized size = 1.

$$\frac{a^2(\sin(dx+c))^3}{3d}-\frac{ab\cos(dx+c)\sin(dx+c)}{d}+abx+\frac{abc}{d}-\frac{b^2\sin(dx+c)}{d}+\frac{b^2\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/3*a^2*sin(d*x+c)^3/d-a*b*cos(d*x+c)*sin(d*x+c)/d+a*b*x+1/d*a*b*c-b^2*sin(
d*x+c)/d+1/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.1129, size = 103, normalized size = 1.18

$$\frac{2\,a^2\sin\left(dx+c\right)^3+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)ab+3\,b^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right)\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^2*sin(d*x + c)^3 + 3*(2*d*x + 2*c - sin(2*d*x + 2*c))*a*b + 3*b^2*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A] time = 0.528936, size = 207, normalized size = 2.38

$$\frac{6\,abdx+3\,b^2\log\left(\sin\left(dx+c\right)+1\right)-3\,b^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2\cos\left(dx+c\right)^2+3\,ab\cos\left(dx+c\right)-a^2+3\,b^2\right)\sin}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(6*a*b*d*x + 3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1)
 - 2*(a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) - a^2 + 3*b^2)*sin(d*x + c))/
d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x), x)

Giac [B] time = 5.72068, size = 7713, normalized size = 88.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] -1/12*a^2*sin(3*d*x + 3*c)/d + 1/4*a^2*sin(d*x + c)/d + 1/2*(2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1

/2*c)*tan(c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c) - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(1/2*c)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan

(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*
tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*a*b*tan(d*x)*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(c) + 2*a*b*tan(d*x)^2*
tan(1/2*c)^2*tan(c) - 2*a*b*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*log(2*
(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*
c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(
1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^
2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2*tan(c)^2
+ b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)
^2*tan(c)^2 - 4*b^2*tan(d*x)^2*tan(1/2*d*x)*tan(c)^2 - b^2*log(2*(tan(1/2*c
)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(c)^2 + b^2*
log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*ta
n(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^
2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2
*tan(c)^2 + 2*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(c)^2 - 4*b^2*tan(d*x)^2*tan(1
/2*c)*tan(c)^2 + 4*b^2*tan(1/2*d*x)^2*tan(1/2*c)*tan(c)^2 - b^2*log(2*(tan(
1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2*tan(c)^2 + b
^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4
*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^
2*tan(c)^2 + 2*a*b*tan(d*x)*tan(1/2*c)^2*tan(c)^2 + 4*b^2*tan(1/2*d*x)*tan(
1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2 + 2*a*b*d*x*tan(1/2*d*x)^2 + 2*a*b
*d*x*tan(1/2*c)^2 + 2*a*b*d*x*tan(c)^2 - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(
1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*ta
n(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x
)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan
(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)^2 - 4*b^2*tan(d*x)^2*tan(1/2*d*x
) - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d

*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1
/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2
*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/
2*d*x)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 +
2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))
*tan(1/2*d*x)^2 - 2*a*b*tan(d*x)*tan(1/2*d*x)^2 - 4*b^2*tan(d*x)^2*tan(1/2*
c) + 4*b^2*tan(1/2*d*x)^2*tan(1/2*c) - b^2*log(2*(tan(1/2*c)^2 + 1)/(tan(1/
2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(
1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^
3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1
/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1)/(t
an(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3
*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*
tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^2 - 2*a*b*tan(d*x)*tan(1/2*c)^
2 + 4*b^2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*a*b*tan(d*x)^2*tan(c) - 2*a*b*tan(1
/2*d*x)^2*tan(c) - 2*a*b*tan(1/2*c)^2*tan(c) - b^2*log(2*(tan(1/2*c)^2 + 1)
/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x
)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1
/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 -
 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(c)^2 + b^2*log(2*(tan(1/2*c)^2 + 1
)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*
x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2
+ 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(c)^2 + 2*a*b*tan(d*x)*tan(c)^2 -
4*b^2*tan(1/2*d*x)*tan(c)^2 - 4*b^2*tan(1/2*c)*tan(c)^2 + 2*a*b*d*x - b^2*l
og(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan
(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2
*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*
d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)) + b^2*log(2*(ta
n(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c)
- 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)) - 2*a*b*tan(d*x) - 4*b^
2*tan(1/2*d*x) - 4*b^2*tan(1/2*c) - 2*a*b*tan(c))/(d*tan(d*x)^2*tan(1/2*d*x
)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*ta
n(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*t
an(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2*tan(1/2*d*x)^2 + d*tan(d
*x)^2*tan(1/2*c)^2 + d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)^2*tan(c)^2
+ d*tan(1/2*d*x)^2*tan(c)^2 + d*tan(1/2*c)^2*tan(c)^2 + d*tan(d*x)^2 + d*ta
n(1/2*d*x)^2 + d*tan(1/2*c)^2 + d*tan(c)^2 + d)

3.239 $\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.115877, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.421, Rules used = {4397, 2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\tan^2(c+dx)\,dx$$
$$= \int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right)\,dx$$
$$= a^2\int \sin^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int \tan^2(c+dx)$$
$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{1}{2}a^2\int 1\,dx-b^2\int 1\,dx+\frac{(2ab}{}$$
$$= \frac{a^2x}{2}-b^2x-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{(2}{}$$
$$= \frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c}{2d}$$

Mathematica [A] time = 0.605068, size = 116, normalized size = 1.51

$$-\frac{-2\left(a^2-2b^2\right)(c+dx)+\tan(c+dx)\left(a^2\cos(2(c+dx))+a^2-4b^2\right)+8ab\sin(c+dx)+8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -(-2*(a^2 - 2*b^2)*(c + d*x) + 8*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*a*b*Sin[c + d*x] +
(a^2 - 4*b^2 + a^2*Cos[2*(c + d*x)])*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.045, size = 99, normalized size = 1.3

$$-\frac{a^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2x}{2}+\frac{a^2c}{2d}-2\frac{ab\sin(dx+c)}{d}+2\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}-b^2x+\frac{b^2\tan(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*x+1/2/d*a^2*c-2*a*b*sin(d*x+c)/d+2
/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-b^2*x+b^2*tan(d*x+c)/d-1/d*b^2*c
```

Maxima [A] time = 1.57076, size = 113, normalized size = 1.47

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2}{4\,d}-\frac{(dx+c-\tan(dx+c))b^2}{d}+\frac{ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - (d*x + c - tan(d*x + c))*b^2/d
 + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))/d
```

Fricas [A] time = 0.525144, size = 279, normalized size = 3.62

$$\frac{\left(a^2-2\,b^2\right)dx\cos(dx+c)+2\,ab\cos(dx+c)\log(\sin(dx+c)+1)-2\,ab\cos(dx+c)\log(-\sin(dx+c)+1)-\left(a^2\cos(dx\right.}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c) +
 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4*a
*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a\sin(c+dx)+b\tan(c+dx))^2\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [B] time = 2.31427, size = 3715, normalized size = 48.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*x - 1/4*a^2*sin(2*d*x + 2*c)/d - (b^2*d*x*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2
*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)
*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/
2*c) + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan(1/2
*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*t
an(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(
1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(c) - b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d*x*tan(d*x)*ta
n(1/2*d*x)^2*tan(c) + b^2*d*x*tan(d*x)*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan
(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2*tan(1/2*c
)^2 + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2
*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 +
 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(
1/2*d*x)^2*tan(1/2*c)^2 + b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b*lo
g(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(
1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d
*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)*tan(1/2
*d*x)^2*tan(c) - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2
- 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(c) - 4*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1
/2*c)*tan(c) + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 +
2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^
4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1
/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) +
 1))*tan(d*x)*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d
*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2
```

*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)*tan(1/2*c)^2*tan(c) - 4*a*b*tan(d*x)*tan(1/2*d*x)*tan(1/2*c)^2*tan(c) + b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*d*x*tan(1/2*d*x)^2 - b^2*d*x*tan(1/2*c)^2 + b^2*d*x*tan(d*x)*tan(c) - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*d*x)^2 + b^2*tan(d*x)*tan(1/2*d*x)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*c) - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(1/2*c)^2 + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(1/2*c)^2 + b^2*tan(d*x)*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1))*tan(d*x)*tan(c) - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1))*tan(d*x)*tan(c) + 4*a*b*tan(d*x)*tan(1/2*d*x)*tan(c) + b^2*tan(1/2*d*x)^2*tan(c) + 4*a*b*tan(d*x)*tan(1/2*c)*tan(c) + b^2*tan(1/2*c)^2*tan(c) - b^2*d*x - a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)) + a*b*log(2*(tan(1/2*c)^2 + 1)/(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)) + b^2*tan(d*x) - 4*a*b*tan(1/2*d*x) - 4*a*b*tan(1/2*c) + b^2*tan(c))/(d*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - d*tan(1/2*d*x)^2*tan(1/2*c)^2 + d*tan(d*x)*tan(1/2*d*x)^2*tan(c) + d*tan(d*x)*tan(1/2*c)^2*tan(c) - d*tan(1/2*d*x)^2 - d*tan(1/2*c)^2 + d*tan(d*x)*tan(c) - d)

3.240 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=90

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2\sin(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}-2abx$$

```
[Out] -2*a*b*x + ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*Sin[c + d*x
])/(2*d) + (a*b*Tan[c + d*x])/d + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rubi [A] time = 0.42642, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4397, 2889, 3048, 3031, 3023, 2735, 3770}

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2\sin(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}-2abx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*Sin[c + d*x
])/(2*d) + (a*b*Tan[c + d*x])/d + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec(c+dx)\tan^2(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (b+a\cos(c+dx)$$
$$= \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\int$$
$$= -\frac{3a^2\sin(c+dx)}{2d} + \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+dx))^2\sec(c+dx)}{2d}$$
$$= -2abx - \frac{3a^2\sin(c+dx)}{2d} + \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+dx))^2\sec(}{2d}$$
$$= -2abx + \frac{\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2\sin(c+dx)}{2d} + \frac{ab\tan(}{d}$$

Mathematica [A] time = 0.192681, size = 75, normalized size = 0.83

$$\frac{\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx)) - 2a^2\sin(c+dx) - 4ab\tan^{-1}(\tan(c+dx)) + 4ab\tan(c+dx) + b^2\tan(c+dx)\sec(c+dx}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (-4*a*b*ArcTan[Tan[c + d*x]] + (2*a^2 - b^2)*ArcTanh[Sin[c + d*x]] - 2*a^2*
Sin[c + d*x] + 4*a*b*Tan[c + d*x] + b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A] time = 0.063, size = 123, normalized size = 1.4

$$-\frac{a^2\sin(dx+c)}{d} + \frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d} - 2abx + 2\frac{ab\tan(dx+c)}{d} - 2\frac{abc}{d} + \frac{b^2(\sin(dx+c))^3}{2d(\cos(dx+c))^2} + \frac{b^2\sin(}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*sin(d*x+c)/d+1/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-2*a*b*x+2*a*b*tan(d*x+c
)/d-2/d*a*b*c+1/2/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^2*sin(d*x+c)/d-1/2/
```

```
d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.71679, size = 138, normalized size = 1.53

$$-\frac{8\,(dx+c-\tan\,(dx+c))ab+b^2\Big(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\,(\sin\,(dx+c)+1)-\log\,(\sin\,(dx+c)-1)\Big)-2\,a^2(\log\,(\sin\,(dx+c)+}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*(d*x + c - tan(d*x + c))*a*b + b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A] time = 0.545411, size = 305, normalized size = 3.39

$$-\frac{8\,abdx\cos\,(dx+c)^2-\left(2\,a^2-b^2\right)\cos\,(dx+c)^2\log\,(\sin\,(dx+c)+1)+\left(2\,a^2-b^2\right)\cos\,(dx+c)^2\log\,(-\sin\,(dx+c)+1)}{4\,d\cos\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a*b*d*x*cos(d*x + c)^2 - (2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x +
 c) + 1) + (2*a^2 - b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^2*c
os(d*x + c)^2 - 4*a*b*cos(d*x + c) - b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin\,(c+dx)+b\tan\,(c+dx))^2\sec\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] time = 2.75911, size = 231, normalized size = 2.57

$$-\frac{4\,(dx+c)ab-\left(2\,a^{2}-b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+\left(2\,a^{2}-b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1}+\frac{2\left(4\,a\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(d*x + c)*a*b - (2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) +
(2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*
c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan
(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.241 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=99

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+a^2(-x)-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{3d}+\frac{\tan(c+dx)\sec^2(c+dx)(a\,\mathrm{c}}{3d}$$

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 - b^2)*Tan[c + d*x])/(3*
d) + (a*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + ((b + a*Cos[c + d*x])^2*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.466797, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4397, 2889, 3048, 3031, 3021, 2735, 3770}

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+a^2(-x)-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{3d}+\frac{\tan(c+dx)\sec^2(c+dx)(a\,\mathrm{c}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 - b^2)*Tan[c + d*x])/(3*
d) + (a*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + ((b + a*Cos[c + d*x])^2*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
```

```
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec^2(c+dx)\tan^2(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^4(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int (b+a\cos(c+$$
$$= \frac{ab\sec(c+dx)\tan(c+dx)}{3d} + \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\tan}{3d}$$
$$= \frac{\left(2a^2-b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+dx)}{3d} + \frac{(b+a\cos(}{}$$
$$= -a^2x + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+dx)}{3d} + \frac{(b+}{}$$
$$= -a^2x - \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{\left(2a^2-b^2\right)\tan(c+dx)}{3d} + \frac{ab\sec(}{}$$

Mathematica [B] time = 1.14578, size = 201, normalized size = 2.03

$$\frac{\sec^3(c+dx)\left(2\sin(c+dx)\left(\left(3a^2-b^2\right)\cos(2(c+dx))+3a^2+6ab\cos(c+dx)+b^2\right)-9a\cos(c+dx)\left(a(c+dx)-b\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(-9*a*Cos[c + d*x]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*a*Cos[3
*(c + d*x)]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(3*a^2 + b^2 + 6*a*b*Cos[c + d
*x] + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*d)
```

Maple [A] time = 0.073, size = 109, normalized size = 1.1

$$-a^2x + \frac{a^2\tan(dx+c)}{d} - \frac{a^2c}{d} + \frac{ab(\sin(dx+c))^3}{d(\cos(dx+c))^2} + \frac{ab\sin(dx+c)}{d} - \frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^2(\sin(dx}{3d(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -a^2*x+a^2*tan(d*x+c)/d-1/d*a^2*c+1/d*a*b*sin(d*x+c)^3/cos(d*x+c)^2+a*b*sin
(d*x+c)/d-1/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/3/d*b^2*sin(d*x+c)^3/cos(d*x+
c)^3
```

Maxima [A] time = 1.69924, size = 111, normalized size = 1.12

$$\frac{2\,b^2\tan\left(dx+c\right)^3-6\left(dx+c-\tan\left(dx+c\right)\right)a^2-3\,ab\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Fricas [A] time = 0.523383, size = 294, normalized size = 2.97

$$-\frac{6\,a^2dx\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-2\left(3\,ab\cos\left(\right.\right.}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) + (3
*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [A] time = 2.90522, size = 213, normalized size = 2.15

$$-\frac{3\,(dx+c)a^2+3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5\right.}{}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*
tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

3.242 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=125

$$-\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{ab\tan(c+dx)\sec^2(c+dx)}{6d}$$

```
[Out] -((4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - (2*a*b*Tan[c + d*x])/(3*d) +
 ((2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*b*Sec[c + d*x]^2*Tan[
c + d*x])/(6*d) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d
)
```

Rubi [A] time = 0.439664, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {4397, 2889, 3048, 3031, 3021, 2748, 3767, 8, 3770}

$$-\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{ab\tan(c+dx)\sec^2(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -((4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(8*d) - (2*a*b*Tan[c + d*x])/(3*d) +
 ((2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*b*Sec[c + d*x]^2*Tan[
c + d*x])/(6*d) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d
)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3031

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[((b*c - a*d)*(A*b^2 - a*b*B + a^2*C)*Cos[e
 + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b^2*f*(m + 1)*(a^2 - b^2)), x] - Dis
t[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m +
 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m +
1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1))))*
Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
 && LtQ[m, -1]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec^3(c+dx)\tan^2(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^5(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{1}{4}\int (b+a\cos(c+d$$
$$= \frac{ab\sec^2(c+dx)\tan(c+dx)}{6d} + \frac{(b+a\cos(c+dx))^2\sec^3(c+dx)\tan($$
$$= \frac{\left(2a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{ab\sec^2(c+dx)\tan(c+dx)}{6d} +$$
$$= \frac{\left(2a^2-b^2\right)\sec(c+dx)\tan(c+dx)}{8d} + \frac{ab\sec^2(c+dx)\tan(c+dx)}{6d} +$$
$$= -\frac{\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{\left(2a^2-b^2\right)\sec(c+dx)\tan(c+d$$
$$= -\frac{\left(4a^2+b^2\right)\tanh^{-1}(\sin(c+dx))}{8d} - \frac{2ab\tan(c+dx)}{3d} + \frac{\left(2a^2-b^2\right)\sec$$

Mathematica [B] time = 0.635513, size = 336, normalized size = 2.69

$$\frac{\sec^4(c+dx)\left(12\left(4a^2+b^2\right)\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^4*(36*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*b^2*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*(4*a^2 + b^2)*Cos[2*(c + d*x)]*
```

```
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) + 3*(4*a^2 + b^2)*Cos[4*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 36*a^2*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 9*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 24*a^2*Sin[c + d*x] + 42*b^2*Sin[c + d*x] + 32*a*b*Sin[2*(c + d*x)]
 + 24*a^2*Sin[3*(c + d*x)] - 6*b^2*Sin[3*(c + d*x)] - 16*a*b*Sin[4*(c + d*x
)]))/(192*d)
```

Maple [A] time = 0.08, size = 169, normalized size = 1.4

$$\frac{a^2 (\sin (dx+c))^3}{2 d (\cos (dx+c))^2}+\frac{a^2 \sin (dx+c)}{2 d}-\frac{a^2 \ln (\sec (dx+c)+\tan (dx+c))}{2 d}+\frac{2 ab (\sin (dx+c))^3}{3 d (\cos (dx+c))^3}+\frac{b^2 (\sin (dx+c))^3}{4 d (\cos (dx+c))^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/2/d*a^2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*a^2*sin(d*x+c)/d-1/2/d*a^2*ln(sec(d
*x+c)+tan(d*x+c))+2/3/d*a*b*sin(d*x+c)^3/cos(d*x+c)^3+1/4/d*b^2*sin(d*x+c)^
3/cos(d*x+c)^4+1/8/d*b^2*sin(d*x+c)^3/cos(d*x+c)^2+1/8*b^2*sin(d*x+c)/d-1/8
/d*b^2*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.12989, size = 174, normalized size = 1.39

$$\frac{32 ab \tan (dx+c)^3+3 b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2 \sin(dx+c)^2+1}-\log (\sin (dx+c)+1)+\log (\sin (dx+c)-1)\right)-12 a^2\left(\frac{2 \sin(dx+c)}{\sin(dx+c)^2-1}+\right.}{48 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*a*b*tan(d*x + c)^3 + 3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c)
+ 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [A] time = 0.524828, size = 325, normalized size = 2.6

$$-\frac{3\left(4a^2+b^2\right)\cos(dx+c)^4\log(\sin(dx+c)+1)-3\left(4a^2+b^2\right)\cos(dx+c)^4\log(-\sin(dx+c)+1)+2\left(16ab\cos(dx+\right.}{48d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*a^2 + b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 + b^
2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*a*b*cos(d*x + c)^3 - 16*a*
b*cos(d*x + c) - 3*(4*a^2 - b^2)*cos(d*x + c)^2 - 6*b^2)*sin(d*x + c))/(d*c
os(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.90036, size = 305, normalized size = 2.44

$$-\frac{3\left(4a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(4a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*
b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*tan(1/2*d*x + 1/2*c)^5 - 64*a*b*tan(1/2
```

```
*d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*
c)^3 + 64*a*b*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a
^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 - 1)^4)/d
```

3.243 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)(a\cos(c+dx)+b)^4}{4a^3d}+\frac{(a\cos(c+dx)+b)^6}{6a^3d}-\frac{2b(a\cos(c+dx)+b)^5}{5a^3d}$$

```
[Out] -((a^2 - b^2)*(b + a*Cos[c + d*x])^4)/(4*a^3*d) - (2*b*(b + a*Cos[c + d*x])
^5)/(5*a^3*d) + (b + a*Cos[c + d*x])^6/(6*a^3*d)
```

Rubi [A] time = 0.188703, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4397, 2668, 697}

$$-\frac{(a^2-b^2)(a\cos(c+dx)+b)^4}{4a^3d}+\frac{(a\cos(c+dx)+b)^6}{6a^3d}-\frac{2b(a\cos(c+dx)+b)^5}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a^2 - b^2)*(b + a*Cos[c + d*x])^4)/(4*a^3*d) - (2*b*(b + a*Cos[c + d*x])
^5)/(5*a^3*d) + (b + a*Cos[c + d*x])^6/(6*a^3*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 697

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int (b+x)^3\left(a^2-x^2\right)\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)(b+x)^3+2b(b+x)^4-(b+x)^5\right)\,dx, x, a\cos\right.}{a^3d}$$
$$= -\frac{\left(a^2-b^2\right)(b+a\cos(c+dx))^4}{4a^3d} - \frac{2b(b+a\cos(c+dx))^5}{5a^3d} + \frac{(b+a\,\text{c}}{}$$

Mathematica [A] time = 0.248718, size = 114, normalized size = 1.48

$$\frac{-360b\left(a^2+2b^2\right)\cos(c+dx)-45\left(a^3+8ab^2\right)\cos(2(c+dx))-60a^2b\cos(3(c+dx))+36a^2b\cos(5(c+dx))+5a^3\cos(6}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-360*b*(a^2 + 2*b^2)*Cos[c + d*x] - 45*(a^3 + 8*a*b^2)*Cos[2*(c + d*x)] -
60*a^2*b*Cos[3*(c + d*x)] + 80*b^3*Cos[3*(c + d*x)] + 90*a*b^2*Cos[4*(c + d
*x)] + 36*a^2*b*Cos[5*(c + d*x)] + 5*a^3*Cos[6*(c + d*x)])/(960*d)
```

Maple [A] time = 0.078, size = 109, normalized size = 1.4

$$\frac{1}{d}\left(a^3\left(-\frac{(\sin(dx+c))^2(\cos(dx+c))^4}{6}-\frac{(\cos(dx+c))^4}{12}\right)+3a^2b\left(-1/5\,(\sin(dx+c))^2(\cos(dx+c))^3-2/15\,(\cos(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a^2*b*(-1/5*s
in(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3/4*a*b^2*sin(d*x+c)^4-1/3*b^3*
(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A] time = 1.16065, size = 128, normalized size = 1.66

$$\frac{45\,ab^2\sin(dx+c)^4 - 5\left(2\,\sin(dx+c)^6 - 3\,\sin(dx+c)^4\right)a^3 + 12\left(3\,\cos(dx+c)^5 - 5\,\cos(dx+c)^3\right)a^2b + 20\left(\cos(dx+c)^3 - 3\,\cos(dx+c)\right)b^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(45*a*b^2*sin(d*x + c)^4 - 5*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3
 + 12*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b + 20*(cos(d*x + c)^3 - 3*
cos(d*x + c))*b^3)/d
```

Fricas [A] time = 0.508109, size = 240, normalized size = 3.12

$$\frac{10\,a^3\cos(dx+c)^6 + 36\,a^2b\cos(dx+c)^5 - 90\,ab^2\cos(dx+c)^2 - 15\left(a^3 - 3\,ab^2\right)\cos(dx+c)^4 - 60\,b^3\cos(dx+c) - 20\left(3\,a^2b - b^3\right)\cos(dx+c)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*a^3*cos(d*x + c)^6 + 36*a^2*b*cos(d*x + c)^5 - 90*a*b^2*cos(d*x +
c)^2 - 15*(a^3 - 3*a*b^2)*cos(d*x + c)^4 - 60*b^3*cos(d*x + c) - 20*(3*a^2*
b - b^3)*cos(d*x + c)^3)/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.244 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=120

$$-\frac{a\left(a^2-3b^2\right)\cos^3(c+dx)}{3d}-\frac{b\left(3a^2-b^2\right)\cos^2(c+dx)}{2d}+\frac{3a^2b\cos^4(c+dx)}{4d}+\frac{a^3\cos^5(c+dx)}{5d}-\frac{3ab^2\cos(c+dx)}{d}-\frac{b^3\text{lo}}{}$$

```
[Out] (-3*a*b^2*Cos[c + d*x])/d - (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/(2*d) - (a*(a^
2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Cos[c + d*x]^4)/(4*d) + (a^3*Co
s[c + d*x]^5)/(5*d) - (b^3*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.194266, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos^3(c+dx)}{3d}-\frac{b\left(3a^2-b^2\right)\cos^2(c+dx)}{2d}+\frac{3a^2b\cos^4(c+dx)}{4d}+\frac{a^3\cos^5(c+dx)}{5d}-\frac{3ab^2\cos(c+dx)}{d}-\frac{b^3\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*b^2*Cos[c + d*x])/d - (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/(2*d) - (a*(a^
2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Cos[c + d*x]^4)/(4*d) + (a^3*Co
s[c + d*x]^5)/(5*d) - (b^3*Log[Cos[c + d*x]])/d
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin^2(c+dx)\tan(c+dx)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{a(b+x)^3\left(a^2-x^2\right)}{x}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3\left(a^2-x^2\right)}{x}\,dx, x, a\cos(c+dx)\right)}{a^2d}$$

$$= -\frac{\text{Subst}\left(\int \left(3a^2b^2+\frac{a^2b^3}{x}+b\left(3a^2-b^2\right)x+\left(a^2-3b^2\right)x^2-3bx^3-\right.\right.}{a^2d}$$

$$= -\frac{3ab^2\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\cos^2(c+dx)}{2d}-\frac{a\left(a^2-3b^2\right)\cos^3(}{3d}$$

Mathematica [A] time = 0.185859, size = 106, normalized size = 0.88

$$-\frac{\frac{1}{3}a\left(a^2-3b^2\right)\cos^3(c+dx)+\frac{1}{2}b\left(3a^2-b^2\right)\cos^2(c+dx)-\frac{3}{4}a^2b\cos^4(c+dx)-\frac{1}{5}a^3\cos^5(c+dx)+3ab^2\cos(c+dx)+b}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((3*a*b^2*Cos[c + d*x] + (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/2 + (a*(a^2 - 3*
b^2)*Cos[c + d*x]^3)/3 - (3*a^2*b*Cos[c + d*x]^4)/4 - (a^3*Cos[c + d*x]^5)/
5 + b^3*Log[Cos[c + d*x]])/d)
```

Maple [A] time = 0.076, size = 128, normalized size = 1.1

$$-\frac{a^3(\sin(dx+c))^2(\cos(dx+c))^3}{5d}-\frac{2a^3(\cos(dx+c))^3}{15d}+\frac{3a^2b(\sin(dx+c))^4}{4d}-\frac{\cos(dx+c)(\sin(dx+c))^2ab^2}{d}-2\frac{ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/5/d*a^3*sin(d*x+c)^2*cos(d*x+c)^3-2/15*a^3*cos(d*x+c)^3/d+3/4/d*a^2*b*si
n(d*x+c)^4-1/d*cos(d*x+c)*sin(d*x+c)^2*a*b^2-2*a*b^2*cos(d*x+c)/d-1/2/d*b^3
*sin(d*x+c)^2-b^3*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.16498, size = 127, normalized size = 1.06

$$\frac{45\,a^2b\sin\left(dx+c\right)^4+4\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^3+60\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)ab^2-30\left(\sin\left(dx+c\right)^2\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(45*a^2*b*sin(d*x + c)^4 + 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3
 + 60*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^2 - 30*(sin(d*x + c)^2 + log(si
n(d*x + c)^2 - 1))*b^3)/d
```

Fricas [A] time = 0.530426, size = 247, normalized size = 2.06

$$\frac{12\,a^3\cos\left(dx+c\right)^5+45\,a^2b\cos\left(dx+c\right)^4-180\,ab^2\cos\left(dx+c\right)-20\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3-60\,b^3\log\left(-\cos\left(dx+c\right)\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 180*a*b^2*cos(d*x +
 c) - 20*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 60*b^3*log(-cos(d*x + c)) - 30*(3
*a^2*b - b^3)*cos(d*x + c)^2)/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**2, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.245 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=112

$$-\frac{a\left(a^2-3b^2\right)\cos^2(c+dx)}{2d}-\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d}+\frac{a^2b\cos^3(c+dx)}{d}+\frac{a^3\cos^4(c+dx)}{4d}-\frac{3ab^2\log(\cos(c+dx))}{d}+b^3$$

```
[Out] -((b*(3*a^2 - b^2)*Cos[c + d*x])/d) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^2)/(2*d
) + (a^2*b*Cos[c + d*x]^3)/d + (a^3*Cos[c + d*x]^4)/(4*d) - (3*a*b^2*Log[Co
s[c + d*x]])/d + (b^3*Sec[c + d*x])/d
```

Rubi [A] time = 0.177274, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4397, 2837, 12, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos^2(c+dx)}{2d}-\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d}+\frac{a^2b\cos^3(c+dx)}{d}+\frac{a^3\cos^4(c+dx)}{4d}-\frac{3ab^2\log(\cos(c+dx))}{d}+b^3$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*Cos[c + d*x])/d) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^2)/(2*d
) + (a^2*b*Cos[c + d*x]^3)/d + (a^3*Cos[c + d*x]^4)/(4*d) - (3*a*b^2*Log[Co
s[c + d*x]])/d + (b^3*Sec[c + d*x])/d
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^2(b+x)^3(a^2-x^2)}{x^2}\,dx, x, a\cos(c+dx)\right)}{a^3 d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^2}\,dx, x, a\cos(c+dx)\right)}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(3a^2b\left(1-\frac{b^2}{3a^2}\right)+\frac{a^2b^3}{x^2}+\frac{3a^2b^2}{x}+\left(a^2-3b^2\right)x-3bx^2-x^3\right.\right.}{ad}$$
$$= -\frac{b\left(3a^2-b^2\right)\cos(c+dx)}{d}-\frac{a\left(a^2-3b^2\right)\cos^2(c+dx)}{2d}+\frac{a^2b\cos^3(c}{d}$$

Mathematica [A] time = 0.271683, size = 98, normalized size = 0.88

$$\frac{8b\left(4b^2-9a^2\right)\cos(c+dx)-4\left(a^3-6ab^2\right)\cos(2(c+dx))+8a^2b\cos(3(c+dx))+a^3\cos(4(c+dx))-96ab^2\log(\cos(c+}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (8*b*(-9*a^2 + 4*b^2)*Cos[c + d*x] - 4*(a^3 - 6*a*b^2)*Cos[2*(c + d*x)] + 8
*a^2*b*Cos[3*(c + d*x)] + a^3*Cos[4*(c + d*x)] - 96*a*b^2*Log[Cos[c + d*x]]
 + 32*b^3*Sec[c + d*x])/(32*d)
```

Maple [A] time = 0.06, size = 147, normalized size = 1.3

$$\frac{a^3(\sin(dx+c))^4}{4d}-\frac{\cos(dx+c)(\sin(dx+c))^2a^2b}{d}-2\frac{a^2b\cos(dx+c)}{d}-\frac{3ab^2(\sin(dx+c))^2}{2d}-3\frac{ab^2\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/4/d*a^3*sin(d*x+c)^4-1/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b-2*a^2*b*cos(d*x+c)
/d-3/2/d*a*b^2*sin(d*x+c)^2-3*a*b^2*ln(cos(d*x+c))/d+1/d*b^3*sin(d*x+c)^4/c
os(d*x+c)+1/d*b^3*cos(d*x+c)*sin(d*x+c)^2+2*b^3*cos(d*x+c)/d
```

Maxima [A] time = 1.11962, size = 117, normalized size = 1.04

$$\frac{a^3\sin(dx+c)^4+4\left(\cos(dx+c)^3-3\cos(dx+c)\right)a^2b-6\left(\sin(dx+c)^2+\log\left(\sin(dx+c)^2-1\right)\right)ab^2+4b^3\left(\frac{1}{\cos(dx+c)}+c\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3*sin(d*x + c)^4 + 4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b - 6*(si
n(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a*b^2 + 4*b^3*(1/cos(d*x + c) + cos
(d*x + c)))/d
```

Fricas [A] time = 0.537816, size = 311, normalized size = 2.78

$$\frac{8a^3\cos(dx+c)^5+32a^2b\cos(dx+c)^4-96ab^2\cos(dx+c)\log(-\cos(dx+c))-16\left(a^3-3ab^2\right)\cos(dx+c)^3+32b^3-}{32d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*a^3*cos(d*x + c)^5 + 32*a^2*b*cos(d*x + c)^4 - 96*a*b^2*cos(d*x + c
)*log(-cos(d*x + c)) - 16*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 32*b^3 - 32*(3*a
^2*b - b^3)*cos(d*x + c)^2 + (5*a^3 - 24*a*b^2)*cos(d*x + c))/(d*cos(d*x +
c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.246 $\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=116

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{3ab^2\sec(c+dx)}{d}+ b^3$$

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.103942, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4397, 2721, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{3ab^2\sec(c+dx)}{d}+ b^3$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
```

```
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^3}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)+\frac{a^2b^3}{x^3}+\frac{3a^2b^2}{x^2}+\frac{3a^2b-b^3}{x}-3bx-x^2\right)dx, x, a\cos(c+dx$$
$$= -\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b\left(3a^2-b^2\right)}{}$$

Mathematica [A] time = 0.310056, size = 102, normalized size = 0.88

$$\frac{-9a\left(a^2-4b^2\right)\cos(c+dx)+9a^2b\cos(2(c+dx))-36a^2b\log(\cos(c+dx))+a^3\cos(3(c+dx))+36ab^2\sec(c+dx)+6b}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
+ d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^2*
Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

Maple [A] time = 0.056, size = 164, normalized size = 1.4

$$-\frac{\cos(dx+c)(\sin(dx+c))^2a^3}{3d}-\frac{2a^3\cos(dx+c)}{3d}-\frac{3a^2b(\sin(dx+c))^2}{2d}-3\frac{a^2b\ln(\cos(dx+c))}{d}+3\frac{ab^2(\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/3/d*cos(d*x+c)*sin(d*x+c)^2*a^3-2/3*a^3*cos(d*x+c)/d-3/2/d*a^2*b*sin(d*x
+c)^2-3*a^2*b*ln(cos(d*x+c))/d+3/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)+3/d*cos(d*
x+c)*sin(d*x+c)^2*a*b^2+6*a*b^2*cos(d*x+c)/d+1/2/d*b^3*tan(d*x+c)^2+b^3*ln(
cos(d*x+c))/d
```

Maxima [A] time = 1.04589, size = 153, normalized size = 1.32

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{3}}{3\,d}-\frac{3\left(\sin\left(dx+c\right)^{2}+\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)a^{2}b}{2\,d}-\frac{b^{3}\left(\frac{1}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)^{2}-\right.\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d - 3/2*(sin(d*x + c)^2 + log(sin
(d*x + c)^2 - 1))*a^2*b/d - 1/2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x +
 c)^2 - 1))/d + 3*a*b^2*(1/cos(d*x + c) + cos(d*x + c))/d
```

Fricas [A] time = 0.536226, size = 300, normalized size = 2.59

$$\frac{4\,a^{3}\cos\left(dx+c\right)^{5}+18\,a^{2}b\cos\left(dx+c\right)^{4}-9\,a^{2}b\cos\left(dx+c\right)^{2}+36\,ab^{2}\cos\left(dx+c\right)-12\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}-12\left(3\,a\right.}{12\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c)
^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^2*
b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

3.247 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=115

$$\frac{b(3a^2-b^2)\sec(c+dx)}{d}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}+\frac{3ab^2\sec^2(c+dx)}{2d}+\frac{b^3 s}{}$$

```
[Out] (3*a^2*b*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*Lo
g[Cos[c + d*x]])/d + (b*(3*a^2 - b^2)*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d*
x]^2)/(2*d) + (b^3*Sec[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.24159, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4397, 2837, 12, 894}

$$\frac{b(3a^2-b^2)\sec(c+dx)}{d}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}+\frac{3ab^2\sec^2(c+dx)}{2d}+\frac{b^3 s}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*Lo
g[Cos[c + d*x]])/d + (b*(3*a^2 - b^2)*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d*
x]^2)/(2*d) + (b^3*Sec[c + d*x]^3)/(3*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^4(b+x)^3(a^2-x^2)}{x^4}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^4}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \left(-3b+\frac{a^2b^3}{x^4}+\frac{3a^2b^2}{x^3}+\frac{3a^2b-b^3}{x^2}+\frac{a^2-3b^2}{x}-x\right)dx, x, a\cos\right.}{d}$$
$$= \frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}-\frac{a\left(a^2-3b^2\right)\log(\cos(c+dx))}{d}$$

Mathematica [A] time = 0.54442, size = 100, normalized size = 0.87

$$\frac{2\left(-6b\left(b^2-3a^2\right)\sec(c+dx)-6a\left(a^2-3b^2\right)\log(\cos(c+dx))+9ab^2\sec^2(c+dx)+2b^3\sec^3(c+dx)\right)+36a^2b\cos(c+}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (36*a^2*b*Cos[c + d*x] + 3*a^3*Cos[2*(c + d*x)] + 2*(-6*a*(a^2 - 3*b^2)*Log
[Cos[c + d*x]] - 6*b*(-3*a^2 + b^2)*Sec[c + d*x] + 9*a*b^2*Sec[c + d*x]^2 +
 2*b^3*Sec[c + d*x]^3))/(12*d)
```

Maple [A] time = 0.078, size = 213, normalized size = 1.9

$$-\frac{a^3(\sin(dx+c))^2}{2d}-\frac{a^3\ln(\cos(dx+c))}{d}+3\frac{a^2b(\sin(dx+c))^4}{d\cos(dx+c)}+3\frac{\cos(dx+c)(\sin(dx+c))^2a^2b}{d}+6\frac{a^2b\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3*sin(d*x+c)^2-a^3*ln(cos(d*x+c))/d+3/d*a^2*b*sin(d*x+c)^4/cos(d*x
+c)+3/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b+6*a^2*b*cos(d*x+c)/d+3/2/d*a*b^2*tan(
d*x+c)^2+3*a*b^2*ln(cos(d*x+c))/d+1/3/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/3/d
*b^3*sin(d*x+c)^4/cos(d*x+c)-1/3/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/3*b^3*cos(
d*x+c)/d
```

Maxima [A] time = 1.11744, size = 147, normalized size = 1.28

$$-\frac{3\left(\sin\left(dx+c\right)^2+\log\left(\sin\left(dx+c\right)^2-1\right)\right)a^3+9\,ab^2\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)-18\,a^2b\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+\right.\right.}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(sin(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a^3 + 9*a*b^2*(1/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 18*a^2*b*(1/cos(d*x + c) + cos(d
*x + c)) + 2*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

Fricas [A] time = 0.549714, size = 297, normalized size = 2.58

$$\frac{6\,a^3\cos\left(dx+c\right)^5+36\,a^2b\cos\left(dx+c\right)^4-3\,a^3\cos\left(dx+c\right)^3-12\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(-\cos\left(dx+c\right)\right)+18\,ab^2\,\mathrm{co}}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^3*cos(d*x + c)^5 + 36*a^2*b*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3
 - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*a*b^2*cos(d*x
+ c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 100.792, size = 691, normalized size = 6.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(6*(a^3 - 3*a*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))
 - 6*(a^3 - 3*a*b^2)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) -
 3*(3*a^3 - 12*a^2*b - 9*a*b^2 - 10*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 12*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a*b^2*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 9*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 1)^2 + (11*a^3 + 36*a^2*b - 33*a*b^2 - 8*b^3 + 33*a^3*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 72*a^2*b*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 135*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 24*b^3*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 33*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 36*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 135*a*b^2*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 11*a^3*(cos(d*x + c) - 1)^3/(cos
(d*x + c) + 1)^3 - 33*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)/d
```

3.248 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{b(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a(a^2-3b^2)\sec(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}+\frac{ab^2\sec^3(c+dx)}{d}+b^3\,\mathrm{se}$$

```
[Out] (a^3*Cos[c + d*x])/d + (3*a^2*b*Log[Cos[c + d*x]])/d + (a*(a^2 - 3*b^2)*Sec
[c + d*x])/d + (b*(3*a^2 - b^2)*Sec[c + d*x]^2)/(2*d) + (a*b^2*Sec[c + d*x]
^3)/d + (b^3*Sec[c + d*x]^4)/(4*d)
```

Rubi [A] time = 0.254024, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$\frac{b(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a(a^2-3b^2)\sec(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a^3\cos(c+dx)}{d}+\frac{ab^2\sec^3(c+dx)}{d}+b^3\,\mathrm{se}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x])/d + (3*a^2*b*Log[Cos[c + d*x]])/d + (a*(a^2 - 3*b^2)*Sec
[c + d*x])/d + (b*(3*a^2 - b^2)*Sec[c + d*x]^2)/(2*d) + (a*b^2*Sec[c + d*x]
^3)/d + (b^3*Sec[c + d*x]^4)/(4*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec^2(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^5(b+x)^3\left(a^2-x^2\right)}{x^5}\,dx, x, a\cos(c+dx)\right)}{a^3 d}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{(b+x)^3\left(a^2-x^2\right)}{x^5}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(-1+\frac{a^2b^3}{x^5}+\frac{3a^2b^2}{x^4}+\frac{3a^2b-b^3}{x^3}+\frac{a^2-3b^2}{x^2}-\frac{3b}{x}\right) dx, x, a\,\text{co}\right.}{d}$$
$$= \frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a\left(a^2-3b^2\right)\sec(c+dx)}{d}$$

Mathematica [A] time = 1.97822, size = 97, normalized size = 0.87

$$\frac{\left(6a^2b-2b^3\right)\sec^2(c+dx)+4a\left(a^2-3b^2\right)\sec(c+dx)+12a^2b\log(\cos(c+dx))+4a^3\cos(c+dx)+4ab^2\sec^3(c+dx)+}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (4*a^3*Cos[c + d*x] + 12*a^2*b*Log[Cos[c + d*x]] + 4*a*(a^2 - 3*b^2)*Sec[c
+ d*x] + (6*a^2*b - 2*b^3)*Sec[c + d*x]^2 + 4*a*b^2*Sec[c + d*x]^3 + b^3*Se
c[c + d*x]^4)/(4*d)
```

Maple [A] time = 0.09, size = 204, normalized size = 1.8

$$\frac{a^3(\sin(dx+c))^4}{d\cos(dx+c)}+\frac{\cos(dx+c)(\sin(dx+c))^2a^3}{d}+2\frac{a^3\cos(dx+c)}{d}+\frac{3a^2b(\tan(dx+c))^2}{2d}+3\frac{a^2b\ln(\cos(dx+c))}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*sin(d*x+c)^4/cos(d*x+c)+1/d*cos(d*x+c)*sin(d*x+c)^2*a^3+2*a^3*cos(d
*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+3*a^2*b*ln(cos(d*x+c))/d+1/d*a*b^2*sin(d*x
+c)^4/cos(d*x+c)^3-1/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)-1/d*cos(d*x+c)*sin(d*x
+c)^2*a*b^2-2*a*b^2*cos(d*x+c)/d+1/4/d*b^3*sin(d*x+c)^4/cos(d*x+c)^4
```

Maxima [A] time = 1.0863, size = 130, normalized size = 1.17

$$\frac{b^3 \tan(dx+c)^4 - 6\,a^2 b\left(\frac{1}{\sin(dx+c)^2-1} - \log\left(\sin(dx+c)^2-1\right)\right) + 4\,a^3\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right) - \frac{4\left(3\,\cos(dx+c)^2-1\right)ab^2}{\cos(dx+c)^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 - 6*a^2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
)^2 - 1)) + 4*a^3*(1/cos(d*x + c) + cos(d*x + c)) - 4*(3*cos(d*x + c)^2 - 1
)*a*b^2/cos(d*x + c)^3)/d
```

Fricas [A] time = 0.542339, size = 258, normalized size = 2.32

$$\frac{4\,a^3\cos(dx+c)^5 + 12\,a^2 b\cos(dx+c)^4\log(-\cos(dx+c)) + 4\,ab^2\cos(dx+c) + 4\left(a^3-3\,ab^2\right)\cos(dx+c)^3 + b^3 + 2\,(3}{4\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*cos(d*x + c)^5 + 12*a^2*b*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*
a*b^2*cos(d*x + c) + 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + b^3 + 2*(3*a^2*b -
b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 105.274, size = 567, normalized size = 5.11

$$-\frac{12\,a^2b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-12\,a^2b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{4\left(2\,a^3+3\,a^2b-\frac{3\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}-\frac{8\,a^3-25\,a^2b-16\,ab^2+\frac{24\,a^3(\cos(dx+}{\cos(dx+c)+}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(12*a^2*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 12*a^
2*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 4*(2*a^3 + 3*a^2
*b - 3*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 1) - (8*a^3 - 25*a^2*b - 16*a*b^2 + 24*a^3*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 124*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 64*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24*a^3*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - 198*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)
^2 - 48*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 16*b^3*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 8*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c)
+ 1)^3 - 124*a^2*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 25*a^2*b*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 1)^4)/d
```

3.249 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=119

$$\frac{b\left(3a^2-b^2\right)\sec^3(c+dx)}{3d}+\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}+\frac{3ab^2\sec^4(c+dx)}{4d}+\frac{b^3}{}$$

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^2*b*Sec[c + d*x])/d + (a*(a^2 - 3*b^2)*Sec
[c + d*x]^2)/(2*d) + (b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(3*d) + (3*a*b^2*Sec[
c + d*x]^4)/(4*d) + (b^3*Sec[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.219906, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$\frac{b\left(3a^2-b^2\right)\sec^3(c+dx)}{3d}+\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a^3\log(\cos(c+dx))}{d}+\frac{3ab^2\sec^4(c+dx)}{4d}+\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^2*b*Sec[c + d*x])/d + (a*(a^2 - 3*b^2)*Sec
[c + d*x]^2)/(2*d) + (b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(3*d) + (3*a*b^2*Sec[
c + d*x]^4)/(4*d) + (b^3*Sec[c + d*x]^5)/(5*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec^3(c+dx)\tan^3(c+dx)\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{a^6(b+x)^3(a^2-x^2)}{x^6}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^6}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{a^2b^3}{x^6}+\frac{3a^2b^2}{x^5}+\frac{3a^2b-b^3}{x^4}+\frac{a^2-3b^2}{x^3}-\frac{3b}{x^2}-\frac{1}{x}\right)dx, x, a\cos$$

$$= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a\left(a^2-3b^2\right)\sec^2(c+dx)}{2d}$$

Mathematica [A] time = 0.3156, size = 99, normalized size = 0.83

$$\frac{-20b\left(b^2-3a^2\right)\sec^3(c+dx)+30a\left(a^2-3b^2\right)\sec^2(c+dx)-180a^2b\sec(c+dx)+60a^3\log(\cos(c+dx))+45ab^2\sec^4(c}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (60*a^3*Log[Cos[c + d*x]] - 180*a^2*b*Sec[c + d*x] + 30*a*(a^2 - 3*b^2)*Sec
[c + d*x]^2 - 20*b*(-3*a^2 + b^2)*Sec[c + d*x]^3 + 45*a*b^2*Sec[c + d*x]^4
+ 12*b^3*Sec[c + d*x]^5)/(60*d)
```

Maple [B] time = 0.096, size = 252, normalized size = 2.1

$$\frac{a^3(\tan(dx+c))^2}{2d}+\frac{a^3\ln(\cos(dx+c))}{d}+\frac{a^2b(\sin(dx+c))^4}{d(\cos(dx+c))^3}-\frac{a^2b(\sin(dx+c))^4}{d\cos(dx+c)}-\frac{\cos(dx+c)(\sin(dx+c))^2a^2b}{d}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/2/d*a^3*tan(d*x+c)^2+a^3*ln(cos(d*x+c))/d+1/d*a^2*b*sin(d*x+c)^4/cos(d*x+
c)^3-1/d*a^2*b*sin(d*x+c)^4/cos(d*x+c)-1/d*cos(d*x+c)*sin(d*x+c)^2*a^2*b-2*
a^2*b*cos(d*x+c)/d+3/4/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)^4+1/5/d*b^3*sin(d*x+
c)^4/cos(d*x+c)^5+1/15/d*b^3*sin(d*x+c)^4/cos(d*x+c)^3-1/15/d*b^3*sin(d*x+c
)^4/cos(d*x+c)-1/15/d*b^3*cos(d*x+c)*sin(d*x+c)^2-2/15*b^3*cos(d*x+c)/d
```

Maxima [A] time = 1.12428, size = 173, normalized size = 1.45

$$-\frac{30\,a^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)-\frac{45\left(2\,\sin(dx+c)^2-1\right)ab^2}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+\frac{60\left(3\,\cos(dx+c)^2-1\right)a^2b}{\cos(dx+c)^3}+\frac{4\left(5\,\cos(dx+c)^2-3\right)b^3}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(30*a^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 45*(2*si
n(d*x + c)^2 - 1)*a*b^2/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 60*(3*cos
(d*x + c)^2 - 1)*a^2*b/cos(d*x + c)^3 + 4*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

Fricas [A] time = 0.535732, size = 270, normalized size = 2.27

$$\frac{60\,a^3\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-180\,a^2b\cos\left(dx+c\right)^4+45\,ab^2\cos\left(dx+c\right)+30\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+12\,b^3}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^2*b*cos(d*x + c)^4 +
 45*a*b^2*cos(d*x + c) + 30*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 12*b^3 + 20*(3
*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 104.493, size = 579, normalized size = 4.87

$$-\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{137\,a^3+240\,a^2b+16\,b^3+\frac{805\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1200\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{80\,b^3(\cos\ldots}{\cos(\ldots}}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3
*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (137*a^3 + 240*a^2*
b + 16*b^3 + 805*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1200*a^2*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 80*b^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1730*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1680*a^2*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 720*a*b^2*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 - 80*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 173
0*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 720*a^2*b*(cos(d*x + c) -
 1)^3/(cos(d*x + c) + 1)^3 - 720*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 + 240*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a^3*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c) - 1)^5/(cos(d*x
+ c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^5)/d
```

3.250 $\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=113

$$-\frac{b^4\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)}-\frac{b\cos(c+dx)}{a^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos^2(c+dx)}{2ad}$$

```
[Out] -((b*Cos[c + d*x])/(a^2*d)) + Cos[c + d*x]^2/(2*a*d) + Log[1 - Cos[c + d*x]
]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) - (b^4*Log[b + a*Cos[
c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rubi [A] time = 0.341056, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 1629}

$$-\frac{b^4\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)}-\frac{b\cos(c+dx)}{a^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/(a^2*d)) + Cos[c + d*x]^2/(2*a*d) + Log[1 - Cos[c + d*x]
]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) - (b^4*Log[b + a*Cos[
c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\cos^3(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{a^4(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{x^4}{(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{a^3d}\\ &= -\frac{\text{Subst}\left(\int \left(b+\frac{a^3}{2(a+b)(a-x)}-x-\frac{a^3}{2(a-b)(a+x)}+\frac{b^4}{(a-b)(a+b)(b+x)}\right)dx, x, a\cos(c+dx)\right)}{a^3d}\\ &= -\frac{b\cos(c+dx)}{a^2d}+\frac{\cos^2(c+dx)}{2ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\end{aligned}$$

Mathematica [A] time = 0.375596, size = 100, normalized size = 0.88

$$\frac{4\left(\frac{b^4\log(a\cos(c+dx)+b)}{a^3(b^2-a^2)}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}\right)-\frac{4b\cos(c+dx)}{a^2}+\frac{\cos(2(c+dx))}{a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] ((-4*b*Cos[c + d*x])/a^2 + Cos[2*(c + d*x)]/a + 4*(Log[Cos[(c + d*x)/2]]/(a
 - b) + (b^4*Log[b + a*Cos[c + d*x]])/(a^3*(-a^2 + b^2)) + Log[Sin[(c + d*x
)/2]]/(a + b)))/(4*d)
```

Maple [A] time = 0.124, size = 111, normalized size = 1.

$$\frac{(\cos(dx+c))^2}{2\,ad}-\frac{b\cos(dx+c)}{a^2d}+\frac{\ln(\cos(dx+c)+1)}{d(2\,a-2\,b)}+\frac{\ln(-1+\cos(dx+c))}{d(2\,a+2\,b)}-\frac{b^4\ln(b+a\cos(dx+c))}{da^3(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/2*cos(d*x+c)^2/a/d-b*cos(d*x+c)/a^2/d+1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/
(2*a+2*b)*ln(-1+cos(d*x+c))-1/d/a^3*b^4/(a+b)/(a-b)*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.64243, size = 254, normalized size = 2.25

$$-\frac{\frac{b^4\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^5-a^3b^2}+\frac{2\left(b+\frac{(a+b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{\left(a^2+b^2\right)\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^4*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^5 - a^3*b
^2) + 2*(b + (a + b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 + 2*a^2*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)
- log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + (a^2 + b^2)*log(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)/a^3)/d
```

Fricas [A] time = 0.633103, size = 288, normalized size = 2.55

$$-\frac{2\,b^4\log(a\cos(dx+c)+b)-\left(a^4-a^2b^2\right)\cos(dx+c)^2+2\left(a^3b-ab^3\right)\cos(dx+c)-\left(a^4+a^3b\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\left(a^5-a^3b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^4*log(a*cos(d*x + c) + b) - (a^4 - a^2*b^2)*cos(d*x + c)^2 + 2*(a
^3*b - a*b^3)*cos(d*x + c) - (a^4 + a^3*b)*log(1/2*cos(d*x + c) + 1/2) - (a
^4 - a^3*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - a^3*b^2)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2592, size = 409, normalized size = 3.62

$$-\frac{\frac{2\,b^4\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^5-a^3b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\left(a^2+b^2\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{3\,a^2-4\,ab+3\,b^2-\frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,ab(\cos(d}{\cos(dx+}}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^4*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^5 - a^3*b^2) - log(abs(-cos(d*x +
c) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*(a^2 + b^2)*log(abs(-(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (3*a^2 - 4*a*b + 3*b^2 - 2*a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^2))/d
```

3.251 $\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{b^3\log(a\cos(c+dx)+b)}{a^2d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos(c+dx)}{ad}$$

```
[Out] Cos[c + d*x]/(a*d) + Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c +
d*x]]/(2*(a - b)*d) + (b^3*Log[b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rubi [A] time = 0.278922, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 1629}

$$\frac{b^3\log(a\cos(c+dx)+b)}{a^2d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Cos[c + d*x]/(a*d) + Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c +
d*x]]/(2*(a - b)*d) + (b^3*Log[b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^2(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x^3}{a^3(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{a^2}{2(a+b)(a-x)}+\frac{a^2}{2(a-b)(a+x)}+\frac{b^3}{(-a+b)(a+b)(b+x)}\right)dx, x, a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\cos(c+dx)}{ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b^3\log(b+a\cos(c+}{a^2\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.225309, size = 80, normalized size = 0.87

$$\frac{\frac{b^3\log(a\cos(c+dx)+b)}{a^4-a^2b^2}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{b-a}+\frac{\cos(c+dx)}{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]/a + Log[Cos[(c + d*x)/2]]/(-a + b) + (b^3*Log[b + a*Cos[c + d
*x]])/(a^4 - a^2*b^2) + Log[Sin[(c + d*x)/2]]/(a + b))/d
```

Maple [A] time = 0.12, size = 93, normalized size = 1.

$$\frac{\cos(dx+c)}{ad}-\frac{\ln(\cos(dx+c)+1)}{d(2a-2b)}+\frac{\ln(-1+\cos(dx+c))}{d(2a+2b)}+\frac{b^3\ln(b+a\cos(dx+c))}{a^2d(a+b)(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] cos(d*x+c)/a/d-1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/(2*a+2*b)*ln(-1+cos(d*x+c
))+1/d/a^2*b^3/(a+b)/(a-b)*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.65883, size = 174, normalized size = 1.89

$$\frac{\frac{b^3\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^4-a^2b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{b\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^2}+\frac{2}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^4 - a^2*b^
2) + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + b*log(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)/a^2 + 2/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
)/d
```

Fricas [A] time = 0.589357, size = 239, normalized size = 2.6

$$\frac{2\,b^3\log\left(a\cos\left(dx+c\right)+b\right)+2\left(a^3-ab^2\right)\cos\left(dx+c\right)-\left(a^3+a^2b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^3-a^2b\right)\log\left(-\frac{1}{2}\cos\left(dx\right.\right.}{2\left(a^4-a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*log(a*cos(d*x + c) + b) + 2*(a^3 - a*b^2)*cos(d*x + c) - (a^3 +
a^2*b)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - a^2*b)*log(-1/2*cos(d*x + c) +
1/2))/((a^4 - a^2*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [B] time = 1.24704, size = 257, normalized size = 2.79

$$\frac{\frac{2\,b^3\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-a^2b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\left(2\,a-b+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - a^2*b^2) + log(abs(-cos(d*x + c
) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*b*log(abs(-(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1))/a^2 - 2*(2*a - b + b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1))/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

3.252 $\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{b^2\log(a\cos(c+dx)+b)}{ad\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (b^2*Log[b + a*Cos[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rubi [A] time = 0.236899, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4397, 2837, 12, 1629}

$$-\frac{b^2\log(a\cos(c+dx)+b)}{ad\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (b^2*Log[b + a*Cos[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\cos(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \frac{x^2}{a^2(b+x)\left(a^2-x^2\right)}\,dx,x,a\cos(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+x)\left(a^2-x^2\right)}\,dx,x,a\cos(c+dx)\right)}{ad}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{a}{2(a+b)(a-x)}-\frac{a}{2(a-b)(a+x)}+\frac{b^2}{(a-b)(a+b)(b+x)}\right)dx,x,a\cos(c+dx)\right)}{ad}\\ &= \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{b^2\log(b+a\cos(c+dx))}{a\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.100196, size = 70, normalized size = 0.88

$$\frac{b^2(-\log(a\cos(c+dx)+b))+a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+a(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a + b)*Log[Cos[(c + d*x)/2]] - b^2*Log[b + a*Cos[c + d*x]] + a*(a - b)*
Log[Sin[(c + d*x)/2]])/(a*(a - b)*(a + b)*d)
```

Maple [A] time = 0.101, size = 80, normalized size = 1.

$$\frac{\ln(\cos(dx+c)+1)}{d(2a-2b)}+\frac{\ln(-1+\cos(dx+c))}{d(2a+2b)}-\frac{b^2\ln(b+a\cos(dx+c))}{d(a+b)(a-b)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/(2*a+2*b)*ln(-1+cos(d*x+c))-1/d*b^2/(a+b
)/(a-b)/a*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.56268, size = 138, normalized size = 1.72

$$-\frac{\frac{b^2\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^3-ab^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^3 - a*b^2
) - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + log(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)/a)/d
```

Fricas [A] time = 0.564225, size = 190, normalized size = 2.38

$$-\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1/
2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cos\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A] time = 1.21627, size = 180, normalized size = 2.25

$$-\frac{\frac{2\,b^2\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^3-ab^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^3 - a*b^2) - log(abs(-cos(d*x + c)
 + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*log(abs(-(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1))/a)/d
```

3.253 $\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.0798118, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {4397, 2721, 801}

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cot(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(a-x)}+\frac{1}{2(a-b)(a+x)}+\frac{b}{(-a+b)(a+b)(b+x)}\right)dx, x, a\cos(c+dx)\right)}{d}$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{b\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.0845769, size = 63, normalized size = 0.85

$$\frac{(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+b\log(a\cos(c+dx)+b)}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
```

```
[Out] (-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Log
[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

Maple [A] time = 0.099, size = 75, normalized size = 1.

$$-\frac{\ln(\cos(dx+c)+1)}{d(2a-2b)}+\frac{\ln(-1+\cos(dx+c))}{d(2a+2b)}+\frac{b\ln(b+a\cos(dx+c))}{d(a-b)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/(2*a+2*b)*ln(-1+cos(d*x+c))+1/d*b/(a-b)
/(a+b)*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.13642, size = 96, normalized size = 1.3

$$\frac{\frac{b\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) + l
og(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

Fricas [A] time = 0.534073, size = 173, normalized size = 2.34

$$\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+(a-b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (a
 - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A] time = 1.21295, size = 135, normalized size = 1.82

$$\frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) + 1)
/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.254 $\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=75

$$-\frac{a\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (a*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.185584, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4397, 2668, 706, 31, 633}

$$-\frac{a\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (a*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 706

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
```

```
0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 633

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[-(
a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x), x], x] + Dist[e/2 - (c
*d)/(2*q), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
-(a*c)]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\csc(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d} \\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cos(c+dx)\right)}{\left(a^2-b^2\right)d} - \frac{a\,\text{Subst}\left(\int \frac{-b+x}{a^2-x^2}\,dx, x, a\cos(c+dx)\right)}{\left(a^2-b^2\right)d} \\
&= -\frac{a\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d} - \frac{\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, a\cos(c+dx)\right)}{2(a-b)d} - \frac{\text{Subst}\left(\int \frac{1}{a-x}\,d\right.}{2(} \\
&= \frac{\log(1-\cos(c+dx))}{2(a+b)d} + \frac{\log(1+\cos(c+dx))}{2(a-b)d} - \frac{a\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A] time = 0.0624528, size = 64, normalized size = 0.85

$$
\frac{(a-b)\log(1-\cos(c+dx))+(a+b)\log(\cos(c+dx)+1)-2a\log(a\cos(c+dx)+b)}{2d(a-b)(a+b)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] ((a - b)*Log[1 - Cos[c + d*x]] + (a + b)*Log[1 + Cos[c + d*x]] - 2*a*Log[b
+ a*Cos[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A] time = 0.123, size = 75, normalized size = 1.

$$\frac{\ln\left(\cos\left(dx+c\right)+1\right)}{d\left(2a-2b\right)}+\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{d\left(2a+2b\right)}-\frac{a\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a-b\right)\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/(2*a+2*b)*ln(-1+cos(d*x+c))-1/d*a/(a-b)/
(a+b)*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.06011, size = 99, normalized size = 1.32

$$-\frac{\frac{a\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) -
log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

Fricas [A] time = 0.538659, size = 174, normalized size = 2.32

$$-\frac{2\,a\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-(a-b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) - (
a - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A] time = 1.20544, size = 136, normalized size = 1.81

$$-\frac{\frac{2\,a\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) - log(abs(-cos(d*x + c) + 1
)/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.255 $\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{a^2\log(a\cos(c+dx)+b)}{bd\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}-\frac{\log(\cos(c+dx))}{bd}$$

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[Cos[c + d*x]]/(b*d) - Log[1 + Cos
[c + d*x]]/(2*(a - b)*d) + (a^2*Log[b + a*Cos[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rubi [A] time = 0.270216, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$\frac{a^2\log(a\cos(c+dx)+b)}{bd\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}-\frac{\log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[Cos[c + d*x]]/(b*d) - Log[1 + Cos
[c + d*x]]/(2*(a - b)*d) + (a^2*Log[b + a*Cos[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\csc(c+dx)\sec(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{a}{x(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1}{x(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{2a^2(a+b)(a-x)}+\frac{1}{a^2bx}+\frac{1}{2a^2(a-b)(a+x)}+\frac{1}{b(-a+b)(a+b)(b+x)}\right) dx, x, a\cos(c+\right.}{d}$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(\cos(c+dx))}{bd}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{a^2\log(b+a\,\text{co}}{b\left(a^2-b\right.}$$

Mathematica [A] time = 0.150821, size = 103, normalized size = 1.1

$$2\left(-\frac{a^2\log(a\cos(c+dx)+b)}{2bd\left(b^2-a^2\right)}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d(a+b)}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d(b-a)}-\frac{\log(\cos(c+dx))}{2bd}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] 2*(Log[Cos[(c + d*x)/2]]/(2*(-a + b)*d) - Log[Cos[c + d*x]]/(2*b*d) - (a^2*
Log[b + a*Cos[c + d*x]])/(2*b*(-a^2 + b^2)*d) + Log[Sin[(c + d*x)/2]]/(2*(a
 + b)*d))
```

Maple [A] time = 0.137, size = 95, normalized size = 1.

$$-\frac{\ln\left(\cos\left(dx+c\right)+1\right)}{d\left(2a-2b\right)}+\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{d\left(2a+2b\right)}-\frac{\ln\left(\cos\left(dx+c\right)\right)}{db}+\frac{a^{2}\ln\left(b+a\cos\left(dx+c\right)\right)}{db\left(a+b\right)\left(a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/(2*a+2*b)*ln(-1+cos(d*x+c))-ln(cos(d*x+
c))/d/b+1/d*a^2/b/(a+b)/(a-b)*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.15958, size = 169, normalized size = 1.8

$$\frac{\frac{a^{2}\log\left(a+b-\frac{(a-b)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a^{2}b-b^{3}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b - b^3)
 - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/b + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

Fricas [A] time = 0.769623, size = 236, normalized size = 2.51

$$\frac{2a^{2}\log\left(a\cos\left(dx+c\right)+b\right)-2\left(a^{2}-b^{2}\right)\log\left(-\cos\left(dx+c\right)\right)-\left(ab+b^{2}\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(ab-b^{2}\right)\log\left(-\frac{1}{2}\cos\right.}{2\left(a^{2}b-b^{3}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*log(a*cos(d*x + c) + b) - 2*(a^2 - b^2)*log(-cos(d*x + c)) - (a*
b + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a*b - b^2)*log(-1/2*cos(d*x + c) +
```

```
1/2))/((a^2*b - b^3)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A] time = 1.26786, size = 180, normalized size = 1.91

$$\frac{\frac{2a^2\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2b-b^3}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}-\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b - b^3) + log(abs(-cos(d*x + c)
+ 1)/abs(cos(d*x + c) + 1))/(a + b) - 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 1))/b)/d
```

3.256 $\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{a^3\log(a\cos(c+dx)+b)}{b^2d\left(a^2-b^2\right)}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\sec(c+dx)}{bd}$$

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + (a*Log[Cos[c + d*x]])/(b^2*d) + Log[1
 + Cos[c + d*x]]/(2*(a - b)*d) - (a^3*Log[b + a*Cos[c + d*x]])/(b^2*(a^2 -
b^2)*d) + Sec[c + d*x]/(b*d)
```

Rubi [A] time = 0.284128, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 12, 894}

$$-\frac{a^3\log(a\cos(c+dx)+b)}{b^2d\left(a^2-b^2\right)}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + (a*Log[Cos[c + d*x]])/(b^2*d) + Log[1
 + Cos[c + d*x]]/(2*(a - b)*d) - (a^3*Log[b + a*Cos[c + d*x]])/(b^2*(a^2 -
b^2)*d) + Sec[c + d*x]/(b*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\csc(c+dx)\sec^2(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{a^2}{x^2(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{1}{x^2(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{2a^3(a+b)(a-x)}+\frac{1}{a^2bx^2}-\frac{1}{a^2b^2x}-\frac{1}{2a^3(a-b)(a+x)}-\frac{1}{b^2(-a+b)(a+b)(b+x)}\right)dx, x\right.}{d}$$

$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{a^3\log(b+a}{b^2\left(a^2\right.}$$

Mathematica [A] time = 0.292099, size = 92, normalized size = 0.85

$$\frac{\frac{a^3\log(a\cos(c+dx)+b)}{b^4-a^2b^2}+\frac{a\log(\cos(c+dx))}{b^2}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}+\frac{\sec(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Log[Cos[(c + d*x)/2]]/(a - b) + (a*Log[Cos[c + d*x]])/b^2 + (a^3*Log[b + a
*Cos[c + d*x]])/(-(a^2*b^2) + b^4) + Log[Sin[(c + d*x)/2]]/(a + b) + Sec[c
+ d*x]/b)/d
```

Maple [A] time = 0.132, size = 110, normalized size = 1.

$$\frac{\ln(\cos(dx+c)+1)}{d(2a-2b)}+\frac{\ln(-1+\cos(dx+c))}{d(2a+2b)}+\frac{a\ln(\cos(dx+c))}{b^2d}+\frac{1}{db\cos(dx+c)}-\frac{a^3\ln(b+a\cos(dx+c))}{d(a+b)(a-b)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] 1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/(2*a+2*b)*ln(-1+cos(d*x+c))+a*ln(cos(d*x
+c))/b^2/d+1/d/b/cos(d*x+c)-1/d*a^3/(a+b)/(a-b)/b^2*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.0753, size = 213, normalized size = 1.97

$$-\frac{\frac{a^3\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2b^2-b^4}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b^2 - b
^4) - a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(
cos(d*x + c) + 1) - 1)/b^2 - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) -
 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [A] time = 0.911594, size = 360, normalized size = 3.33

$$-\frac{2\,a^3\cos(dx+c)\log(a\cos(dx+c)+b)-2\,a^2b+2\,b^3-2\left(a^3-ab^2\right)\cos(dx+c)\log(-\cos(dx+c))-\left(ab^2+b^3\right)\cos(d}{2\left(a^2b^2-b^4\right)d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)*log(a*cos(d*x + c) + b) - 2*a^2*b + 2*b^3 - 2*(a^3
 - a*b^2)*cos(d*x + c)*log(-cos(d*x + c)) - (a*b^2 + b^3)*cos(d*x + c)*log(
1/2*cos(d*x + c) + 1/2) - (a*b^2 - b^3)*cos(d*x + c)*log(-1/2*cos(d*x + c)
+ 1/2))/((a^2*b^2 - b^4)*d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A] time = 1.26384, size = 257, normalized size = 2.38

$$-\frac{\frac{2\,a^3\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2b^2-b^4}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}-\frac{2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}+\frac{2\left(a-2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b^2 - b^4) - log(abs(-cos(d*x +
c) + 1)/abs(cos(d*x + c) + 1))/(a + b) - 2*a*log(abs(-(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1))/b^2 + 2*(a - 2*b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

3.257 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=243

$$-\frac{b^5\sin(c+dx)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2bx}{a^3}-$$

```
[Out] (2*b*x)/a^3 + (2*b^6*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (2*b^4*(5*a^2 - 3*b^2)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) -
 Sin[c + d*x]/(a^2*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - S
in[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (b^5*Sin[c + d*x])/(a^2*(a
^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.610712, antiderivative size = 243, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4397, 2897, 2648, 2637, 2664, 12, 2659, 208}

$$-\frac{b^5\sin(c+dx)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2bx}{a^3}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b*x)/a^3 + (2*b^6*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (2*b^4*(5*a^2 - 3*b^2)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) -
 Sin[c + d*x]/(a^2*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - S
in[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (b^5*Sin[c + d*x])/(a^2*(a
^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int \left(-\frac{2b}{a^3} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{\cos(c+}{a^2}\right.$$

$$= \frac{2bx}{a^3} - \frac{\int \cos(c+dx)\,dx}{a^2} + \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(b^4\left(5a^2-3b^2\right)\right.}{a^3\left(a\right.}$$

$$= \frac{2bx}{a^3} - \frac{\sin(c+dx)}{a^2d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))}$$

$$= \frac{2bx}{a^3} + \frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{a^2d} - \frac{\sin(c}{2(a+b)^2d(1-}$$

$$= \frac{2bx}{a^3} + \frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{a^2d} - \frac{\sin(c}{2(a+b)^2d(1-}$$

$$= \frac{2bx}{a^3} + \frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 3.06031, size = 164, normalized size = 0.67

$$-\frac{\frac{2b^5\sin(c+dx)}{a^2(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{4b^4\left(5a^2-2b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3\left(a^2-b^2\right)^{5/2}} - \frac{4b(c+dx)}{a^3} + \frac{2\sin(c+dx)}{a^2} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -((-4*b*(c + d*x))/a^3 + (4*b^4*(5*a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(c +
d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]/(a +
b)^2 + (2*Sin[c + d*x])/a^2 + (2*b^5*Sin[c + d*x])/(a^2*(a - b)^2*(a + b)^2
*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2]/(a - b)^2)/(2*d)
```

Maple [A] time = 0.186, size = 291, normalized size = 1.2

$$-\frac{1}{2d\left(a^2-2ab+b^2\right)}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{\tan\left(1/2\,dx+c/2\right)}{a^2d\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}+4\frac{b\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{a^3d}+2\frac{}{d\left(a+b\right)^2(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d/a^2*tan(1/2*d*x+1/2*c)/(1+tan
(1/2*d*x+1/2*c)^2)+4/d/a^3*b*arctan(tan(1/2*d*x+1/2*c))+2/d*b^5/(a+b)^2/(a-
b)^2/a^2*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-
a-b)+10/d*b^4/(a+b)^2/(a-b)^2/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d
*x+1/2*c)/((a+b)*(a-b))^(1/2))-4/d*b^6/(a+b)^2/(a-b)^2/a^3/((a+b)*(a-b))^(1
/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/2/d/(a+b)^2/tan
(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.70098, size = 1839, normalized size = 7.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^7*b - 6*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7 - 2*(a^8 - 3*a^6*b^2 + 3*a
^4*b^4 - a^2*b^6)*cos(d*x + c)^3 + (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*b^
```

```
6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(3
*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2*a*b^7)*cos(d*x + c)^2 + 2*(2*a^8 - 5*a^6
*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c) - 4*((a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x)*
sin(d*x + c))/(((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c) + (
a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)*sin(d*x + c)), -(2*a^7*b - 3*a^
5*b^3 + 3*a^3*b^5 - 2*a*b^7 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d
*x + c)^3 - (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt(-
a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d
*x + c)))*sin(d*x + c) - (3*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2*a*b^7)*cos(d*
x + c)^2 + (2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c) - 2*((a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4
 + 3*a^2*b^6 - b^8)*d*x)*sin(d*x + c))/(((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^
4*b^6)*d*cos(d*x + c) + (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)*sin(d*
x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29026, size = 639, normalized size = 2.63

$$-\frac{\frac{4\left(5a^2b^4-2b^6\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}+\frac{5a^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-7a^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-5a^3b^2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(5*a^2*b^4 - 2*b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b
) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^
2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/
(a^2 - 2*a*b + b^2) + (5*a^5*tan(1/2*d*x + 1/2*c)^4 - 7*a^4*b*tan(1/2*d*x +
 1/2*c)^4 - 5*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 7*a^2*b^3*tan(1/2*d*x + 1/2*
c)^4 + 4*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 8*b^5*tan(1/2*d*x + 1/2*c)^4 - 4*a^
5*tan(1/2*d*x + 1/2*c)^2 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*b^2*tan(
1/2*d*x + 1/2*c)^2 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 4*a*b^4*tan(1/2*d*x
 + 1/2*c)^2 - 8*b^5*tan(1/2*d*x + 1/2*c)^2 - a^5 + a^4*b + a^3*b^2 - a^2*b^
3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^5 - b*tan(1/2*d*x +
 1/2*c)^5 - 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2
*d*x + 1/2*c))) - 4*(d*x + c)*b/a^3)/d
```

3.258 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-\frac{}{2d(a}$$

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rubi [A] time = 0.491171, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4397, 2897, 2648, 2664, 12, 2659, 208}

$$\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-\frac{}{2d(a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```


```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} + \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{}{a^2\left(a^2-b\right.}\right.$$

$$= -\frac{x}{a^2} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{b^4\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2\left(a^2-b^2\right)} + \frac{\left(2b^3\left(2a^2\right.\right.}{}$$

$$= -\frac{x}{a^2} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{b^4 \mathrm{s}}{a\left(a^2-b^2\right)^2}$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} +$$

$$= -\frac{x}{a^2} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} +$$

$$= -\frac{x}{a^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3\left(2a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A] time = 2.16558, size = 151, normalized size = 0.67

$$\frac{-\frac{4b^3\left(b^2-4a^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2\left(a^2-b^2\right)^{5/2}} - \frac{2(c+dx)}{a^2} + \frac{2b^4\sin(c+dx)}{a(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-2*(c + d*x))/a^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)) - Cot[(c + d*x)/2]/(a + b)^2
 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + Tan[
(c + d*x)/2]/(a - b)^2)/(2*d)
```

Maple [A] time = 0.171, size = 255, normalized size = 1.1

$$\frac{1}{2\,d\left(a^{2}-2\,ab+b^{2}\right)}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\,\frac{\arctan\left(\tan\left(1/2\,dx+c/2\right)\right)}{a^{2}d}-2\,\frac{b^{4}\tan\left(1/2\,dx+c/2\right)}{d\left(a+b\right)^{2}\left(a-b\right)^{2}a\left(\left(\tan\left(1/2\,dx+c/2\right)\right)^{2}a-(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2/d/a^2*arctan(tan(1/2*d*x+1/2*c))
-2/d*b^4/(a+b)^2/(a-b)^2/a*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1
/2*d*x+1/2*c)^2*b-a-b)-8/d*b^3/(a+b)^2/(a-b)^2/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*b^5/(a+b)^2/(a-b)^2/a^2/(
(a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/
2/d/(a+b)^2/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.634744, size = 1507, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
```

```
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(
a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
- 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4
- a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*si
n(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 -
 a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^6)*cos(d*x
+ c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)
 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.22763, size = 447, normalized size = 1.97

$$\frac{\frac{4\left(4a^2b^3-b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-2a^4b^2+a^2b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}-\frac{a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^2\tan(}{\left(a^5-2a^3b^2+ab^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^
2 - 2*a*b + b^2) - (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x + 1/2*
```

```
c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2 + 4*
b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2*a^3*b
^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) - 2*(d*x + c)/a^2)/d
```

3.259 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$-\frac{b^3\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{s}{2d(a+b)}$$

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2
)*(a + b)^(5/2)*d) + (2*b^2*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a
+ b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x
])) - (b^3*Sin[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.384874, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4397, 2897, 2648, 2659, 208, 2664, 12}

$$-\frac{b^3\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{s}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2
)*(a + b)^(5/2)*d) + (2*b^2*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a
+ b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x
])) - (b^3*Sin[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
```

```
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int\left(-\frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{b^2(}{a\left(a^2-b^2\right)^2(}\right.$$

$$= \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{b^3\int \frac{1}{(b+a\cos(c+dx))^2}\,dx}{a\left(a^2-b^2\right)} - \frac{\left(b^2\left(3a^2-b^2\right)\right)\int}{a\left(a^2-\right.}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} - \frac{b^3\sin(c+}{\left(a^2-b^2\right)^2d(b+a}$$

$$= \frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{}{2(a-}$$

$$= \frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{}{2(a-}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} + \frac{2b^2\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{}{2(a}$$

Mathematica [A] time = 1.2941, size = 131, normalized size = 0.6

$$\frac{\frac{\csc(c+dx)\left(-2a\left(a^2-b^2\right)\cos(c+dx)+\left(2a^2b+b^3\right)\cos(2(c+dx))-3b^3\right)}{a\cos(c+dx)+b} - \frac{12ab^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{2d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-12*a*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2
- b^2] + ((-3*b^3 - 2*a*(a^2 - b^2)*Cos[c + d*x] + (2*a^2*b + b^3)*Cos[2*(c
 + d*x)])*Csc[c + d*x])/(b + a*Cos[c + d*x]))/(2*(a - b)^2*(a + b)^2*d)
```

Maple [A] time = 0.145, size = 155, normalized size = 0.7

$$\frac{1}{d}\left(-\frac{1}{2a^2-4ab+2b^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{b^2}{(a-b)^2(a+b)^2}\left(-\frac{\tan(1/2\,dx+c/2)\,b}{(\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b}-3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2*b^2/(a-b)^2/(a+b)^2*(-tan(1/
2*d*x+1/2*c)*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-3*a/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/2/
(a+b)^2/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.595528, size = 1134, normalized size = 5.18

$$\left[-\frac{2a^4b+2a^2b^3-4b^5-3\left(a^2b^2\cos(dx+c)+ab^3\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(d\right.}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}}{2\left(\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos(dx+c)+\left(a^6b-3\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b + 2*a^2*b^3 - 4*b^5 - 3*(a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
```

```
c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^4*b - a^2*b^3 - b^5
)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7)*d)*sin(d*x + c)), -(a^4*b + a^2*b^3 - 2*b^5 - 3*(a^2*b^2*cos(d*x + c)
 + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin
(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.21165, size = 381, normalized size = 1.74

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab^2}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3ab^2\tan\left(\frac{1}{2}dx+\cdots\right.}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b^2/((a^4 -
2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b + b^
2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*a*b^2
*tan(1/2*d*x + 1/2*c)^2 - 5*b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b + a*b^
2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x
 + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

3.260 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos}$$

```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.399241, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {4397, 2731, 2648, 2664, 12, 2659, 208}

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```



```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
```

```
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{2(a+b)^2(-1+\cos(c+dx))} + \frac{1}{2(a-b)^2(1+\cos(c+dx))} - \frac{\quad}{\left(-a^2+b^2\right)(b\cdots}\right.$$

$$= \frac{\int \frac{1}{1+\cos(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(2a^2b\right)\int \frac{1}{b+a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{b^2\int \frac{1}{(b+a\cos(c\cdots}}{a^2-b\cdots}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{ab^2\sin(\cdots}{\left(a^2-b^2\right)^2d(b+\cdots}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(\cdots}{2(a-b)^2d(1\cdots}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(\cdots}{2(a-b)^2d(1\cdots}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin\cdots}{2(a+b)^2d\cdots}$$

Mathematica [A] time = 1.24578, size = 128, normalized size = 0.63

$$\frac{\frac{4b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```

```
[Out] ((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/((a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [A] time = 0.148, size = 162, normalized size = 0.8

$$\frac{1}{d}\left(\frac{1}{2a^2-4ab+2b^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{b}{(a-b)^2(a+b)^2}\left(-\frac{ab\tan(1/2\,dx+c/2)}{(\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b}-\frac{2}{\sqrt{(a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2*b/(a-b)^2/(a+b)^2*(-a*b*tan(1
/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2+b^
2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.590717, size = 1164, normalized size = 5.73

$$\left[\frac{6a^3b^2-6ab^4+\left(2a^2b^2+b^4+\left(2a^3b+ab^3\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right.}{2\left(\left(a^7-3a^5b^2+3a^3b^4-ab^6\right)d\cos(dx+c)+\left(a^6b-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
```

```
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*b^2
- 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^5 + a^3*b
^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-2), x)
```

Giac [A] time = 1.12958, size = 390, normalized size = 1.92

$$\frac{\frac{4(2a^2b+b^3)\Big(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\Big(\frac{a\tan(\frac{1}{2}dx+\frac{1}{2}c)-b\tan(\frac{1}{2}dx+\frac{1}{2}c)}{\sqrt{-a^2+b^2}}\Big)\Big)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan(\frac{1}{2}dx+\frac{1}{2}c)}{a^2-2ab+b^2}-\frac{a^3\tan(\frac{1}{2}dx+\frac{1}{2}c)^2-3a^2b\tan(\frac{1}{2}dx+\frac{1}{2}c)^2+7ab^2\tan(}{(a^4-2a^2b^2+b^4)\big(a\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-b\tan(\frac{1}{2}dx}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + a
rctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*
a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 +
 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
+ a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

3.261 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$-\frac{\csc(c+dx)\left(a^2-3ab\cos(c+dx)+2b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (2*a*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(5/2)*(a + b)^(5/2)*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c
 + d*x])) - ((a^2 + 2*b^2 - 3*a*b*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^
2*d)
```

Rubi [A] time = 0.317527, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {4397, 2864, 2866, 12, 2659, 208}

$$-\frac{\csc(c+dx)\left(a^2-3ab\cos(c+dx)+2b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{2a\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```


```
[Out] (2*a*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(5/2)*(a + b)^(5/2)*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c
 + d*x])) - ((a^2 + 2*b^2 - 3*a*b*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^
2*d)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2864

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[((b*c
 - a*d)*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(f*g*(a^2 -
```

```
b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*(a
 + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p +
2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cot(c+dx)\csc(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{\int \frac{(-a+2b\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^2\,d} + \frac{\int$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^2\,d} + \frac{(a$$

$$= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(c+dx)}{(a^2-b^2)^2\,d} + \frac{(2$$

$$= \frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+$$

Mathematica [A] time = 1.13716, size = 127, normalized size = 0.93

$$-\frac{\frac{4a(a^2+2b^2)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{\frac{2a^2b\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -((4*a*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
(a^2 - b^2)^(5/2) + Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^2*b*Sin[c + d*x])/((
a + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [A] time = 0.17, size = 162, normalized size = 1.2

$$\frac{1}{d}\left(-\frac{1}{2a^2-4ab+2b^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{a}{(a-b)^2(a+b)^2}\left(-\frac{ab\tan(1/2\,dx+c/2)}{(\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b}-\frac{}{\sqrt{(}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-2*a/(a-b)^2/(a+b)^2*(-a*b*tan(
1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(a^2+2*b
^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2
)))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.599259, size = 1164, normalized size = 8.56

$$\left[-\frac{4\,a^4b-2\,a^2b^3-2\,b^5-\left(a^3b+2\,ab^3+\left(a^4+2\,a^2b^2\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\,\cdots}{a^2\cos(dx+c)^2+2\,ab\cos(d\cdots}\right.}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)+\left(a^6b\,\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^4*b - 2*a^2*b^3 - 2*b^5 - (a^3*b + 2*a*b^3 + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*
x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^
2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^4*b
 - a^2*b^3)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c)), -(2*a^4*b - a^2*b^3 - b^5 - (a^3*b + 2*a*
b^3 + (a^4 + 2*a^2*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4
```

```
*b - a^2*b^3)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [B] time = 1.21431, size = 389, normalized size = 2.86

$$-\frac{\frac{4\left(a^3+2ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3ab^2\tan\cdots}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx\cdots\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))
/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2
*a*b + b^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b*tan(1/2*d*x + 1/2*c)^2
+ 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
 + a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(
1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

3.262 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)^2}-\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (-6*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + (a*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c + d*x]))
 + ((3*a*b - (2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^2*d)
```

Rubi [A] time = 0.334015, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {4397, 2694, 2866, 12, 2659, 208}

$$\frac{a\csc(c+dx)}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{\csc(c+dx)\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)}{d\left(a^2-b^2\right)^2}-\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (-6*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + (a*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c + d*x]))
 + ((3*a*b - (2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^2*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
```

```
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\int \frac{(-b+2a\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{\left(a^2-b^2\right)^2 d} +$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{\left(a^2-b^2\right)^2 d} -$$

$$= \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\left(3ab-\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{\left(a^2-b^2\right)^2 d} -$$

$$= -\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\csc(c+dx)}{\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\left(3ab-\left(2a\right.\right.}{}$$

Mathematica [A] time = 0.762765, size = 121, normalized size = 0.92

$$\frac{\frac{12a^2b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{\frac{2a^3\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((12*a^2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2
)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^3*Sin[c + d*x])/((a + b)^2*(b
+ a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [A] time = 0.188, size = 155, normalized size = 1.2

$$\frac{1}{d}\left(\frac{1}{2a^2-4ab+2b^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{a^2}{(a-b)^2(a+b)^2}\left(-\frac{a\tan(1/2\,dx+c/2)}{(\tan(1/2\,dx+c/2))^2a-(\tan(1/2\,dx+c/2))^2b-a-b}-3-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2*a^2/(a-b)^2/(a+b)^2*(-a*tan(1
/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-3*b/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/2/(
a+b)^2/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.583734, size = 1131, normalized size = 8.63

$$\Bigg[\frac{2\,a^5+2\,a^3b^2-4\,ab^4+3\left(a^3b\cos\left(dx+c\right)+a^2b^2\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)+\left(a^6b-3\,a^4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^5 + 2*a^3*b^2 - 4*a*b^4 + 3*(a^3*b*cos(d*x + c) + a^2*b^2)*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^5 - a^3*b^2 - a*b^4)
*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d)*sin(d*x + c)), (a^5 + a^3*b^2 - 2*a*b^4 - 3*(a^3*b*cos(d*x + c) + a
^2*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
```

```
c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin(d
*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [B] time = 1.20543, size = 383, normalized size = 2.92

$$\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^2b}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}-\frac{5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3ab^2\tan\left(\frac{1}{2}d\right.}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2*b/((a^4 - 2
*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b + b^2
) - (5*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*a*b^
2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b + a*b^2
 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x
+ 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

3.263 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=231

$$-\frac{a^4\sin(c+dx)}{bd\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{}{2d(a+b}$$

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (2*a^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - (2*a^3*(a^2 - 3*b^2)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^2*(a + b
)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]
/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (a^4*Sin[c + d*x])/(b*(a^2 - b^2)^2*d
*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.43695, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4397, 2897, 2648, 2664, 12, 2659, 208, 3770}

$$-\frac{a^4\sin(c+dx)}{bd\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{}{2d(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```


```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (2*a^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - (2*a^3*(a^2 - 3*b^2)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^2*(a + b
)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]
/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (a^4*Sin[c + d*x])/(b*(a^2 - b^2)^2*d
*(b + a*Cos[c + d*x]))
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\sec(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int\left(-\frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx))} + \frac{}{b\left(a^2-b^2\right)(-}\right.$$

$$= \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \sec(c+dx)\,dx}{b^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{\left(a^3\left(a^2-3b^2\right)\right)\int \frac{}{-b-a}}{b^2\left(a^2-b^2\right)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx)}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^2(a+b)^{5/2}d} - \frac{\sin(c}{2(a+b)^2d(1}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^2(a+b)^{5/2}d} - \frac{\sin(c}{2(a+b)^2d(1}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2a^3\left(a^2-3b^2\right)\tanh^{-1}\left(\right.}{(a-b)^{5/2}b^2(a}$$

Mathematica [A] time = 2.00197, size = 196, normalized size = 0.85

$$-\frac{-\frac{4\left(a^5-4a^3b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2\left(a^2-b^2\right)^{5/2}} + \frac{2a^4\sin(c+dx)}{b(a-b)^2(a+b)^2(a\cos(c+dx)+b)} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^2} - }{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -((-4*(a^5 - 4*a^3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]
])/(b^2*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]/(a + b)^2 + (2*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]])/b^2 - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]])/b^2 + (2*a^4*Sin[c + d*x])/((a - b)^2*b*(a + b)^2*(b + a*Cos[c + d*x]))
```

```
+ Tan[(c + d*x)/2]/(a - b)^2)/(2*d)
```

Maple [A] time = 0.186, size = 276, normalized size = 1.2

$$-\frac{1}{2\,d\left(a^2-2\,ab+b^2\right)}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\,\frac{a^4\tan(1/2\,dx+c/2)}{d\,(a-b)^2\,(a+b)^2\,b\left((\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b\right)}-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] -1/2/d/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2/d*a^4/(a-b)^2/(a+b)^2/b*tan(1/2
*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-2/d*a^5/(a-
b)^2/(a+b)^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b
)*(a-b))^(1/2))+8/d*a^3/(a-b)^2/(a+b)^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*t
an(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+1/d/b^2*ln(tan(1/2*d*x+1/2*c)+1)-1/d
/b^2*ln(tan(1/2*d*x+1/2*c)-1)-1/2/d/(a+b)^2/tan(1/2*d*x+1/2*c)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.54428, size = 1906, normalized size = 8.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^6*b - 2*b^7 + (a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b^2)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 +
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^6*b + a^4*b^3
 - 2*a^2*b^5)*cos(d*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*
x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a^5*b^2 -
 2*a^3*b^4 + a*b^6)*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8)*d*cos(d*x + c) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*sin(d*x + c)
), -1/2*(2*a^6*b - 2*b^7 + 2*(a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b^2)*cos(d*x
 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2
 - b^2)*sin(d*x + c)))*sin(d*x + c) - 2*(a^6*b + a^4*b^3 - 2*a^2*b^5)*cos(d
*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x
+ c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)
*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c) +
 (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [A] time = 1.29365, size = 478, normalized size = 2.07

$$\frac{\frac{4\left(a^5-4a^3b^2\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4b^2-2a^2b^4+b^6\right)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}+\frac{4a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3a^2b^2\tan(\cdots}{\left(a^4b-2a^2b^3+b^5\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3\cdots\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^4*b^2 - 2*a^2*b^4 + b^6)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a^
2 - 2*a*b + b^2) + (4*a^4*tan(1/2*d*x + 1/2*c)^2 - a^3*b*tan(1/2*d*x + 1/2*
c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 +
b^4*tan(1/2*d*x + 1/2*c)^2 + a^3*b - a^2*b^2 - a*b^3 + b^4)/((a^4*b - 2*a^2
*b^3 + b^5)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/b^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2)/d
```

3.264 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=248

$$\frac{b^6}{2a^3d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2b^5\left(3a^2-b^2\right)}{a^3d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^4\left(-4a^2b^2+15a^4+b^4\right)\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)^4}$$

```
[Out] b^6/(2*a^3*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*b^5*(3*a^2 - b^2))/
(a^3*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 +
 b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((2*a + 5*b)*Log[
1 - Cos[c + d*x]])/(4*(a + b)^4*d) - ((2*a - 5*b)*Log[1 + Cos[c + d*x]])/(4
*(a - b)^4*d) - (b^4*(15*a^4 - 4*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/(a
^3*(a^2 - b^2)^4*d)
```

Rubi [A] time = 0.901639, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2837, 12, 1647, 1629}

$$\frac{b^6}{2a^3d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2b^5\left(3a^2-b^2\right)}{a^3d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^4\left(-4a^2b^2+15a^4+b^4\right)\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```


```
[Out] b^6/(2*a^3*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*b^5*(3*a^2 - b^2))/
(a^3*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 +
 b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((2*a + 5*b)*Log[
1 - Cos[c + d*x]])/(4*(a + b)^4*d) - ((2*a - 5*b)*Log[1 + Cos[c + d*x]])/(4
*(a - b)^4*d) - (b^4*(15*a^4 - 4*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/(a
^3*(a^2 - b^2)^4*d)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^6}{a^6(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{a^3 d}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^6b^4\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3}+\cdots}{\cdots}\right)}{\cdots}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(-\frac{a^5(2a+5b)}{2(a+b)^4(a-\cdots}\cdots\right)\right)}{\cdots}$$

$$= \frac{b^6}{2a^3\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{2b^5\left(3a^2-b^2\right)}{a^3\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} - \frac{\left(a\left(a^2\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 6.34652, size = 713, normalized size = 2.88

$$\frac{b^6\tan^3(c+dx)(a\cos(c+dx)+b)}{2a^3d(b-a)^2(a+b)^2(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{2b^5\left(b^2-3a^2\right)\tan^3(c+dx)(a\cos(c+dx)+b)^2}{a^3d(b-a)^3(a+b)^3(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{2i\left(-4a^3b^2+\cdots\right.}{d(a\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^6*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*a^3*(-a + b)^2*(a + b)^2*d*(a*
Sin[c + d*x] + b*Tan[c + d*x])^3) - (2*b^5*(-3*a^2 + b^2)*(b + a*Cos[c + d*
x])^2*Tan[c + d*x]^3)/(a^3*(-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c
 + d*x])^3) - ((2*I)*(a^5 - 4*a^3*b^2 - 9*a*b^4)*(c + d*x)*(b + a*Cos[c + d
*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d
*x])^3) - ((I/2)*(-2*a - 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*T
an[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(
-2*a + 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((-
a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*C
```

```
sc[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c
+ d*x])^3) + ((-2*a + 5*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*T
an[c + d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((-15
*a^4*b^4 + 4*a^2*b^6 - b^8)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*
Tan[c + d*x]^3)/(a^3*(-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
+ ((-2*a - 5*b)*(b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]
^3)/(4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((b + a*Cos[c + d
*x])^3*Sec[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] +
 b*Tan[c + d*x])^3)
```

Maple [A] time = 0.18, size = 333, normalized size = 1.3

$$-\frac{1}{4\,d\,(a-b)^{3}\,(\cos(dx+c)+1)}-\frac{a\ln(\cos(dx+c)+1)}{2\,(a-b)^{4}\,d}+\frac{5\,b\ln(\cos(dx+c)+1)}{4\,(a-b)^{4}\,d}+\frac{1}{4\,d\,(a+b)^{3}\,(-1+\cos(dx+c))}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/4/d/(a-b)^3/(cos(d*x+c)+1)-1/2*a*ln(cos(d*x+c)+1)/(a-b)^4/d+5/4*b*ln(cos
(d*x+c)+1)/(a-b)^4/d+1/4/d/(a+b)^3/(-1+cos(d*x+c))-1/2/d/(a+b)^4*ln(-1+cos(
d*x+c))*a-5/4/d/(a+b)^4*ln(-1+cos(d*x+c))*b+1/2/d*b^6/a^3/(a+b)^2/(a-b)^2/(
b+a*cos(d*x+c))^2-15/d*b^4/(a+b)^4/(a-b)^4*a*ln(b+a*cos(d*x+c))+4/d*b^6/(a+
b)^4/(a-b)^4/a*ln(b+a*cos(d*x+c))-1/d*b^8/(a+b)^4/(a-b)^4/a^3*ln(b+a*cos(d*
x+c))-6/d*b^5/a/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+2/d*b^7/a^3/(a+b)^3/(a-b)^
3/(b+a*cos(d*x+c))
```

Maxima [B] time = 1.89728, size = 923, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a + b - (a - b)*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2)/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8) +
 4*(2*a + 5*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4) + (a^8 - 2*a^7*b - a^6*b^2 + 4*a^5*b^3 - a^4*b^4 - 2*a
```

```
^3*b^5 + a^2*b^6 - 2*(a^8 - 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 44*a^3*b^5 -
49*a^2*b^6 + 8*a*b^7 + 8*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^8 -
6*a^7*b + 15*a^6*b^2 - 20*a^5*b^3 + 15*a^4*b^4 - 102*a^3*b^5 + 81*a^2*b^6 +
 32*a*b^7 - 16*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^11 + a^10*b -
4*a^9*b^2 - 4*a^8*b^3 + 6*a^7*b^4 + 6*a^6*b^5 - 4*a^5*b^6 - 4*a^4*b^7 + a^3
*b^8 + a^2*b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^11 - a^10*b - 4*
a^9*b^2 + 4*a^8*b^3 + 6*a^7*b^4 - 6*a^6*b^5 - 4*a^5*b^6 + 4*a^4*b^7 + a^3*b
^8 - a^2*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^11 - 3*a^10*b + 8*a^
8*b^3 - 6*a^7*b^4 - 6*a^6*b^5 + 8*a^5*b^6 - 3*a^3*b^8 + a^2*b^9)*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*(cos(d*x + c) + 1)^2) - 8*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/a^
3)/d
```

Fricas [B] time = 1.80159, size = 2583, normalized size = 10.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^8*b^2 + 4*a^6*b^4 + 16*a^4*b^6 - 28*a^2*b^8 + 6*b^10 - 2*(3*a^9*b
- 2*a^7*b^3 + 11*a^5*b^5 - 16*a^3*b^7 + 4*a*b^9)*cos(d*x + c)^3 + 2*(a^10 -
 4*a^8*b^2 + a^6*b^4 - 9*a^4*b^6 + 14*a^2*b^8 - 3*b^10)*cos(d*x + c)^2 + 2*
(2*a^9*b + a^7*b^3 + 8*a^5*b^5 - 15*a^3*b^7 + 4*a*b^9)*cos(d*x + c) + 4*(15
*a^4*b^6 - 4*a^2*b^8 + b^10 - (15*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(d*x +
c)^4 - 2*(15*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)^3 + (15*a^6*b^4 - 19
*a^4*b^6 + 5*a^2*b^8 - b^10)*cos(d*x + c)^2 + 2*(15*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (2*a^8*b^2 + 3*a^7*b^3 - 8*a^
6*b^4 - 22*a^5*b^5 - 18*a^4*b^6 - 5*a^3*b^7 - (2*a^10 + 3*a^9*b - 8*a^8*b^2
 - 22*a^7*b^3 - 18*a^6*b^4 - 5*a^5*b^5)*cos(d*x + c)^4 - 2*(2*a^9*b + 3*a^8
*b^2 - 8*a^7*b^3 - 22*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x + c)^3 + (2
*a^10 + 3*a^9*b - 10*a^8*b^2 - 25*a^7*b^3 - 10*a^6*b^4 + 17*a^5*b^5 + 18*a^
4*b^6 + 5*a^3*b^7)*cos(d*x + c)^2 + 2*(2*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 22
*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2
) + (2*a^8*b^2 - 3*a^7*b^3 - 8*a^6*b^4 + 22*a^5*b^5 - 18*a^4*b^6 + 5*a^3*b^
7 - (2*a^10 - 3*a^9*b - 8*a^8*b^2 + 22*a^7*b^3 - 18*a^6*b^4 + 5*a^5*b^5)*co
s(d*x + c)^4 - 2*(2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6*b^4 - 18*a^5*b^5
 + 5*a^4*b^6)*cos(d*x + c)^3 + (2*a^10 - 3*a^9*b - 10*a^8*b^2 + 25*a^7*b^3
- 10*a^6*b^4 - 17*a^5*b^5 + 18*a^4*b^6 - 5*a^3*b^7)*cos(d*x + c)^2 + 2*(2*a
^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6*b^4 - 18*a^5*b^5 + 5*a^4*b^6)*cos(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^
```

```
7*b^6 + a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*
a^6*b^7 + a^4*b^9)*d*cos(d*x + c)^3 - (a^13 - 5*a^11*b^2 + 10*a^9*b^4 - 10*
a^7*b^6 + 5*a^5*b^8 - a^3*b^10)*d*cos(d*x + c)^2 - 2*(a^12*b - 4*a^10*b^3 +
 6*a^8*b^5 - 4*a^6*b^7 + a^4*b^9)*d*cos(d*x + c) - (a^11*b^2 - 4*a^9*b^4 +
6*a^7*b^6 - 4*a^5*b^8 + a^3*b^10)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48901, size = 1145, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)*l
og(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1)))/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^
8) - (a + b + 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 10*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 4*(45*a^6*b^4 + 66*a^5*b^5 - 15*a^4*b^
6 - 44*a^3*b^7 - a^2*b^8 + 10*a*b^9 + 3*b^10 + 90*a^6*b^4*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 24*a^5*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
18*a^4*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 28*a^3*b^7*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 34*a^2*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*a*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*b^10*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 45*a^6*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 90*a^5*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^6*(cos(d
```

```
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 24*a^3*b^7*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 9*a^2*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a
*b^9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*b^10*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2)/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^
8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))^2) - 8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1))/a^3)/d
```

3.265 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$-\frac{b^5}{2a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^4\left(5a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{2b^3\left(5a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{\text{csc}}{}$$

```
[Out] -b^5/(2*a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^4*(5*a^2 - b^2))/(
a^2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*
b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((a + 4*b)*Log[1 -
 Cos[c + d*x]])/(4*(a + b)^4*d) + ((a - 4*b)*Log[1 + Cos[c + d*x]])/(4*(a -
 b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.761775, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2837, 12, 1647, 1629}

$$-\frac{b^5}{2a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^4\left(5a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{2b^3\left(5a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{\text{csc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -b^5/(2*a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^4*(5*a^2 - b^2))/(
a^2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*
b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((a + 4*b)*Log[1 -
 Cos[c + d*x]])/(4*(a + b)^4*d) + ((a - 4*b)*Log[1 + Cos[c + d*x]])/(4*(a -
 b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^5}{a^5(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^5}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{a^2 d}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^6b^3\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3} -}{}\right.}{}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(-\frac{a^4(a+4b)}{2(a+b)^4(a-}\right.\right.}{}$$

$$= -\frac{b^5}{2a^2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^4\left(5a^2-b^2\right)}{a^2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} + \frac{(b}{}$$

Mathematica [A] time = 6.17891, size = 204, normalized size = 0.88

$$\frac{-\frac{4b^5}{a^2(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{8b^4\left(b^2-5a^2\right)}{a^2(b-a)^3(a+b)^3(a\cos(c+dx)+b)} + \frac{16b^3\left(5a^2+b^2\right)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^4} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3} - \frac{4(a+4b}{}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] ((-4*b^5)/(a^2*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (8*b^4*(-5*a^2
 + b^2))/(a^2*(-a + b)^3*(a + b)^3*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]
^2/(a + b)^3 + (4*(a - 4*b)*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (16*b^3*(5*a
^2 + b^2)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 - (4*(a + 4*b)*Log[Sin[(c
+ d*x)/2]])/(a + b)^4 + Sec[(c + d*x)/2]^2/(a - b)^3)/(8*d)
```

Maple [A] time = 0.174, size = 295, normalized size = 1.3

$$\frac{1}{4\,d\,(a-b)^3\,(\cos(dx+c)+1)}+\frac{a\ln(\cos(dx+c)+1)}{4\,(a-b)^4\,d}-\frac{b\ln(\cos(dx+c)+1)}{(a-b)^4\,d}+\frac{1}{4\,d\,(a+b)^3\,(-1+\cos(dx+c))}-\ln(-1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/4/d/(a-b)^3/(cos(d*x+c)+1)+1/4*a*ln(cos(d*x+c)+1)/(a-b)^4/d-b*ln(cos(d*x+
c)+1)/(a-b)^4/d+1/4/d/(a+b)^3/(-1+cos(d*x+c))-1/4/d/(a+b)^4*ln(-1+cos(d*x+c
))*a-1/d/(a+b)^4*ln(-1+cos(d*x+c))*b-1/2/d*b^5/a^2/(a+b)^2/(a-b)^2/(b+a*cos
(d*x+c))^2+10/d*b^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*a^2+2/d*b^5/(a+b)^4/
(a-b)^4*ln(b+a*cos(d*x+c))+5/d*b^4/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-1/d*b^6
/(a+b)^3/(a-b)^3/a^2/(b+a*cos(d*x+c))
```

Maxima [B] time = 1.2071, size = 795, normalized size = 3.43

$$\frac{\frac{16\left(5\,a^2b^3+b^5\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{4\,(a+4\,b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-\frac{2\left(a^6-4\,a^5b+5\,a^4b^2+3\cdots\right.}{(\cos\cdots}}{\frac{\left(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4\,a^7b^2+\cdots\right.}{\cdots}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(16*(5*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(a + 4*b)*log(s
in(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
 - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6
- 4*a^5*b + 5*a^4*b^2 + 35*a^2*b^4 + 44*a*b^5 - b^6)*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 95*a^2*b^4 - 70*
a*b^5 - 15*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b - 4*a^7*
b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b
^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^
6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*
a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
 sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2))/d
```

Fricas [B] time = 1.19362, size = 2279, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^6*b^3 + 14*a^4*b^5 - 22*a^2*b^7 + 2*b^9 - 2*(a^9 + 2*a^7*b^2 + 7*
a^5*b^4 - 12*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^3 + 2*(a^8*b - 6*a^6*b^3 - 4*a
^4*b^5 + 10*a^2*b^7 - b^9)*cos(d*x + c)^2 + 2*(5*a^7*b^2 + 4*a^5*b^4 - 11*a
^3*b^6 + 2*a*b^8)*cos(d*x + c) + 8*(5*a^4*b^5 + a^2*b^7 - (5*a^6*b^3 + a^4*
b^5)*cos(d*x + c)^4 - 2*(5*a^5*b^4 + a^3*b^6)*cos(d*x + c)^3 + (5*a^6*b^3 -
 4*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + 2*(5*a^5*b^4 + a^3*b^6)*cos(d*x + c)
)*log(a*cos(d*x + c) + b) + (a^7*b^2 - 10*a^5*b^4 - 20*a^4*b^5 - 15*a^3*b^6
 - 4*a^2*b^7 - (a^9 - 10*a^7*b^2 - 20*a^6*b^3 - 15*a^5*b^4 - 4*a^4*b^5)*cos
(d*x + c)^4 - 2*(a^8*b - 10*a^6*b^3 - 20*a^5*b^4 - 15*a^4*b^5 - 4*a^3*b^6)*
cos(d*x + c)^3 + (a^9 - 11*a^7*b^2 - 20*a^6*b^3 - 5*a^5*b^4 + 16*a^4*b^5 +
15*a^3*b^6 + 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^8*b - 10*a^6*b^3 - 20*a^5*b^4
 - 15*a^4*b^5 - 4*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^7
*b^2 - 10*a^5*b^4 + 20*a^4*b^5 - 15*a^3*b^6 + 4*a^2*b^7 - (a^9 - 10*a^7*b^2
 + 20*a^6*b^3 - 15*a^5*b^4 + 4*a^4*b^5)*cos(d*x + c)^4 - 2*(a^8*b - 10*a^6*
b^3 + 20*a^5*b^4 - 15*a^4*b^5 + 4*a^3*b^6)*cos(d*x + c)^3 + (a^9 - 11*a^7*b
^2 + 20*a^6*b^3 - 5*a^5*b^4 - 16*a^4*b^5 + 15*a^3*b^6 - 4*a^2*b^7)*cos(d*x
+ c)^2 + 2*(a^8*b - 10*a^6*b^3 + 20*a^5*b^4 - 15*a^4*b^5 + 4*a^3*b^6)*cos(d
*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*
a^6*b^6 + a^4*b^8)*d*cos(d*x + c)^4 + 2*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4
*a^5*b^7 + a^3*b^9)*d*cos(d*x + c)^3 - (a^12 - 5*a^10*b^2 + 10*a^8*b^4 - 10
*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^2 - 2*(a^11*b - 4*a^9*b^3 +
 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*cos(d*x + c) - (a^10*b^2 - 4*a^8*b^4 +
6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] time = 1.37794, size = 913, normalized size = 3.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(a + 4*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(5*a^2*b^3 + b^5)*log(abs(-a - b
- a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 2*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos
(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(
d*x + c) + 1)) + 8*(15*a^4*b^3 + 20*a^3*b^4 - 2*a^2*b^5 - 4*a*b^6 + 3*b^7 +
 30*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^3*b^4*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 26*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 10*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^7*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 15*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 - 30*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^2*b^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^8 - 4*
a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

3.266 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$\frac{b^4}{2ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{4ab^3}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{6ab^2\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc^2(c+$$

```
[Out] b^4/(2*a*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (4*a*b^3)/((a^2 - b^2)^3
*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Cos[c + d*x]
)*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - (3*b*Log[1 - Cos[c + d*x]])/(4*(a +
 b)^4*d) + (3*b*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) - (6*a*b^2*(a^2 + b^
2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.667958, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4397, 2837, 12, 1647, 1629}

$$\frac{b^4}{2ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{4ab^3}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{6ab^2\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```



```
[Out] b^4/(2*a*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (4*a*b^3)/((a^2 - b^2)^3
*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Cos[c + d*x]
)*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - (3*b*Log[1 - Cos[c + d*x]])/(4*(a +
 b)^4*d) + (3*b*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) - (6*a*b^2*(a^2 + b^
2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^4}{a^4(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{ad}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^4b^4(3a^2+b^2)}{(a^2-b^2)^3}\cdots}{\cdots}\right.}{\cdots}$$

$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(-\frac{3a^3b}{2(a+b)^4(a\cdots}\right.\right.}{\cdots}$$

$$= \frac{b^4}{2a\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{4ab^3}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} - \frac{\left(a\left(a^2+\cdots\right.\right.}{\cdots}$$

Mathematica [A] time = 5.47795, size = 184, normalized size = 0.87

$$\frac{-\frac{48ab^2\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^4} + \frac{4b^4}{a(a-b)^2(a+b)^2(a\cos(c+dx)+b)^2} + \frac{32ab^3}{(b-a)^3(a+b)^3(a\cos(c+dx)+b)} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{(b-a)^3} - \frac{12b\log\left(\text{si}\cdots\right.}{(a\cdots}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] ((4*b^4)/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (32*a*b^3)/((-a +
 b)^3*(a + b)^3*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(a + b)^3 + (12*
b*Log[Cos[(c + d*x)/2]])/(a - b)^4 - (48*a*b^2*(a^2 + b^2)*Log[b + a*Cos[c
+ d*x]])/(a^2 - b^2)^4 - (12*b*Log[Sin[(c + d*x)/2]])/(a + b)^4 + Sec[(c +
d*x)/2]^2/(-a + b)^3)/(8*d)
```

Maple [A] time = 0.142, size = 220, normalized size = 1.

$$-\frac{1}{4 d (a-b)^3 (\cos(dx+c)+1)}+\frac{3 b \ln(\cos(dx+c)+1)}{4 (a-b)^4 d}+\frac{1}{4 d (a+b)^3 (-1+\cos(dx+c))}-\frac{3 \ln(-1+\cos(dx+c)) b}{4 d (a+b)^4}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/4/d/(a-b)^3/(cos(d*x+c)+1)+3/4*b*ln(cos(d*x+c)+1)/(a-b)^4/d+1/4/d/(a+b)^
3/(-1+cos(d*x+c))-3/4/d/(a+b)^4*ln(-1+cos(d*x+c))*b+1/2/d*b^4/(a+b)^2/(a-b)
^2/a/(b+a*cos(d*x+c))^2-4/d*a*b^3/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-6/d*a^3*
b^2/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))-6/d*b^4/(a+b)^4/(a-b)^4*a*ln(b+a*cos
(d*x+c))
```

Maxima [B] time = 1.30881, size = 801, normalized size = 3.8

$$-\frac{\frac{48\left(a^3b^2+ab^4\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{12b\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(a^6-4a^5b+5a^4b^2-32a\cdots\right.}{(\cdots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4a^7b^2\cdots\right.}{\cdots}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(48*(a^3*b^2 + a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*b*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (a^
6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6 - 4*a^
5*b + 5*a^4*b^2 - 32*a^3*b^3 - 37*a^2*b^4 - 4*a*b^5 - 9*b^6)*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 84*a^3*b^3 + 63*a^2*b
^4 - 6*a*b^5 + 17*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b -
 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*
b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2
 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b
^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) +
1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^
2))/d
```

Fricas [B] time = 1.10824, size = 2115, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^6*b^2 + 18*a^4*b^4 - 18*a^2*b^6 - 2*b^8 - 6*(a^7*b + 2*a^5*b^3 - 3
*a^3*b^5)*cos(d*x + c)^3 + 2*(a^8 - 4*a^6*b^2 - 6*a^4*b^4 + 8*a^2*b^6 + b^8
)*cos(d*x + c)^2 + 2*(2*a^7*b + 9*a^5*b^3 - 12*a^3*b^5 + a*b^7)*cos(d*x + c
) + 24*(a^4*b^4 + a^2*b^6 - (a^6*b^2 + a^4*b^4)*cos(d*x + c)^4 - 2*(a^5*b^3
 + a^3*b^5)*cos(d*x + c)^3 + (a^6*b^2 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^5*b^
3 + a^3*b^5)*cos(d*x + c))*log(a*cos(d*x + c) + b) - 3*(a^5*b^3 + 4*a^4*b^4
 + 6*a^3*b^5 + 4*a^2*b^6 + a*b^7 - (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b
^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b
^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3 - 5*a^3*b^5 -
 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4
*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(a^5*b^3
- 4*a^4*b^4 + 6*a^3*b^5 - 4*a^2*b^6 + a*b^7 - (a^7*b - 4*a^6*b^2 + 6*a^5*b^
3 - 4*a^4*b^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 - 4*a^5*b^3 + 6*a^4*b^
4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b - 4*a^6*b^2 + 5*a^5*b^3 -
5*a^3*b^5 + 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 - 4*a^5*b^3 + 6*
a^4*b^4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/
((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^4 + 2*
(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^3 - (
a^11 - 5*a^9*b^2 + 10*a^7*b^4 - 10*a^5*b^6 + 5*a^3*b^8 - a*b^10)*d*cos(d*x
+ c)^2 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x
 + c) - (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] time = 1.3658, size = 932, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(6*b*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^3*b^2 + a*b^4)*log(abs(-a - b - a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 6*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 8*(9*a^5*b^2 + 10*a^4*b^3 + 2*a^3*b
^4 + 8*a^2*b^5 + 5*a*b^6 - 2*b^7 + 18*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 8*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^4*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 16*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^7*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 9*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 18*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^3*b^4
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

3.267 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=229

$$-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{b\left(8a^2b^2+3a^4+b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\cdots$$

```
[Out] -b^3/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a^2
 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos
[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c +
 d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d)
 + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.480423, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {4397, 2721, 1647, 1629}

$$-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{b\left(8a^2b^2+3a^4+b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

```
[Out] -b^3/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a^2
 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos
[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c +
 d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d)
 + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
```

```
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{-\frac{a^4b^3\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3} - \frac{a^2}{}}{}\right.}{}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(\frac{a^2(a-2b)}{2(a+b)^4(a-x)} + \right.\right.}{}$$

$$= -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^2\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} + \frac{\left(b\left(3a^2\right.\right.}{}$$

Mathematica [C] time = 6.3109, size = 696, normalized size = 3.04

$$-\frac{2i\left(8a^2b^3+3a^4b+b^5\right)(c+dx)\tan^3(c+dx)(a\cos(c+dx)+b)^3}{d(a-b)^4(a+b)^4(a\sin(c+dx)+b\tan(c+dx))^3}-\frac{b^2\left(3a^2+b^2\right)\tan^3(c+dx)(a\cos(c+dx)+b)^2}{d(b-a)^3(a+b)^3(a\sin(c+dx)+b\tan(c+dx))^3}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

```
[Out] -(b^3*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*(-a + b)^2*(a + b)^2*d*(a*Sin
[c + d*x] + b*Tan[c + d*x])^3) - (b^2*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^2*
Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3
) - ((2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x)*(b + a*Cos[c + d*x])^3*Tan
[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) -
((I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^
3)/((-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(a - 2*b)*Ar
cTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((a + b)^4*d*(a*S
in[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(c + d*x)/2]
^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((
-a - 2*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4
*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((3*a^4*b + 8*a^2*b^3
+ b^5)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*Tan[c + d*x]^3)/((-a^
2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((a - 2*b)*(b + a*Cos[c
 + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4*(a + b)^4*d*(a*Sin[c
+ d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Sec[(c + d*x)/2]^2*Ta
n[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
```

Maple [A] time = 0.151, size = 322, normalized size = 1.4

$$\frac{1}{4\,d\,(a-b)^3\,(\cos(dx+c)+1)}-\frac{a\ln(\cos(dx+c)+1)}{4\,(a-b)^4\,d}-\frac{b\ln(\cos(dx+c)+1)}{2\,(a-b)^4\,d}+\frac{1}{4\,d\,(a+b)^3\,(-1+\cos(dx+c))}+\frac{\ln(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/4/d/(a-b)^3/(cos(d*x+c)+1)-1/4*a*ln(cos(d*x+c)+1)/(a-b)^4/d-1/2*b*ln(cos(
d*x+c)+1)/(a-b)^4/d+1/4/d/(a+b)^3/(-1+cos(d*x+c))+1/4/d/(a+b)^4*ln(-1+cos(d
*x+c))*a-1/2/d/(a+b)^4*ln(-1+cos(d*x+c))*b-1/2/d*b^3/(a+b)^2/(a-b)^2/(b+a*c
os(d*x+c))^2+3/d*b/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*a^4+8/d*b^3/(a+b)^4/(
```

```
a-b)^4*ln(b+a*cos(d*x+c))*a^2+1/d*b^5/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+3/
d*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))*a^2+1/d*b^4/(a+b)^3/(a-b)^3/(b+a*cos
(d*x+c))
```

Maxima [B] time = 1.21293, size = 811, normalized size = 3.54

$$\frac{\frac{8\left(3a^4b+8a^2b^3+b^5\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{4(a-2b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(a^6-4a^5b+29a^4b\cdots\right.}{\cdots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4\cdots\right.}{\cdots}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 4*(a - 2
*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4) - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6
- 2*(a^6 - 4*a^5*b + 29*a^4*b^2 + 24*a^3*b^3 + 11*a^2*b^4 + 20*a*b^5 - b^6)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 63*a^4*b^2 - 52*a^3
*b^3 + 31*a^2*b^4 - 38*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((
a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4
*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*
b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 +
 a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*
b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d
*x + c) + 1)^2))/d
```

Fricas [B] time = 1.22552, size = 2344, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*a
*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x +
```

```
c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3 + 8
*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*a^5*
b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2*b^5
- b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c))*log
(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^5 + 9*a
*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4 + 2*a^2
*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a
^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 + 10*a^4*b^3
 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b + 6*
a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 - 16*a^2*b^5 +
9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 + 9*a^3*b^4 - 2*
a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 +
9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a^5*b^2 - 10*a^4*
b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b -
 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a
^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b
^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a\sin{(c+dx)}+b\tan{(c+dx)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-3), x)
```

Giac [B] time = 1.23048, size = 1080, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^3*b^4 +
11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^6*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^2
*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

3.268 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=231

$$\frac{ab^2}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2ab\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{a\left(8a^2b^2+a^4+3b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc}{}$$

```
[Out] (a*b^2)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*a*b*(a^2 + b^2))/((
a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*
Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((2*a - b)*Log[1 - Cos[
c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4
*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*
d)
```

Rubi [A] time = 0.620968, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {4397, 2837, 12, 1647, 1629}

$$\frac{ab^2}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2ab\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{a\left(8a^2b^2+a^4+3b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```


```
[Out] (a*b^2)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*a*b*(a^2 + b^2))/((
a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*
Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((2*a - b)*Log[1 - Cos[
c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4
*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*
d)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^2}{a^2(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x^2}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \frac{\frac{a^2b^4(3a^2+b^2)}{\left(a^2-b^2\right)^3}}{}\right.}{}$$
$$= -\frac{\left(a\left(a^2+3b^2\right)-b\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int\left(\frac{a(2a-b)}{2(a+b)^4(a-}\right.\right.}{}$$
$$= \frac{ab^2}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{2ab\left(a^2+b^2\right)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} - \frac{\left(a\left(a^2+\right.\right.}{}$$

Mathematica [C] time = 6.31286, size = 703, normalized size = 3.04

$$\frac{2i\left(8a^3b^2+a^5+3ab^4\right)(c+dx)\tan^3(c+dx)(a\cos(c+dx)+b)^3}{d(a-b)^4(a+b)^4(a\sin(c+dx)+b\tan(c+dx))^3} + \frac{\left(-8a^3b^2-a^5-3ab^4\right)\tan^3(c+dx)(a\cos(c+dx)+b}{d\left(b^2-a^2\right)^4(a\sin(c+dx)+b\tan(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*b^2*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*(-a + b)^2*(a + b)^2*d*(a*Si
n[c + d*x] + b*Tan[c + d*x])^3) + (2*a*b*((-I)*a + b)*(I*a + b)*(b + a*Cos[
c + d*x])^2*Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan
[c + d*x])^3) + ((2*I)*(a^5 + 8*a^3*b^2 + 3*a*b^4)*(c + d*x)*(b + a*Cos[c +
 d*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c +
 d*x])^3) - ((I/2)*(2*a - b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Ta
n[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(2
*a + b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((-a +
b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(
```

```
c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*
x])^3) + ((2*a + b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*Tan[c +
d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((-a^5 - 8*a
^3*b^2 - 3*a*b^4)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*Tan[c + d*
x]^3)/((-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((2*a - b)*(
b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4*(a + b)^4*
d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[(c + d
*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^
3)
```

Maple [A] time = 0.177, size = 324, normalized size = 1.4

$$-\frac{1}{4\,d\,(a-b)^3\,(\cos(dx+c)+1)}+\frac{b\ln(\cos(dx+c)+1)}{4\,(a-b)^4\,d}+\frac{a\ln(\cos(dx+c)+1)}{2\,(a-b)^4\,d}+\frac{1}{4\,d\,(a+b)^3\,(-1+\cos(dx+c))}+\frac{\ln(\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/4/d/(a-b)^3/(cos(d*x+c)+1)+1/4*b*ln(cos(d*x+c)+1)/(a-b)^4/d+1/2*a*ln(cos
(d*x+c)+1)/(a-b)^4/d+1/4/d/(a+b)^3/(-1+cos(d*x+c))+1/2/d/(a+b)^4*ln(-1+cos(
d*x+c))*a-1/4/d/(a+b)^4*ln(-1+cos(d*x+c))*b-1/d*a^5/(a+b)^4/(a-b)^4*ln(b+a*
cos(d*x+c))-8/d*a^3*b^2/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))-3/d*b^4/(a+b)^4/
(a-b)^4*a*ln(b+a*cos(d*x+c))+1/2/d*b^2/(a+b)^2*a/(a-b)^2/(b+a*cos(d*x+c))^2
-2/d*a^3*b/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-2/d*a*b^3/(a+b)^3/(a-b)^3/(b+a*
cos(d*x+c))
```

Maxima [B] time = 1.19394, size = 813, normalized size = 3.52

$$-\frac{\frac{8\left(a^5+8a^3b^2+3ab^4\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4(2a-b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(a^6-20a^5b-11\cdots\right.}{\cdots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b\cdots\right.}{\cdots}}}{8a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(2*a
```

```
- b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6
 - 2*(a^6 - 20*a^5*b - 11*a^4*b^2 - 24*a^3*b^3 - 29*a^2*b^4 + 4*a*b^5 - b^6
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 38*a^5*b + 31*a^4*b^2 - 52*a
^3*b^3 + 63*a^2*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(
(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
4*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8
*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7
+ a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6
*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(
d*x + c) + 1)^2))/d
```

Fricas [B] time = 1.17415, size = 2338, normalized size = 10.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a^5*b^2 + 8*a^3*b^4 - 16*a*b^6 - 2*(7*a^6*b - 2*a^4*b^3 - 5*a^2*b^5)
*cos(d*x + c)^3 + 2*(a^7 - 7*a^5*b^2 - a^3*b^4 + 7*a*b^6)*cos(d*x + c)^2 +
2*(6*a^6*b + a^4*b^3 - 8*a^2*b^5 + b^7)*cos(d*x + c) + 4*(a^5*b^2 + 8*a^3*b
^4 + 3*a*b^6 - (a^7 + 8*a^5*b^2 + 3*a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b + 8*
a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^3 + (a^7 + 7*a^5*b^2 - 5*a^3*b^4 - 3*a*b^
6)*cos(d*x + c)^2 + 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c))*log(a*c
os(d*x + c) + b) - (2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b
^6 + b^7 - (2*a^7 + 9*a^6*b + 16*a^5*b^2 + 14*a^4*b^3 + 6*a^3*b^4 + a^2*b^5
)*cos(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2
*b^5 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 9*a^6*b + 14*a^5*b^2 + 5*a^4*b^3 -
10*a^3*b^4 - 13*a^2*b^5 - 6*a*b^6 - b^7)*cos(d*x + c)^2 + 2*(2*a^6*b + 9*a^
5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^5 + a*b^6)*cos(d*x + c))*log(1/2*
cos(d*x + c) + 1/2) - (2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5 + 6*
a*b^6 - b^7 - (2*a^7 - 9*a^6*b + 16*a^5*b^2 - 14*a^4*b^3 + 6*a^3*b^4 - a^2*
b^5)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*
a^2*b^5 - a*b^6)*cos(d*x + c)^3 + (2*a^7 - 9*a^6*b + 14*a^5*b^2 - 5*a^4*b^3
 - 10*a^3*b^4 + 13*a^2*b^5 - 6*a*b^6 + b^7)*cos(d*x + c)^2 + 2*(2*a^6*b - 9
*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*a^2*b^5 - a*b^6)*cos(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b
^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9
)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^
```

```
8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
 a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^1
0)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] time = 1.3857, size = 1081, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(2*a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) + 4*(3*a^7 - 2*a^6*b + 11*a^5*b^2 + 28*a^4*b^3 + 9*a
^3*b^4 + 6*a^2*b^5 + 9*a*b^6 + 6*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 38*a^5*b^2*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 4*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 26*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^2*b^5*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 18*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a^6*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 27*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 48*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^3*b^4*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(cos(d*x + c) - 1)^2
```

```
/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

3.269 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=212

$$\frac{3a^2\left(a^2+3b^2\right)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{3a^2b}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{6a^2b\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{\csc^2(c+}{2d\left(a^2-b\right.}$$

```
[Out] (-3*a^2*b)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*a^2*(a^2 + 3*b^2
))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b - a*Cos[c + d*x])*Csc[c +
 d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (3*a*Log[1 - Cos[c + d*
x]])/(4*(a + b)^4*d) - (3*a*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) + (6*a^2
*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.363985, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {4397, 2837, 12, 823, 801}

$$\frac{3a^2\left(a^2+3b^2\right)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{3a^2b}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{6a^2b\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{\csc^2(c+}{2d\left(a^2-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```


```
[Out] (-3*a^2*b)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*a^2*(a^2 + 3*b^2
))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b - a*Cos[c + d*x])*Csc[c +
 d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (3*a*Log[1 - Cos[c + d*
x]])/(4*(a + b)^4*d) - (3*a*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) + (6*a^2
*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```


Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
```

```
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot(c+dx)\csc^2(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$
$$= -\frac{a^3\operatorname{Subst}\left(\int \frac{x}{a(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\operatorname{Subst}\left(\int \frac{x}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \frac{-3a^2b+3a^2x}{(b+x)^3(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{2(a^2-b^2)\,d}$$
$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{\operatorname{Subst}\left(\int \left(\frac{3a(a-b)}{2(a+b)^3(a-x)} + \frac{3a(a+b)}{2(a-b)^3(a+x)} - \frac{6a^2b}{(a^2-b^2)(b}\right.\right.}{2}$$
$$= -\frac{3a^2b}{2(a^2-b^2)^2\,d(b+a\cos(c+dx))^2} + \frac{3a^2(a^2+3b^2)}{2(a^2-b^2)^3\,d(b+a\cos(c+dx))} + \frac{(b-a}{2(a^2-}$$

Mathematica [A] time = 6.27284, size = 217, normalized size = 1.02

$$-\frac{a^2(a^2+3b^2)}{d(b-a)^3(a+b)^3(a\cos(c+dx)+b)} + \frac{6(a^2b^3+a^4b)\log(a\cos(c+dx)+b)}{d(b^2-a^2)^4} - \frac{a^2b}{2d(b-a)^2(a+b)^2(a\cos(c+dx)+b)^2} -$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -(a^2*b)/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) - (a^2*(a^2 + 3*
b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(8
*(a + b)^3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) + (6*(a^4*b +
a^2*b^3)*Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + (3*a*Log[Sin[(c + d*
x)/2]])/(2*(a + b)^4*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

Maple [A] time = 0.194, size = 251, normalized size = 1.2

$$\frac{1}{4d(a-b)^3(\cos(dx+c)+1)} - \frac{3a\ln(\cos(dx+c)+1)}{4(a-b)^4d} + \frac{1}{4d(a+b)^3(-1+\cos(dx+c))} + \frac{3\ln(-1+\cos(dx+c))a}{4d(a+b)^4} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] 1/4/d/(a-b)^3/(cos(d*x+c)+1)-3/4*a*ln(cos(d*x+c)+1)/(a-b)^4/d+1/4/d/(a+b)^3
/(-1+cos(d*x+c))+3/4/d/(a+b)^4*ln(-1+cos(d*x+c))*a-1/2/d*b*a^2/(a+b)^2/(a-b
)^2/(b+a*cos(d*x+c))^2+1/d*a^4/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+3/d*b^2/(a+
b)^3/(a-b)^3/(b+a*cos(d*x+c))*a^2+6/d*b/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*
a^4+6/d*b^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*a^2
```

Maxima [B] time = 1.26719, size = 805, normalized size = 3.8

$$\frac{\frac{48\left(a^4b+a^2b^3\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{12a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(9a^6+4a^5b+37a^4b^2+\cdots\right.}{\cdots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4a^7b^2\cdots\right.}{\cdots}}\cdots}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(48*(a^4*b + a^2*b^3)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*a*log(sin(d*x
+ c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^6
 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(9*a^6 + 4*a
^5*b + 37*a^4*b^2 + 32*a^3*b^3 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + (17*a^6 - 6*a^5*b + 63*a^4*b^2 - 84*a^3*b^3 + 15*a^2
*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b -
4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b
^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2
+ 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^
4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2
))/d
```

Fricas [B] time = 1.11246, size = 2021, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^6*b + 18*a^4*b^3 - 18*a^2*b^5 - 2*b^7 - 6*(a^7 + 2*a^5*b^2 - 3*a^
3*b^4)*cos(d*x + c)^3 - 24*(a^4*b^3 - a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^7 +
9*a^5*b^2 - 12*a^3*b^4 + a*b^6)*cos(d*x + c) + 24*(a^4*b^3 + a^2*b^5 - (a^6
*b + a^4*b^3)*cos(d*x + c)^4 - 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 + (a^6*
b - a^2*b^5)*cos(d*x + c)^2 + 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos
(d*x + c) + b) - 3*(a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6 - (
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b
+ 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 + (a^7 + 4*a^
6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b
+ 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) + 3*(a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6 - (a
^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 + (a^7 - 4*a^6
*b + 5*a^5*b^2 - 5*a^3*b^4 + 4*a^2*b^5 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*log(-1/2*cos(d*
x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(
d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*
x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*
d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*
cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] time = 1.39573, size = 930, normalized size = 4.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(6*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^4*b + a^2*b^3)*log(abs(-a - b - a*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))
)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 6*a*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) + 8*(2*a^7 - 5*a^6*b - 8*a^5*b^2 - 2*a
^4*b^3 - 10*a^3*b^4 - 9*a^2*b^5 + 2*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 16*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^5*b^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 8*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^2*b^5*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^4*b^3*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^3*b^4*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 9*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

3.270 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=228

$$-\frac{ab\left(11a^2+b^2\right)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{a\left(2a^2+b^2\right)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2a^3\left(a^2+5b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc^2(c}{2d\left(a^2-\right.}$$

```
[Out] (a*(2*a^2 + b^2))/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (a*b*(11*a^2
 + b^2))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a - b*Cos[c + d*x])*C
sc[c + d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((4*a + b)*Log[1
- Cos[c + d*x]])/(4*(a + b)^4*d) + ((4*a - b)*Log[1 + Cos[c + d*x]])/(4*(a
- b)^4*d) - (2*a^3*(a^2 + 5*b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.40793, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2668, 741, 801}

$$-\frac{ab\left(11a^2+b^2\right)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{a\left(2a^2+b^2\right)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2a^3\left(a^2+5b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc^2(c}{2d\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + b^2))/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (a*b*(11*a^2
 + b^2))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a - b*Cos[c + d*x])*C
sc[c + d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((4*a + b)*Log[1
- Cos[c + d*x]])/(4*(a + b)^4*d) + ((4*a - b)*Log[1 + Cos[c + d*x]])/(4*(a
- b)^4*d) - (2*a^3*(a^2 + 5*b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2668

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
```

```
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 741

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp
[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2
+ a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[
c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^
2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] &&
 LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\csc^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\,\text{Subst}\left(\int \frac{1}{(b+x)^3\left(a^2-x^2\right)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{a\,\text{Subst}\left(\int \frac{-4a^2+b^2+3bx}{(b+x)^3\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx\right.}{2\left(a^2-b^2\right)d}$$

$$= -\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{a\,\text{Subst}\left(\int \left(\frac{(-4a-b)(a-b)}{2a(a+b)^3(a-x)} + \frac{(4a-b)(a+b)}{2a(a-b)^3(a+x)} - \right.\right.}{}$$

$$= \frac{a\left(2a^2+b^2\right)}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{ab\left(11a^2+b^2\right)}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} - \frac{(a-b}{2\left(a^2-\right.}$$

Mathematica [A] time = 6.26157, size = 217, normalized size = 0.95

$$-\frac{2\left(5a^3b^2+a^5\right)\log(a\cos(c+dx)+b)}{d\left(b^2-a^2\right)^4} + \frac{4a^3b}{d(b-a)^3(a+b)^3(a\cos(c+dx)+b)} + \frac{a^3}{2d(b-a)^2(a+b)^2(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] a^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (4*a^3*b)/((-a + b)
^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) +
 ((4*a - b)*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) - (2*(a^5 + 5*a^3*b^2)*
Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + ((4*a + b)*Log[Sin[(c + d*x)/
2]])/(2*(a + b)^4*d) + Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

Maple [A] time = 0.183, size = 256, normalized size = 1.1

$$-\frac{1}{4d(a-b)^3(\cos(dx+c)+1)}+\frac{a\ln(\cos(dx+c)+1)}{(a-b)^4 d}-\frac{b\ln(\cos(dx+c)+1)}{4(a-b)^4 d}+\frac{1}{4d(a+b)^3(-1+\cos(dx+c))}+\frac{\ln(\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] -1/4/d/(a-b)^3/(cos(d*x+c)+1)+a*ln(cos(d*x+c)+1)/(a-b)^4/d-1/4*b*ln(cos(d*x
+c)+1)/(a-b)^4/d+1/4/d/(a+b)^3/(-1+cos(d*x+c))+1/d/(a+b)^4*ln(-1+cos(d*x+c)
)*a+1/4/d/(a+b)^4*ln(-1+cos(d*x+c))*b+1/2/d*a^3/(a+b)^2/(a-b)^2/(b+a*cos(d*
x+c))^2-4/d*a^3*b/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-2/d*a^5/(a+b)^4/(a-b)^4*
ln(b+a*cos(d*x+c))-10/d*a^3*b^2/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))
```

Maxima [B] time = 1.23796, size = 798, normalized size = 3.5

$$-\frac{\frac{16\left(a^5+5a^3b^2\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4(4a+b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(a^6-44a^5b-35a^4\ldots\right)}{\ldots}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4a^7b\ldots\right)}{\ldots}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(16*(a^5 + 5*a^3*b^2)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(4*a + b)*log(
sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6
 - 44*a^5*b - 35*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x + c)^2/(cos(d
```

```
*x + c) + 1)^2 - (15*a^6 + 70*a^5*b - 95*a^4*b^2 + 20*a^3*b^3 - 15*a^2*b^4
+ 6*a*b^5 - b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b - 4*a^7
*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 +
b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a
^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6
*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)
+ sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2))/d
```

Fricas [B] time = 1.16053, size = 2142, normalized size = 9.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^7 - 22*a^5*b^2 + 14*a^3*b^4 + 6*a*b^6 + 2*(11*a^6*b - 10*a^4*b^3
- a^2*b^5)*cos(d*x + c)^3 - 4*(a^7 - 7*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*x
 + c)^2 - 2*(10*a^6*b - 7*a^4*b^3 - 4*a^2*b^5 + b^7)*cos(d*x + c) - 8*(a^5*
b^2 + 5*a^3*b^4 - (a^7 + 5*a^5*b^2)*cos(d*x + c)^4 - 2*(a^6*b + 5*a^4*b^3)*
cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c)^2 + 2*(a^6*b +
5*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^5*b^2 + 15*a^4*b^3
+ 20*a^3*b^4 + 10*a^2*b^5 - b^7 - (4*a^7 + 15*a^6*b + 20*a^5*b^2 + 10*a^4*b
^3 - a^2*b^5)*cos(d*x + c)^4 - 2*(4*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 10*a^
3*b^4 - a*b^6)*cos(d*x + c)^3 + (4*a^7 + 15*a^6*b + 16*a^5*b^2 - 5*a^4*b^3
- 20*a^3*b^4 - 11*a^2*b^5 + b^7)*cos(d*x + c)^2 + 2*(4*a^6*b + 15*a^5*b^2 +
 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + (4*a^5*b^2 - 15*a^4*b^3 + 20*a^3*b^4 - 10*a^2*b^5 + b^7 - (4*a^7 - 15*a^
6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(4*a^6*b - 15*a
^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (4*a^7 - 15*a^6*
b + 16*a^5*b^2 + 5*a^4*b^3 - 20*a^3*b^4 + 11*a^2*b^5 - b^7)*cos(d*x + c)^2
+ 2*(4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a
^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*
b^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8
+ b^10)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] time = 1.39256, size = 911, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(4*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(a^5 + 5*a^3*b^2)*log(abs(-a - b -
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 8*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(
d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d
*x + c) + 1)) + 8*(3*a^7 - 4*a^6*b - 2*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 +
4*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^6*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 26*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 10*a
^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 30*a^3*b^4*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*
a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^5*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 - 30*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 15*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

3.271 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=155

$$-\frac{b\left(3a^2-b^2\right)\cos^m(c+dx)}{dm}-\frac{a\left(a^2-3b^2\right)\cos^{m+1}(c+dx)}{d(m+1)}+\frac{3a^2b\cos^{m+2}(c+dx)}{d(m+2)}+\frac{a^3\cos^{m+3}(c+dx)}{d(m+3)}+\frac{3ab^2\cos^{m-1}(c}{d(1-m)}$$

```
[Out] (b^3*Cos[c + d*x]^(-2 + m))/(d*(2 - m)) + (3*a*b^2*Cos[c + d*x]^(-1 + m))/(
d*(1 - m)) - (b*(3*a^2 - b^2)*Cos[c + d*x]^m)/(d*m) - (a*(a^2 - 3*b^2)*Cos[
c + d*x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*Cos[c + d*x]^(2 + m))/(d*(2 + m))
+ (a^3*Cos[c + d*x]^(3 + m))/(d*(3 + m))
```

Rubi [A] time = 0.384805, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {4397, 2837, 948}

$$-\frac{b\left(3a^2-b^2\right)\cos^m(c+dx)}{dm}-\frac{a\left(a^2-3b^2\right)\cos^{m+1}(c+dx)}{d(m+1)}+\frac{3a^2b\cos^{m+2}(c+dx)}{d(m+2)}+\frac{a^3\cos^{m+3}(c+dx)}{d(m+3)}+\frac{3ab^2\cos^{m-1}(c}{d(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^3*Cos[c + d*x]^(-2 + m))/(d*(2 - m)) + (3*a*b^2*Cos[c + d*x]^(-1 + m))/(
d*(1 - m)) - (b*(3*a^2 - b^2)*Cos[c + d*x]^m)/(d*m) - (a*(a^2 - 3*b^2)*Cos[
c + d*x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*Cos[c + d*x]^(2 + m))/(d*(2 + m))
+ (a^3*Cos[c + d*x]^(3 + m))/(d*(3 + m))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int \cos^{-3+m}(c+dx)(b+a\cos(c+dx))^3\sin^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{x}{a}\right)^{-3+m}(b+x)^3\left(a^2-x^2\right)dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \left(a^2b^3\left(\frac{x}{a}\right)^{-3+m}+3a^3b^2\left(\frac{x}{a}\right)^{-2+m}+a^2b\left(3a^2-b^2\right)\left(\frac{x}{a}\right)^{-1+m}\right.\right.}{}$$
$$= \frac{b^3\cos^{-2+m}(c+dx)}{d(2-m)}+\frac{3ab^2\cos^{-1+m}(c+dx)}{d(1-m)}-\frac{b\left(3a^2-b^2\right)\cos^m(c+}{dm}$$

Mathematica [A] time = 1.51235, size = 246, normalized size = 1.59

$$\frac{\cos^{m+1}(c+dx)(a+b\sec(c+dx))^3\left(-am\left(m^3-m^2-4m+4\right)\left(a^2(m+9)-12b^2(m+3)\right)\cos^3(c+dx)+\left(m^3-2m^2-m\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*(-4*b^3*m*(-6 - 5*m + 5*m^2 + 5*m^3 + m^4) - 12*a*b^2
*m*(-12 - 16*m - m^2 + 4*m^3 + m^4)*Cos[c + d*x] - a*m*(4 - 4*m - m^2 + m^3
)*(-12*b^2*(3 + m) + a^2*(9 + m))*Cos[c + d*x]^3 + (2 - m - 2*m^2 + m^3)*Co
s[c + d*x]^2*(2*b*(3 + m)*(2*b^2*(2 + m) - 3*a^2*(4 + m)) + 6*a^2*b*m*(3 +
m)*Cos[2*(c + d*x)] + a^3*m*(2 + m)*Cos[3*(c + d*x)]))*(a + b*Sec[c + d*x])
^3)/(4*d*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*(3 + m)*(b + a*Cos[c + d*x])^3
)
```

Maple [F] time = 3.616, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (a\sin(dx+c)+b\tan(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

Maxima [A] time = 1.12904, size = 243, normalized size = 1.57

$$\frac{\frac{\left((m+1)\cos(dx+c)^3-(m+3)\cos(dx+c)\right)a^3\cos(dx+c)^m}{m^2+4\,m+3}+\frac{3\left(m\cos(dx+c)^2-m-2\right)a^2b\cos(dx+c)^m}{m^2+2\,m}+\frac{3\left((m-1)\cos(dx+c)^2-m-1\right)ab^2\cos(dx+c)^m}{\left(m^2-1\right)\cos(dx+c)}+\frac{\left((m-2)\cos\right.}{\left(m\right.}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (((m + 1)*cos(d*x + c)^3 - (m + 3)*cos(d*x + c))*a^3*cos(d*x + c)^m/(m^2 +
4*m + 3) + 3*(m*cos(d*x + c)^2 - m - 2)*a^2*b*cos(d*x + c)^m/(m^2 + 2*m) +
3*((m - 1)*cos(d*x + c)^2 - m - 1)*a*b^2*cos(d*x + c)^m/((m^2 - 1)*cos(d*x
+ c)) + ((m - 2)*cos(d*x + c)^2 - m)*b^3*cos(d*x + c)^m/((m^2 - 2*m)*cos(d*
x + c)^2))/d
```

Fricas [B] time = 0.612053, size = 876, normalized size = 5.65

$$-\frac{\left(b^3m^5+5\,b^3m^4+5\,b^3m^3-\left(a^3m^5-5\,a^3m^3+4\,a^3m\right)\cos\left(dx+c\right)^5-5\,b^3m^2-3\left(a^2bm^5+a^2bm^4-7\,a^2bm^3-a^2bm^2+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(b^3*m^5 + 5*b^3*m^4 + 5*b^3*m^3 - (a^3*m^5 - 5*a^3*m^3 + 4*a^3*m)*cos(d*x
 + c)^5 - 5*b^3*m^2 - 3*(a^2*b*m^5 + a^2*b*m^4 - 7*a^2*b*m^3 - a^2*b*m^2 +
6*a^2*b*m)*cos(d*x + c)^4 - 6*b^3*m + ((a^3 - 3*a*b^2)*m^5 + 2*(a^3 - 3*a*b
^2)*m^4 - 7*(a^3 - 3*a*b^2)*m^3 - 8*(a^3 - 3*a*b^2)*m^2 + 12*(a^3 - 3*a*b^2
)*m)*cos(d*x + c)^3 + ((3*a^2*b - b^3)*m^5 + 3*(3*a^2*b - b^3)*m^4 - 5*(3*a
^2*b - b^3)*m^3 + 36*a^2*b - 12*b^3 - 15*(3*a^2*b - b^3)*m^2 + 4*(3*a^2*b -
 b^3)*m)*cos(d*x + c)^2 + 3*(a*b^2*m^5 + 4*a*b^2*m^4 - a*b^2*m^3 - 16*a*b^2
*m^2 - 12*a*b^2*m)*cos(d*x + c))*cos(d*x + c)^m/((d*m^6 + 3*d*m^5 - 5*d*m^4
```

```
- 15*d*m^3 + 4*d*m^2 + 12*d*m)*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)

[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] Timed out
```

3.272 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=264

$$-\frac{\left(a^2(1-m)-b^2(m+2)\right)\sin(c+dx)\cos^{m-1}(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\cos^2(c+dx)\right)}{d(1-m)m(m+2)\sqrt{\sin^2(c+dx)}}-\frac{2ab\sin(c+d\ldots}{\ldots}$$

```
[Out] ((a^2 - 2*b^2)*Cos[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*m*(2 + m)) - (2*a*b*C
os[c + d*x]^m*Sin[c + d*x])/(d*(2 + 3*m + m^2)) - (Cos[c + d*x]^(-1 + m)*(b
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*(2 + m)) - ((a^2*(1 - m) - b^2*(2 + m
))*Cos[c + d*x]^(-1 + m)*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - m)*m*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a
*b*Cos[c + d*x]^m*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Si
n[c + d*x])/(d*m*(1 + m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.764108, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {4397, 2889, 3050, 3033, 3023, 2748, 2643}

$$-\frac{\left(a^2(1-m)-b^2(m+2)\right)\sin(c+dx)\cos^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\cos^2(c+dx)\right)}{d(1-m)m(m+2)\sqrt{\sin^2(c+dx)}}+\frac{\left(a^2-2b^2\right)\sin(c+dx)\cos^{m-1}(c\ldots}{dm(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*b^2)*Cos[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*m*(2 + m)) - (2*a*b*C
os[c + d*x]^m*Sin[c + d*x])/(d*(2 + 3*m + m^2)) - (Cos[c + d*x]^(-1 + m)*(b
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*(2 + m)) - ((a^2*(1 - m) - b^2*(2 + m
))*Cos[c + d*x]^(-1 + m)*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - m)*m*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a
*b*Cos[c + d*x]^m*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Si
n[c + d*x])/(d*m*(1 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int \cos^{-2+m}(c+dx)(b+a\cos(c+dx))^2\sin^2(c+dx)\,dx$$

$$= \int \cos^{-2+m}(c+dx)(b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\,dx$$

$$= -\frac{\cos^{-1+m}(c+dx)(b+a\cos(c+dx))^2\sin(c+dx)}{d(2+m)} + \frac{\int \cos^{-2+m}(c \cdot}{}$$

$$= -\frac{2ab\cos^m(c+dx)\sin(c+dx)}{d\left(2+3m+m^2\right)} - \frac{\cos^{-1+m}(c+dx)(b+a\cos(c+d}{d(2+m)}$$

$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c+dx)\sin(c+}{d\left(2+3m+m^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c+dx)\sin(c+}{d\left(2+3m+m^2\right)}$$

$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c+dx)\sin(c+}{d\left(2+3m+m^2\right)}$$

Mathematica [A] time = 12.386, size = 401, normalized size = 1.52

$$\frac{\sin(c+dx)\cos^{m-1}(c+dx)\left(\frac{2048a^2\cos^4(c+dx)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+3}{2},\frac{m+5}{2},\cos^2(c+dx)\right)}{m+3} - \frac{2048a^2\cos^2(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m+1}{2}\right.}{m+1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]^(-1 + m)*(-((b^2*Hypergeometric2F1[-1/2, (-1 + m)/2, (1 + m)/
2, Cos[c + d*x]^2])/(-1 + m)) - (2*a*b*Cos[c + d*x]*Hypergeometric2F1[-1/2,
 m/2, (2 + m)/2, Cos[c + d*x]^2])/m - (2048*a^2*Cos[c + d*x]^2*Hypergeometr
```

```
ic2F1[-1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2])/(1 + m) - (4095*b^2*Hype
rgeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[c + d*x]^2])/(-1 + m) - (8190
*a*b*Cos[c + d*x]*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2])/m
 - (2048*a^2*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Co
s[c + d*x]^2])/(1 + m) + (4095*b^2*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1
 + m)/2, (3 + m)/2, Cos[c + d*x]^2])/(1 + m) + (8190*a*b*Cos[c + d*x]^3*Hyp
ergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[c + d*x]^2])/(2 + m) + (2048*
a^2*Cos[c + d*x]^4*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x
]^2])/(3 + m))*Sin[c + d*x])/(4096*d*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 1.85, size = 0, normalized size = 0.

$$\int (\cos (dx + c))^m (a \sin (dx + c) + b \tan (dx + c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sin (dx + c) + b \tan (dx + c))^2 \cos (dx + c)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(a^2 \cos (dx + c)^2 - 2\, ab \sin (dx + c) \tan (dx + c) - b^2 \tan (dx + c)^2 - a^2\right) \cos (dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)*tan(d*x + c) - b^2*tan(d
*x + c)^2 - a^2)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+b\tan(dx+c))^2\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

3.273 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=39

$$-\frac{a\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b\cos^m(c+dx)}{dm}$$

```
[Out] -((b*Cos[c + d*x]^m)/(d*m)) - (a*Cos[c + d*x]^(1 + m))/(d*(1 + m))
```

Rubi [A] time = 0.073123, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4377, 12, 2565, 30}

$$-\frac{a\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b\cos^m(c+dx)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x]^m)/(d*m)) - (a*Cos[c + d*x]^(1 + m))/(d*(1 + m))
```

Rule 4377

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = a\int \cos^m(c+dx)\sin(c+dx)\,dx+\int b\cos^{-1+m}(c+dx)\sin(c+dx)$$

$$= b\int \cos^{-1+m}(c+dx)\sin(c+dx)\,dx-\frac{a\,\text{Subst}\left(\int x^m\,dx,x,\cos(c+\right.}{d}$$

$$= -\frac{a\cos^{1+m}(c+dx)}{d(1+m)}-\frac{b\,\text{Subst}\left(\int x^{-1+m}\,dx,x,\cos(c+dx)\right)}{d}$$

$$= -\frac{b\cos^m(c+dx)}{dm}-\frac{a\cos^{1+m}(c+dx)}{d(1+m)}$$

Mathematica [A] time = 0.0790838, size = 35, normalized size = 0.9

$$-\frac{\cos^m(c+dx)(am\cos(c+dx)+bm+b)}{dm(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x]^m*(b + b*m + a*m*Cos[c + d*x]))/(d*m*(1 + m)))
```

Maple [A] time = 0.056, size = 40, normalized size = 1.

$$-\frac{b\left(\cos\left(dx+c\right)\right)^m}{dm}-\frac{a\left(\cos\left(dx+c\right)\right)^{1+m}}{d\left(1+m\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] -b*cos(d*x+c)^m/d/m-a*cos(d*x+c)^(1+m)/d/(1+m)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.495067, size = 81, normalized size = 2.08

$$-\frac{(am\cos(dx+c)+bm+b)\cos(dx+c)^m}{dm^2+dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*m*cos(d*x + c) + b*m + b)*cos(d*x + c)^m/(d*m^2 + d*m)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(c+dx)+b\tan(c+dx))\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**m, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sin(dx+c)+b\tan(dx+c))\cos(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))*cos(d*x + c)^m, x)
```

3.274 $\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=144

$$-\frac{a^2\cos^{m+2}(c+dx)\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{a\cos(c+dx)}{b}\right)}{bd(m+2)\left(a^2-b^2\right)}+\frac{\cos^{m+2}(c+dx)\text{Hypergeometric2F1}(1,m+2}{2d(m+2)(a-b)}$$

```
[Out] (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c + d*x]])/(2
*(a - b)*d*(2 + m)) - (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
 m, Cos[c + d*x]])/(2*(a + b)*d*(2 + m)) - (a^2*Cos[c + d*x]^(2 + m)*Hyperg
eometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)])/(b*(a^2 - b^2)*d*(2 +
m))
```

Rubi [A] time = 0.431573, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {4397, 2837, 961, 64}

$$-\frac{a^2\cos^{m+2}(c+dx)\,{}_2F_1\left(1,m+2;m+3;-\frac{a\cos(c+dx)}{b}\right)}{bd(m+2)\left(a^2-b^2\right)}+\frac{\cos^{m+2}(c+dx)\,{}_2F_1(1,m+2;m+3;-\cos(c+dx))}{2d(m+2)(a-b)}-\frac{\cos^{m+2}(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c + d*x]])/(2
*(a - b)*d*(2 + m)) - (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
 m, Cos[c + d*x]])/(2*(a + b)*d*(2 + m)) - (a^2*Cos[c + d*x]^(2 + m)*Hyperg
eometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)])/(b*(a^2 - b^2)*d*(2 +
m))
```

Rule 4397

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 961

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 64

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(c^n*(b*x
)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*x)/c)])/(b*(m + 1)), x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-(d/(b*c)), 0])))
```

Rubi steps

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^{1+m}(c+dx)\csc(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\,\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{(b+x)\left(a^2-x^2\right)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a\,\text{Subst}\left(\int \left(\frac{\left(\frac{x}{a}\right)^{1+m}}{2a(a+b)(a-x)} - \frac{\left(\frac{x}{a}\right)^{1+m}}{2a(a-b)(a+x)} + \frac{\left(\frac{x}{a}\right)^{1+m}}{(a-b)(a+b)(b+x)}\right)dx, x, a\cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{a+x}\,dx, x, a\cos(c+dx)\right)}{2(a-b)d} - \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{a-x}\,dx, x, a\cos(c+dx)\right)}{2(a+b)d} - -$$

$$= \frac{\cos^{2+m}(c+dx)\,{}_2F_1(1,2+m;3+m;-\cos(c+dx))}{2(a-b)d(2+m)} - \frac{\cos^{2+m}(c+dx)\,{}_2F_1(1,2+m}{2(a+b)d(2}$$

Mathematica [A] time = 0.390116, size = 106, normalized size = 0.74

$$\frac{\cos^{m+2}(c+dx)\left(-2a^2\text{Hypergeometric2F1}\left(1,m+2,m+3,-\frac{a\cos(c+dx)}{b}\right)+b(a+b)\text{Hypergeometric2F1}(1,m+2,m\right.}{2bd(m+2)(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(2 + m)*(b*(a + b)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c
+ d*x]] - (a - b)*b*Hypergeometric2F1[1, 2 + m, 3 + m, Cos[c + d*x]] - 2*a^
2*Hypergeometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)]))/(2*(a - b)*b*
(a + b)*d*(2 + m))
```

Maple [F] time = 0.547, size = 0, normalized size = 0.

$$\int \frac{(\cos(dx+c))^m}{a\sin(dx+c)+b\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{a\sin(dx+c)+b\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cos(dx+c)^m}{a\sin(dx+c)+b\tan(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**m/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cos(dx+c)^m}{a\sin(dx+c)+b\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

3.275 $\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=65

$$\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

```
[Out] (a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (a
*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)
```

Rubi [A] time = 0.0652865, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3109, 2637, 2638, 3074, 206}

$$\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (a
*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx &= \frac{a\int \sin(x)\,dx}{a^2+b^2}+\frac{b\int \cos(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\ &= -\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}+\frac{(ab)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2+b^2}\\ &= \frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}-\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\end{aligned}$$

Mathematica [A] time = 0.13048, size = 61, normalized size = 0.94

$$\frac{b\sin(x)-a\cos(x)}{a^2+b^2}-\frac{2ab\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (-(
a*Cos[x]) + b*Sin[x])/(a^2 + b^2)
```

Maple [A] time = 0.079, size = 81, normalized size = 1.3

$$-2\,\frac{-b\tan(x/2)+a}{\left(a^2+b^2\right)\left((\tan(x/2))^2+1\right)}-4\,\frac{ab}{\left(2\,a^2+2\,b^2\right)\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan(x/2)-2\,b}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -2/(a^2+b^2)*(-b*tan(1/2*x)+a)/(tan(1/2*x)^2+1)-4*a*b/(2*a^2+2*b^2)/(a^2+b^
2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.522599, size = 348, normalized size = 5.35

$$\frac{\sqrt{a^2+b^2}ab\log\left(\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-2\left(a^3+ab^2\right)\cos(x)+2\left(a^2b+b^3\right)\sin(x)}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 -
2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x)
 + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^3 + a*b^2)*cos(x) + 2*(a^2*b + b^3)*
sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Exception raised: AttributeError
```

Giac [A] time = 1.26134, size = 127, normalized size = 1.95

$$\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}+\frac{2\left(b\tan\left(\frac{1}{2}\,x\right)-a\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan(1/2*x) - a)/((a^2 +
b^2)*(tan(1/2*x)^2 + 1))
```

3.276 $\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=92

$$\frac{ax}{2(a^2+b^2)}-\frac{ab^2x}{(a^2+b^2)^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}-\frac{a\sin(x)\cos(x)}{2(a^2+b^2)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

```
[Out] -((a*b^2*x)/(a^2 + b^2)^2) + (a*x)/(2*(a^2 + b^2)) + (a^2*b*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 - (a*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (b*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rubi [A] time = 0.138902, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2564, 30, 2635, 8, 3097, 3133}

$$\frac{ax}{2(a^2+b^2)}-\frac{ab^2x}{(a^2+b^2)^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}-\frac{a\sin(x)\cos(x)}{2(a^2+b^2)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 + b^2)^2) + (a*x)/(2*(a^2 + b^2)) + (a^2*b*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 - (a*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (b*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \sin^2(x)\,dx}{a^2+b^2} + \frac{b\int \cos(x)\sin(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$
$$= -\frac{ab^2x}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)\sin(x)}{2\left(a^2+b^2\right)} + \frac{\left(a^2b\right)\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{a\int 1\,dx}{2\left(a^2+b^2\right)} + \frac{b\,\mathrm{Subst}(\int x\,dx, x,}{a^2+b^2}$$
$$= -\frac{ab^2x}{\left(a^2+b^2\right)^2} + \frac{ax}{2\left(a^2+b^2\right)} + \frac{a^2b\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)\sin(x)}{2\left(a^2+b^2\right)} + \frac{b\sin^2(x)}{2\left(a^2+b^2\right)}$$

Mathematica [C] time = 0.324497, size = 153, normalized size = 1.66

$$-\frac{2b\left(a^2+b^2\right)\cos(2x) - 2ib\left(b^2-3a^2\right)\tan^{-1}(\tan(x)) - 2\left(a^2+b^2\right)(b\log(a\cos(x)+b\sin(x))+ax) - 6ia^2bx - 3a^2b\log\big((}{8\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(-2*a^3*x - (6*I)*a^2*b*x + 6*a*b^2*x + (2*I)*b^3*x - (2*I)*b*(-3*a^2 + b^
2)*ArcTan[Tan[x]] + 2*b*(a^2 + b^2)*Cos[2*x] - 2*(a^2 + b^2)*(a*x + b*Log[a
*Cos[x] + b*Sin[x]]) - 3*a^2*b*Log[(a*Cos[x] + b*Sin[x])^2] + b^3*Log[(a*Co
s[x] + b*Sin[x])^2] + 2*a^3*Sin[2*x] + 2*a*b^2*Sin[2*x])/(8*(a^2 + b^2)^2)
```

Maple [B] time = 0.069, size = 174, normalized size = 1.9

$$\frac{a^2b\ln(a+b\tan(x))}{\left(a^2+b^2\right)^2} - \frac{\tan(x)\,a^3}{2\left(a^2+b^2\right)^2\left((\tan(x))^2+1\right)} - \frac{a\tan(x)\,b^2}{2\left(a^2+b^2\right)^2\left((\tan(x))^2+1\right)} - \frac{a^2b}{2\left(a^2+b^2\right)^2\left((\tan(x))^2+1\right)} - \frac{}{2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] a^2*b/(a^2+b^2)^2*ln(a+b*tan(x))-1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*a^3-1/
2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*a*b^2-1/2/(a^2+b^2)^2/(tan(x)^2+1)*a^2*b-
1/2/(a^2+b^2)^2/(tan(x)^2+1)*b^3-1/2/(a^2+b^2)^2*ln(tan(x)^2+1)*a^2*b+1/2/(
a^2+b^2)^2*arctan(tan(x))*a^3-1/2/(a^2+b^2)^2*arctan(tan(x))*a*b^2
```

Maxima [B] time = 1.64151, size = 285, normalized size = 3.1

$$\frac{a^2b\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}-\frac{a^2b\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3-ab^2\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4}-\frac{\frac{a\sin(x)}{\cos(x)+1}-\frac{2\,b\sin(x)^2}{(\cos(x)+1)^2}}{a^2+b^2+\frac{2\left(a^2+b^2\right)\sin(x)^2}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^2*b*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
2*a^2*b^2 + b^4) - a^2*b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2
+ b^4) + (a^3 - a*b^2)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4)
- (a*sin(x)/(cos(x) + 1) - 2*b*sin(x)^2/(cos(x) + 1)^2 - a*sin(x)^3/(cos(x)
 + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*s
in(x)^4/(cos(x) + 1)^4)
```

Fricas [A] time = 0.513713, size = 221, normalized size = 2.4

$$\frac{a^2b\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\left(a^2b+b^3\right)\cos\left(x\right)^2-\left(a^3+ab^2\right)\cos\left(x\right)\sin\left(x\right)+\left(a^3-ab^2\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^2*b +
 b^3)*cos(x)^2 - (a^3 + a*b^2)*cos(x)*sin(x) + (a^3 - a*b^2)*x)/(a^4 + 2*a^
2*b^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

[Out] Timed out

Giac [A] time = 1.13871, size = 205, normalized size = 2.23

$$\frac{a^2b^2\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{a^2b\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{\left(a^3-ab^2\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{a^2b\tan\left(x\right)^2-a^3\tan\left(x\right)-ab^2\tan\left(x\right)-b^3}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^2*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*a^2*b*log(ta
n(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^3 - a*b^2)*x/(a^4 + 2*a^2*b^2
+ b^4) + 1/2*(a^2*b*tan(x)^2 - a^3*tan(x) - a*b^2*tan(x) - b^3)/((a^4 + 2*a
^2*b^2 + b^4)*(tan(x)^2 + 1))
```

3.277 $\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=122

$$\frac{b\sin^3(x)}{3(a^2+b^2)}+\frac{a^2b\sin(x)}{(a^2+b^2)^2}+\frac{a\cos^3(x)}{3(a^2+b^2)}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab^2\cos(x)}{(a^2+b^2)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

```
[Out] (a^3*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x])/(a^2 + b^2) + (a*Cos[x]^3)/(3*(a^
2 + b^2)) + (a^2*b*Sin[x])/(a^2 + b^2)^2 + (b*Sin[x]^3)/(3*(a^2 + b^2))
```

Rubi [A] time = 0.16644, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2564, 30, 2633, 3099, 3074, 206, 2638}

$$\frac{b\sin^3(x)}{3(a^2+b^2)}+\frac{a^2b\sin(x)}{(a^2+b^2)^2}+\frac{a\cos^3(x)}{3(a^2+b^2)}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab^2\cos(x)}{(a^2+b^2)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```


```
[Out] (a^3*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x])/(a^2 + b^2) + (a*Cos[x]^3)/(3*(a^
2 + b^2)) + (a^2*b*Sin[x])/(a^2 + b^2)^2 + (b*Sin[x]^3)/(3*(a^2 + b^2))
```


Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \sin^3(x)\,dx}{a^2+b^2} + \frac{b\int \cos(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$
$$= \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} - \frac{\left(a^3b\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \sin(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(\right.}{a^2+b^2}$$
$$= \frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)}{a^2+b^2} + \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} + \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b\sin^3(x)}{3\left(a^2+b^2\right)} + \frac{\left(a^3b\right)\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\right.}{\left(a^2+\right.}$$
$$= \frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} + \frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos(x)}{a^2+b^2} + \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} + \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b\sin^3(x}{3\left(a^2+b\right.}$$

Mathematica [A] time = 1.00694, size = 113, normalized size = 0.93

$$\frac{\left(3ab^2-9a^3\right)\cos(x)+a\left(a^2+b^2\right)\cos(3x)-2b\sin(x)\left(\left(a^2+b^2\right)\cos(2x)-7a^2-b^2\right)}{12\left(a^2+b^2\right)^2} - \frac{2a^3b\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a^3*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (
(-9*a^3 + 3*a*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-7*a^2 - b^2 + (a
^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

Maple [A] time = 0.092, size = 166, normalized size = 1.4

$$-2\,\frac{-a^2b\left(\tan(x/2)\right)^5-ab^2\left(\tan(x/2)\right)^4+\left(-10/3\,a^2b-4/3\,b^3\right)\left(\tan(x/2)\right)^3+2\,a^3\left(\tan(x/2)\right)^2-\tan(x/2)\,a^2b+2/3\,a^3-}{\left(a^4+2\,a^2b^2+b^4\right)\left(\left(\tan(x/2)\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -2/(a^4+2*a^2*b^2+b^4)*(-a^2*b*tan(1/2*x)^5-a*b^2*tan(1/2*x)^4+(-10/3*a^2*b
-4/3*b^3)*tan(1/2*x)^3+2*a^3*tan(1/2*x)^2-tan(1/2*x)*a^2*b+2/3*a^3-1/3*a*b^
2)/(tan(1/2*x)^2+1)^3-16*a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arc
tanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.539294, size = 494, normalized size = 4.05

$$\frac{3\sqrt{a^2+b^2}a^3b\log\left(\frac{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)+2\left(a^5+2a^3b^2+ab^4\right)\cos(x)^3-6\left(a^5+a^3\right.}{6\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*si
n(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3
- 6*(a^5 + a^3*b^2)*cos(x) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*
b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20769, size = 257, normalized size = 2.11

$$\frac{a^3 b \log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}} + \frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,x\right)^5 + 3\,ab^2\tan\left(\frac{1}{2}\,x\right)^4 + 10\,a^2b\tan\left(\frac{1}{2}\,x\right)^3 + 4\,b^3\tan\left(\frac{1}{2}\,x\right)^3 - 6\,a^3\tan\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^3*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3
*(3*a^2*b*tan(1/2*x)^5 + 3*a*b^2*tan(1/2*x)^4 + 10*a^2*b*tan(1/2*x)^3 + 4*b
^3*tan(1/2*x)^3 - 6*a^3*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - 2*a^3 + a*b^2)/
((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

3.278 $\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\sin(x)\cos(x)}{2\left(a^2+b^2\right)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] -((a^2*b*x)/(a^2 + b^2)^2) + (b*x)/(2*(a^2 + b^2)) - (a*b^2*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 + (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (a*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rubi [A] time = 0.133434, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {3109, 2635, 8, 2564, 30, 3098, 3133}

$$-\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}+\frac{a\sin^2(x)}{2\left(a^2+b^2\right)}+\frac{b\sin(x)\cos(x)}{2\left(a^2+b^2\right)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*b*x)/(a^2 + b^2)^2) + (b*x)/(2*(a^2 + b^2)) - (a*b^2*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 + (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (a*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos(x)\sin(x)\,dx}{a^2+b^2} + \frac{b\int \cos^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{a^2bx}{\left(a^2+b^2\right)^2} + \frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)} - \frac{\left(ab^2\right)\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{a\,\text{Subst}(\int x\,dx, x, \sin(x))}{a^2+b^2} + \frac{b}{2\left(a\right.}$$

$$= -\frac{a^2bx}{\left(a^2+b^2\right)^2} + \frac{bx}{2\left(a^2+b^2\right)} - \frac{ab^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2} + \frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)} + \frac{a\sin^2(x)}{2\left(a^2+b^2\right)}$$

Mathematica [C] time = 0.303178, size = 82, normalized size = 0.88

$$\frac{b\left(a^2+b^2\right)\sin(2x) - a\left(a^2+b^2\right)\cos(2x) + 4iab^2\tan^{-1}(\tan(x)) - 2b\left(ab\log\left((a\cos(x)+b\sin(x))^2\right) + x(a+ib)^2\right)}{4\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] ((4*I)*a*b^2*ArcTan[Tan[x]] - a*(a^2 + b^2)*Cos[2*x] - 2*b*((a + I*b)^2*x +
 a*b*Log[(a*Cos[x] + b*Sin[x])^2]) + b*(a^2 + b^2)*Sin[2*x])/(4*(a^2 + b^2)
^2)
```

Maple [A] time = 0.069, size = 175, normalized size = 1.9

$$-\frac{ab^2\ln(a+b\tan(x))}{\left(a^2+b^2\right)^2} + \frac{\tan(x)\,a^2b}{2\left(a^2+b^2\right)^2\left((\tan(x))^2+1\right)} + \frac{\tan(x)\,b^3}{2\left(a^2+b^2\right)^2\left((\tan(x))^2+1\right)} - \frac{a^3}{2\left(a^2+b^2\right)^2\left((\tan(x))^2+1\right)} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a*b^2/(a^2+b^2)^2*ln(a+b*tan(x))+1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*a^2*b
+1/2/(a^2+b^2)^2/(tan(x)^2+1)*tan(x)*b^3-1/2/(a^2+b^2)^2/(tan(x)^2+1)*a^3-1
/2/(a^2+b^2)^2/(tan(x)^2+1)*a*b^2+1/2/(a^2+b^2)^2*ln(tan(x)^2+1)*a*b^2-1/2/
(a^2+b^2)^2*arctan(tan(x))*a^2*b+1/2/(a^2+b^2)^2*arctan(tan(x))*b^3
```

Maxima [B] time = 1.6452, size = 286, normalized size = 3.08

$$-\frac{ab^2\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}+\frac{ab^2\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{\left(a^2b-b^3\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\frac{b\sin(x)}{\cos(x)+1}+\frac{2\,a\sin(x)^2}{(\cos(x)+1)}}{a^2+b^2+\frac{2\left(a^2+b^2\right)\sin(x}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
 2*a^2*b^2 + b^4) + a*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2
 + b^4) - (a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4)
 + (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x
) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*
sin(x)^4/(cos(x) + 1)^4)
```

Fricas [A] time = 0.51382, size = 223, normalized size = 2.4

$$-\frac{ab^2\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)+\left(a^3+ab^2\right)\cos\left(x\right)^2-\left(a^2b+b^3\right)\cos\left(x\right)\sin\left(x\right)+\left(a^2b-b^3\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^3 +
a*b^2)*cos(x)^2 - (a^2*b + b^3)*cos(x)*sin(x) + (a^2*b - b^3)*x)/(a^4 + 2*a
^2*b^2 + b^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1426, size = 211, normalized size = 2.27

$$-\frac{ab^3\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{ab^2\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{\left(a^2b-b^3\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{ab^2\tan\left(x\right)^2-a^2b\tan\left(x\right)-b^3\tan\left(x\right)+a^3+}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a*b^3*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a*b^2*log(tan
(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2*b - b^3)*x/(a^4 + 2*a^2*b^2 +
 b^4) - 1/2*(a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + a^3 + 2*a*b^2)/((
a^4 + 2*a^2*b^2 + b^4)*(tan(x)^2 + 1))
```

3.279 $\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=112

$$\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{ab^2\sin(x)}{(a^2+b^2)^2}-\frac{b\cos^3(x)}{3(a^2+b^2)}+\frac{a^2b\cos(x)}{(a^2+b^2)^2}-\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

```
[Out] -((a^2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2
)) + (a^2*b*Cos[x])/(a^2 + b^2)^2 - (b*Cos[x]^3)/(3*(a^2 + b^2)) - (a*b^2*S
in[x])/(a^2 + b^2)^2 + (a*Sin[x]^3)/(3*(a^2 + b^2))
```

Rubi [A] time = 0.196009, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3109, 2565, 30, 2564, 2637, 2638, 3074, 206}

$$\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{ab^2\sin(x)}{(a^2+b^2)^2}-\frac{b\cos^3(x)}{3(a^2+b^2)}+\frac{a^2b\cos(x)}{(a^2+b^2)^2}-\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2
)) + (a^2*b*Cos[x])/(a^2 + b^2)^2 - (b*Cos[x]^3)/(3*(a^2 + b^2)) - (a*b^2*S
in[x])/(a^2 + b^2)^2 + (a*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos(x)\sin^2(x)\,dx}{a^2+b^2} + \frac{b\int \cos^2(x)\sin(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$
$$= -\frac{\left(a^2b\right)\int \sin(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \cos(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{a\,\mathrm{Subst}\left(\int x^2\,dx,x,s\right.}{a^2+b^2}$$
$$= \frac{a^2b\cos(x)}{\left(a^2+b^2\right)^2} - \frac{b\cos^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{a\sin^3(x)}{3\left(a^2+b^2\right)} - \frac{\left(a^2b^2\right)\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos\right.}{\left(a^2+b^2\right)^2}$$
$$= -\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} + \frac{a^2b\cos(x)}{\left(a^2+b^2\right)^2} - \frac{b\cos^3(x)}{3\left(a^2+b^2\right)} - \frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{a\sin^3(x)}{3\left(a^2+b^2\right)}$$

Mathematica [A] time = 0.655489, size = 115, normalized size = 1.03

$$\frac{2a^2b^2\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{\left(3b^3-9a^2b\right)\cos(x)+b\left(a^2+b^2\right)\cos(3x)+2a\sin(x)\left(\left(a^2+b^2\right)\cos(2x)-a^2+5b^2\right)}{12\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a^2*b^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
((-9*a^2*b + 3*b^3)*Cos[x] + b*(a^2 + b^2)*Cos[3*x] + 2*a*(-a^2 + 5*b^2 + (
a^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

Maple [A] time = 0.091, size = 168, normalized size = 1.5

$$2\,\frac{-ab^2\left(\tan\left(x/2\right)\right)^5-b^3\left(\tan\left(x/2\right)\right)^4+\left(4/3\,a^3-2/3\,ab^2\right)\left(\tan\left(x/2\right)\right)^3+2\,a^2b\left(\tan\left(x/2\right)\right)^2-ab^2\tan\left(x/2\right)+2/3\,a^2b-1/3}{\left(a^4+2\,a^2b^2+b^4\right)\left(\left(\tan\left(x/2\right)\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)*(-a*b^2*tan(1/2*x)^5-b^3*tan(1/2*x)^4+(4/3*a^3-2/3*a*
b^2)*tan(1/2*x)^3+2*a^2*b*tan(1/2*x)^2-a*b^2*tan(1/2*x)+2/3*a^2*b-1/3*b^3)/
(tan(1/2*x)^2+1)^3+8*a^2*b^2/(4*a^4+8*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*arctan
h(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.535051, size = 498, normalized size = 4.45

$$\frac{3\sqrt{a^2+b^2}a^2b^2\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(x\right)^3+6\left(a^4b+\right.}{6\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^2*b^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(
x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)
*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)
^3 + 6*(a^4*b + a^2*b^3)*cos(x) + 2*(a^5 - a^3*b^2 - 2*a*b^4 - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22947, size = 259, normalized size = 2.31

$$-\frac{a^2b^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(3\,ab^2\tan\left(\frac{1}{2}\,x\right)^5+3\,b^3\tan\left(\frac{1}{2}\,x\right)^4-4\,a^3\tan\left(\frac{1}{2}\,x\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3-6\,a^2b\,\mathrm{t}\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^2*b^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*
x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) -
2/3*(3*a*b^2*tan(1/2*x)^5 + 3*b^3*tan(1/2*x)^4 - 4*a^3*tan(1/2*x)^3 + 2*a*b
^2*tan(1/2*x)^3 - 6*a^2*b*tan(1/2*x)^2 + 3*a*b^2*tan(1/2*x) - 2*a^2*b + b^3
)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

3.280 $\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=176

$$\frac{bx}{8(a^2+b^2)}-\frac{a^2bx}{2(a^2+b^2)^2}+\frac{a^2b^3x}{(a^2+b^2)^3}+\frac{a\sin^4(x)}{4(a^2+b^2)}-\frac{ab^2\sin^2(x)}{2(a^2+b^2)^2}-\frac{b\sin(x)\cos^3(x)}{4(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{8(a^2+b^2)}+\frac{a^2b\sin(x)\cos}{2(a^2+b^2}$$

```
[Out] (a^2*b^3*x)/(a^2 + b^2)^3 - (a^2*b*x)/(2*(a^2 + b^2)^2) + (b*x)/(8*(a^2 + b
^2)) - (a^3*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (a^2*b*Cos[x]*Sin
[x])/(2*(a^2 + b^2)^2) + (b*Cos[x]*Sin[x])/(8*(a^2 + b^2)) - (b*Cos[x]^3*Si
n[x])/(4*(a^2 + b^2)) - (a*b^2*Sin[x]^2)/(2*(a^2 + b^2)^2) + (a*Sin[x]^4)/(
4*(a^2 + b^2))
```

Rubi [A] time = 0.278453, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3109, 2568, 2635, 8, 2564, 30, 3097, 3133}

$$\frac{bx}{8(a^2+b^2)}-\frac{a^2bx}{2(a^2+b^2)^2}+\frac{a^2b^3x}{(a^2+b^2)^3}+\frac{a\sin^4(x)}{4(a^2+b^2)}-\frac{ab^2\sin^2(x)}{2(a^2+b^2)^2}-\frac{b\sin(x)\cos^3(x)}{4(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{8(a^2+b^2)}+\frac{a^2b\sin(x)\cos}{2(a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^2*b^3*x)/(a^2 + b^2)^3 - (a^2*b*x)/(2*(a^2 + b^2)^2) + (b*x)/(8*(a^2 + b
^2)) - (a^3*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (a^2*b*Cos[x]*Sin
[x])/(2*(a^2 + b^2)^2) + (b*Cos[x]*Sin[x])/(8*(a^2 + b^2)) - (b*Cos[x]^3*Si
n[x])/(4*(a^2 + b^2)) - (a*b^2*Sin[x]^2)/(2*(a^2 + b^2)^2) + (a*Sin[x]^4)/(
4*(a^2 + b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
```

```
), 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos(x)\sin^3(x)\,dx}{a^2+b^2} + \frac{b\int \cos^2(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\cos^3(x)\sin(x)}{4\left(a^2+b^2\right)} - \frac{\left(a^2b\right)\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^2\right)\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{\left(a^2b^2\right)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,d}{\left(a^2+b^2\right)^2}$$

$$= \frac{a^2b^3x}{\left(a^2+b^2\right)^3} + \frac{a^2b\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2} + \frac{b\cos(x)\sin(x)}{8\left(a^2+b^2\right)} - \frac{b\cos^3(x)\sin(x)}{4\left(a^2+b^2\right)} + \frac{a\sin^4(x)}{4\left(a^2+b^2\right)} - \frac{\left(a^3b^2\right)\int}{(}$$

$$= \frac{a^2b^3x}{\left(a^2+b^2\right)^3} - \frac{a^2bx}{2\left(a^2+b^2\right)^2} + \frac{bx}{8\left(a^2+b^2\right)} - \frac{a^3b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3} + \frac{a^2b\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2} +$$

Mathematica [C] time = 0.507061, size = 178, normalized size = 1.01

$$\frac{-32ia^3b^2x + 24a^2b^3x + 8a^2b^3\sin(2x) - 2a^2b^3\sin(4x) + 2a^3b^2\cos(4x) - 4a\left(a^4-b^4\right)\cos(2x) + 32ia^3b^2\tan^{-1}(\tan(x)) - 1}{32\left(a^2 -\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-12*a^4*b*x - (32*I)*a^3*b^2*x + 24*a^2*b^3*x + 4*b^5*x + (32*I)*a^3*b^2*A
rcTan[Tan[x]] - 4*a*(a^4 - b^4)*Cos[2*x] + a^5*Cos[4*x] + 2*a^3*b^2*Cos[4*x
] + a*b^4*Cos[4*x] - 16*a^3*b^2*Log[(a*Cos[x] + b*Sin[x])^2] + 8*a^4*b*Sin[
2*x] + 8*a^2*b^3*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*Sin[4
*x])/(32*(a^2 + b^2)^3)
```

Maple [B] time = 0.075, size = 363, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a^3*b^2/(a^2+b^2)^3*ln(a+b*tan(x))+3/4/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3
*a^2*b^3+1/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3*b^5+5/8/(a^2+b^2)^3/(tan(x
)^2+1)^2*tan(x)^3*a^4*b-1/2/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^2*a^5-1/2/(a^
2+b^2)^3/(tan(x)^2+1)^2*tan(x)^2*a^3*b^2+3/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan
(x)*a^4*b+1/4/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)*a^2*b^3-1/8/(a^2+b^2)^3/(ta
n(x)^2+1)^2*tan(x)*b^5-1/4/(a^2+b^2)^3/(tan(x)^2+1)^2*a^5+1/4/(a^2+b^2)^3/(
tan(x)^2+1)^2*a*b^4-3/8/(a^2+b^2)^3*arctan(tan(x))*a^4*b+3/4/(a^2+b^2)^3*ar
ctan(tan(x))*a^2*b^3+1/8/(a^2+b^2)^3*arctan(tan(x))*b^5+1/2/(a^2+b^2)^3*ln(
tan(x)^2+1)*a^3*b^2
```

Maxima [B] time = 1.78876, size = 582, normalized size = 3.31

$$-\frac{a^3b^2\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{a^3b^2\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(3\,a^4b-6\,a^2b^3-b^5\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{\frac{8\,ab^2\sin(x)^2}{(\cos(x)+1)^2}}{4\left(a^4+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^3*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + a^3*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1)/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b - 6*a^2*b^3 - b^5)*arcta
n(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(8*a*b^2*s
in(x)^2/(cos(x) + 1)^2 - 16*a^3*sin(x)^4/(cos(x) + 1)^4 + 8*a*b^2*sin(x)^6/
(cos(x) + 1)^6 - (3*a^2*b - b^3)*sin(x)/(cos(x) + 1) - (11*a^2*b + 7*b^3)*s
in(x)^3/(cos(x) + 1)^3 + (11*a^2*b + 7*b^3)*sin(x)^5/(cos(x) + 1)^5 + (3*a^
2*b - b^3)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2
*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^4/(c
os(x) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6 + (a^4 + 2
*a^2*b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

Fricas [A] time = 0.55469, size = 396, normalized size = 2.25

$$-\frac{4\,a^3b^2\log\left(2\,ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2\right)-2\left(a^5+2\,a^3b^2+ab^4\right)\cos(x)^4+4\left(a^5+a^3b^2\right)\cos(x)^2+\left(3\,a\right.}{8\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*a^3*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*(
a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^4 + 4*(a^5 + a^3*b^2)*cos(x)^2 + (3*a^4*b -
 6*a^2*b^3 - b^5)*x + (2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 - (5*a^4*b + 6*
a^2*b^3 + b^5)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12061, size = 371, normalized size = 2.11

$$-\frac{a^3b^3\log\left(|b\tan\left(x\right)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{a^3b^2\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{\left(3\,a^4b-6\,a^2b^3-b^5\right)x}{8\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{6\,a^3b^2\tan\left(x\right)^4-5\,a^4b\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^3*b^3*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2
*a^3*b^2*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(3*a^4
*b - 6*a^2*b^3 - b^5)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(6*a^3*b^
2*tan(x)^4 - 5*a^4*b*tan(x)^3 - 6*a^2*b^3*tan(x)^3 - b^5*tan(x)^3 + 4*a^5*t
an(x)^2 + 16*a^3*b^2*tan(x)^2 - 3*a^4*b*tan(x) - 2*a^2*b^3*tan(x) + b^5*tan
(x) + 2*a^5 + 6*a^3*b^2 - 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(ta
n(x)^2 + 1)^2)
```

3.281 $\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=123

$$-\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}+\frac{b\sin(x)}{a^2+b^2}-\frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

```
[Out] (a*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x]^3)/(3*(a^2 + b^2)) - (a^2*b*Sin[x]
)/(a^2 + b^2)^2 + (b*Sin[x])/(a^2 + b^2) - (b*Sin[x]^3)/(3*(a^2 + b^2))
```

Rubi [A] time = 0.157631, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3109, 2633, 2565, 30, 3100, 2637, 3074, 206}

$$-\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}+\frac{b\sin(x)}{a^2+b^2}-\frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}-\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x]^3)/(3*(a^2 + b^2)) - (a^2*b*Sin[x]
)/(a^2 + b^2)^2 + (b*Sin[x])/(a^2 + b^2) - (b*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin(x)\,dx}{a^2+b^2} + \frac{b\int \cos^3(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{\left(a^2b\right)\int \cos(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(ab^3\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} - \frac{a\,\mathrm{Subst}\left(\int x^2\,dx, x, \cos(x)\right)}{a^2+b^2}$$

$$= -\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} - \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b\sin(x)}{a^2+b^2} - \frac{b\sin^3(x)}{3\left(a^2+b^2\right)} + \frac{\left(ab^3\right)\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\right.}{\left(a^2\right.}$$

$$= \frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a\cos^3(x)}{3\left(a^2+b^2\right)} - \frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b\sin(x)}{a^2+b^2} - \frac{b\sin^3(x)}{3\left(a^2+b^2\right.}$$

Mathematica [A] time = 1.06909, size = 112, normalized size = 0.91

$$-\frac{3a\left(a^2+5b^2\right)\cos(x)+a\left(a^2+b^2\right)\cos(3x)-2b\sin(x)\left(\left(a^2+b^2\right)\cos(2x)-a^2+5b^2\right)}{12\left(a^2+b^2\right)^2} - \frac{2ab^3\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
3*a*(a^2 + 5*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-a^2 + 5*b^2 + (a^
2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

Maple [A] time = 0.091, size = 170, normalized size = 1.4

$$-2\,\frac{-b^3\left(\tan(x/2)\right)^5+\left(a^3+2\,ab^2\right)\left(\tan(x/2)\right)^4+\left(4/3\,a^2b-2/3\,b^3\right)\left(\tan(x/2)\right)^3+2\,ab^2\left(\tan(x/2)\right)^2-b^3\tan(x/2)+1/3}{\left(a^4+2\,a^2b^2+b^4\right)\left(\left(\tan(x/2)\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -2/(a^4+2*a^2*b^2+b^4)*(-b^3*tan(1/2*x)^5+(a^3+2*a*b^2)*tan(1/2*x)^4+(4/3*a
^2*b-2/3*b^3)*tan(1/2*x)^3+2*a*b^2*tan(1/2*x)^2-b^3*tan(1/2*x)+1/3*a^3+4/3*
a*b^2)/(tan(1/2*x)^2+1)^3-4*a*b^3/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^(1/2)*a
rctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.53742, size = 494, normalized size = 4.02

$$\frac{3\sqrt{a^2+b^2}ab^3\log\left(\frac{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+\left(a^2-b^2\right)\cos(x)^2+b^2}\right)-2\left(a^5+2a^3b^2+ab^4\right)\cos(x)^3-6\left(a^3b^2+\cdots\right.}{6\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*si
n(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3
- 6*(a^3*b^2 + a*b^4)*cos(x) - 2*(a^4*b - a^2*b^3 - 2*b^5 - (a^4*b + 2*a^2*
b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.21764, size = 271, normalized size = 2.2

$$\frac{ab^3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,b^3\tan\left(\frac{1}{2}\,x\right)^5-3\,a^3\tan\left(\frac{1}{2}\,x\right)^4-6\,ab^2\tan\left(\frac{1}{2}\,x\right)^4-4\,a^2b\tan\left(\frac{1}{2}\,x\right)^3+2\,b^3\tan\left(\right.\right.}{3\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3
*(3*b^3*tan(1/2*x)^5 - 3*a^3*tan(1/2*x)^4 - 6*a*b^2*tan(1/2*x)^4 - 4*a^2*b*
tan(1/2*x)^3 + 2*b^3*tan(1/2*x)^3 - 6*a*b^2*tan(1/2*x)^2 + 3*b^3*tan(1/2*x)
 - a^3 - 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

3.282 $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=175

$$\frac{ax}{8\left(a^2+b^2\right)}-\frac{ab^2x}{2\left(a^2+b^2\right)^2}+\frac{a^3b^2x}{\left(a^2+b^2\right)^3}-\frac{a^2b\sin^2(x)}{2\left(a^2+b^2\right)^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}-\frac{a\sin(x)\cos^3(x)}{4\left(a^2+b^2\right)}+\frac{a\sin(x)\cos(x)}{8\left(a^2+b^2\right)}-\frac{ab^2\sin(x)\,\mathrm{co}}{2\left(a^2+b^2\right.}$$

```
[Out] (a^3*b^2*x)/(a^2 + b^2)^3 - (a*b^2*x)/(2*(a^2 + b^2)^2) + (a*x)/(8*(a^2 + b
^2)) - (b*Cos[x]^4)/(4*(a^2 + b^2)) + (a^2*b^3*Log[a*Cos[x] + b*Sin[x]])/(a
^2 + b^2)^3 - (a*b^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2) + (a*Cos[x]*Sin[x])/(
8*(a^2 + b^2)) - (a*Cos[x]^3*Sin[x])/(4*(a^2 + b^2)) - (a^2*b*Sin[x]^2)/(2*
(a^2 + b^2)^2)
```

Rubi [A] time = 0.278293, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133}

$$\frac{ax}{8\left(a^2+b^2\right)}-\frac{ab^2x}{2\left(a^2+b^2\right)^2}+\frac{a^3b^2x}{\left(a^2+b^2\right)^3}-\frac{a^2b\sin^2(x)}{2\left(a^2+b^2\right)^2}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}-\frac{a\sin(x)\cos^3(x)}{4\left(a^2+b^2\right)}+\frac{a\sin(x)\cos(x)}{8\left(a^2+b^2\right)}-\frac{ab^2\sin(x)\,\mathrm{co}}{2\left(a^2+b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```


```
[Out] (a^3*b^2*x)/(a^2 + b^2)^3 - (a*b^2*x)/(2*(a^2 + b^2)^2) + (a*x)/(8*(a^2 + b
^2)) - (b*Cos[x]^4)/(4*(a^2 + b^2)) + (a^2*b^3*Log[a*Cos[x] + b*Sin[x]])/(a
^2 + b^2)^3 - (a*b^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2) + (a*Cos[x]*Sin[x])/(
8*(a^2 + b^2)) - (a*Cos[x]^3*Sin[x])/(4*(a^2 + b^2)) - (a^2*b*Sin[x]^2)/(2*
(a^2 + b^2)^2)
```


Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin^2(x)\,dx}{a^2+b^2}+\frac{b\int \cos^3(x)\sin(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$
$$= -\frac{a\cos^3(x)\sin(x)}{4\left(a^2+b^2\right)}-\frac{\left(a^2b\right)\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2}-\frac{\left(ab^2\right)\int \cos^2(x)\,dx}{\left(a^2+b^2\right)^2}+\frac{\left(a^2b^2\right)\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}\,d}{\left(a^2+b^2\right)^2}$$
$$= \frac{a^3b^2x}{\left(a^2+b^2\right)^3}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}-\frac{ab^2\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2}+\frac{a\cos(x)\sin(x)}{8\left(a^2+b^2\right)}-\frac{a\cos^3(x)\sin(x)}{4\left(a^2+b^2\right)}+\frac{\left(a^2b^3\right)\int}{(}$$
$$= \frac{a^3b^2x}{\left(a^2+b^2\right)^3}-\frac{ab^2x}{2\left(a^2+b^2\right)^2}+\frac{ax}{8\left(a^2+b^2\right)}-\frac{b\cos^4(x)}{4\left(a^2+b^2\right)}+\frac{a^2b^3\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}-\frac{ab^2 c}{2(}$$

Mathematica [C] time = 0.75795, size = 287, normalized size = 1.64

$$-\frac{-24a^3b^2x-24ia^2b^3x+8a^3b^2\sin(2x)+2a^3b^2\sin(4x)+2a^2b^3\cos(4x)+4b\left(b^4-a^4\right)\cos(2x)-4ib\left(-6a^2b^2+a^4+b^4\right)\text{t}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(-4*a^5*x + (4*I)*a^4*b*x - 24*a^3*b^2*x - (24*I)*a^2*b^3*x + 12*a*b^4*x +
 (4*I)*b^5*x - (4*I)*b*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[x]] + 4*b*(-a^4 +
 b^4)*Cos[2*x] + a^4*b*Cos[4*x] + 2*a^2*b^3*Cos[4*x] + b^5*Cos[4*x] - 4*a^4
*b*Log[a*Cos[x] + b*Sin[x]] - 8*a^2*b^3*Log[a*Cos[x] + b*Sin[x]] - 4*b^5*Lo
g[a*Cos[x] + b*Sin[x]] + 2*a^4*b*Log[(a*Cos[x] + b*Sin[x])^2] - 12*a^2*b^3*
Log[(a*Cos[x] + b*Sin[x])^2] + 2*b^5*Log[(a*Cos[x] + b*Sin[x])^2] + 8*a^3*b
^2*Sin[2*x] + 8*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4*x] + a*b^4*
Sin[4*x])/(32*(a^2 + b^2)^3)
```

Maple [B] time = 0.078, size = 363, normalized size = 2.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] a^2*b^3/(a^2+b^2)^3*ln(a+b*tan(x))+1/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3*
a^5-1/4/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^3*a^3*b^2-3/8/(a^2+b^2)^3/(tan(x)
^2+1)^2*tan(x)^3*a*b^4+1/2/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^2*a^4*b+1/2/(a
^2+b^2)^3/(tan(x)^2+1)^2*tan(x)^2*a^2*b^3-3/4/(a^2+b^2)^3/(tan(x)^2+1)^2*ta
n(x)*a^3*b^2-5/8/(a^2+b^2)^3/(tan(x)^2+1)^2*tan(x)*a*b^4-1/8/(a^2+b^2)^3/(t
an(x)^2+1)^2*tan(x)*a^5+1/4/(a^2+b^2)^3/(tan(x)^2+1)^2*a^4*b-1/4/(a^2+b^2)^
3/(tan(x)^2+1)^2*b^5-1/2/(a^2+b^2)^3*ln(tan(x)^2+1)*a^2*b^3+1/8/(a^2+b^2)^3
*arctan(tan(x))*a^5+3/4/(a^2+b^2)^3*arctan(tan(x))*a^3*b^2-3/8/(a^2+b^2)^3*
arctan(tan(x))*a*b^4
```

Maxima [B] time = 1.65446, size = 572, normalized size = 3.27

$$\frac{a^2b^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{a^2b^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^5+6\,a^3b^2-3\,ab^4\right)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{\frac{8\,b^3\sin(x)^2}{(\cos(x)+1)^2}}{4\left(a^4+2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^2*b^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) - a^2*b^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(a^5 + 6*a^3*b^2 - 3*a*b^4)*arctan
(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(8*b^3*sin(
x)^2/(cos(x) + 1)^2 - 16*a^2*b*sin(x)^4/(cos(x) + 1)^4 + 8*b^3*sin(x)^6/(co
s(x) + 1)^6 - (a^3 + 5*a*b^2)*sin(x)/(cos(x) + 1) + (7*a^3 + 3*a*b^2)*sin(x
)^3/(cos(x) + 1)^3 - (7*a^3 + 3*a*b^2)*sin(x)^5/(cos(x) + 1)^5 + (a^3 + 5*a
*b^2)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2
+ b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^4/(cos(x)
 + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6 + (a^4 + 2*a^2*
```

```
b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

Fricas [A] time = 0.558151, size = 397, normalized size = 2.27

$$\frac{4 a^2b^3 \log \left(2 ab \cos (x) \sin (x) + \left(a^2 - b^2\right) \cos (x)^2 + b^2\right) - 2 \left(a^4b + 2 a^2b^3 + b^5\right) \cos (x)^4 + 4 \left(a^4b + a^2b^3\right) \cos (x)^2 + \left(a^5 + \right.}{8 \left(a^6 + 3 a^4b^2 + 3 a^2b^4 + b^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*b^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*(a
^4*b + 2*a^2*b^3 + b^5)*cos(x)^4 + 4*(a^4*b + a^2*b^3)*cos(x)^2 + (a^5 + 6*
a^3*b^2 - 3*a*b^4)*x - (2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 - 2*a^3
*b^2 - 3*a*b^4)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17323, size = 369, normalized size = 2.11

$$\frac{a^2b^4 \log \left(|b \tan (x) + a|\right)}{a^6b + 3 a^4b^3 + 3 a^2b^5 + b^7} - \frac{a^2b^3 \log \left(\tan (x)^2 + 1\right)}{2 \left(a^6 + 3 a^4b^2 + 3 a^2b^4 + b^6\right)} + \frac{\left(a^5 + 6 a^3b^2 - 3 ab^4\right)x}{8 \left(a^6 + 3 a^4b^2 + 3 a^2b^4 + b^6\right)} + \frac{6 a^2b^3 \tan (x)^4 + a^5 \tan (x)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^2*b^4*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2*
a^2*b^3*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(a^5 +
6*a^3*b^2 - 3*a*b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(6*a^2*b^3
*tan(x)^4 + a^5*tan(x)^3 - 2*a^3*b^2*tan(x)^3 - 3*a*b^4*tan(x)^3 + 4*a^4*b*
tan(x)^2 + 16*a^2*b^3*tan(x)^2 - a^5*tan(x) - 6*a^3*b^2*tan(x) - 5*a*b^4*ta
n(x) + 2*a^4*b + 6*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(t
an(x)^2 + 1)^2)
```

3.283 $\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=193

$$-\frac{b\sin^5(x)}{5\left(a^2+b^2\right)}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a^2b\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3}+\frac{a\cos^5(x)}{5\left(a^2+b^2\right)}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}+\frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^3b^3}{\ldots}$$

```
[Out] (a^3*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
- (a^3*b^2*Cos[x])/(a^2 + b^2)^3 + (a*b^2*Cos[x]^3)/(3*(a^2 + b^2)^2) - (a*
Cos[x]^3)/(3*(a^2 + b^2)) + (a*Cos[x]^5)/(5*(a^2 + b^2)) + (a^2*b^3*Sin[x])
/(a^2 + b^2)^3 - (a^2*b*Sin[x]^3)/(3*(a^2 + b^2)^2) + (b*Sin[x]^3)/(3*(a^2
+ b^2)) - (b*Sin[x]^5)/(5*(a^2 + b^2))
```

Rubi [A] time = 0.358671, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.45, Rules used = {3109, 2564, 14, 2565, 30, 2637, 2638, 3074, 206}

$$-\frac{b\sin^5(x)}{5\left(a^2+b^2\right)}+\frac{b\sin^3(x)}{3\left(a^2+b^2\right)}-\frac{a^2b\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3}+\frac{a\cos^5(x)}{5\left(a^2+b^2\right)}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}+\frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^3b^3}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```


```
[Out] (a^3*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
- (a^3*b^2*Cos[x])/(a^2 + b^2)^3 + (a*b^2*Cos[x]^3)/(3*(a^2 + b^2)^2) - (a*
Cos[x]^3)/(3*(a^2 + b^2)) + (a*Cos[x]^5)/(5*(a^2 + b^2)) + (a^2*b^3*Sin[x])
/(a^2 + b^2)^3 - (a^2*b*Sin[x]^3)/(3*(a^2 + b^2)^2) + (b*Sin[x]^3)/(3*(a^2
+ b^2)) - (b*Sin[x]^5)/(5*(a^2 + b^2))
```


Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
```

```
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin^3(x)\,dx}{a^2+b^2}+\frac{b\int \cos^3(x)\sin^2(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{\left(a^2b\right)\int \cos(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2}-\frac{\left(ab^2\right)\int \cos^2(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2}+\frac{\left(a^2b^2\right)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2}-\frac{a\,\mathrm{Subs}}{}$$

$$= \frac{\left(a^3b^2\right)\int \sin(x)\,dx}{\left(a^2+b^2\right)^3}+\frac{\left(a^2b^3\right)\int \cos(x)\,dx}{\left(a^2+b^2\right)^3}-\frac{\left(a^3b^3\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^3}-\frac{\left(a^2b\right)\mathrm{Subst}\left(\int x^2\,dx,\right.}{\left(a^2+b^2\right)^2}$$

$$= -\frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}+\frac{a\cos^5(x)}{5\left(a^2+b^2\right)}+\frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3}-\frac{a^2b\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{b\,\mathrm{si}}{3\left(a^2\right.}$$

$$= \frac{a^3b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}}-\frac{a^3b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{ab^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a\cos^3(x)}{3\left(a^2+b^2\right)}+\frac{a\cos^5(x)}{5\left(a^2+b^2\right)}+\frac{a^2b^3}{\left(a^2\right.}$$

Mathematica [A] time = 1.55243, size = 223, normalized size = 1.16

$$\frac{240a^2b^3\sin(x)+10a^2b^3\sin(3x)-6a^2b^3\sin(5x)+6a^3b^2\cos(5x)-30a\left(8a^2b^2+a^4-b^4\right)\cos(x)-5a\left(-2a^2b^2+a^4-3b^4\right)}{240\,(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a^3*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) +
 (-30*a*(a^4 + 8*a^2*b^2 - b^4)*Cos[x] - 5*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Cos[
3*x] + 3*a^5*Cos[5*x] + 6*a^3*b^2*Cos[5*x] + 3*a*b^4*Cos[5*x] - 30*a^4*b*Si
n[x] + 240*a^2*b^3*Sin[x] + 30*b^5*Sin[x] + 15*a^4*b*Sin[3*x] + 10*a^2*b^3*
Sin[3*x] - 5*b^5*Sin[3*x] - 3*a^4*b*Sin[5*x] - 6*a^2*b^3*Sin[5*x] - 3*b^5*S
in[5*x])/(240*(a^2 + b^2)^3)
```

Maple [A] time = 0.111, size = 305, normalized size = 1.6

$$-2\frac{1}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\left(a^{2}+b^{2}\right)\left(\left(\tan\left(x/2\right)\right)^{2}+1\right)^{5}}\left(-a^{2}b^{3}\left(\tan\left(x/2\right)\right)^{9}-ab^{4}\left(\tan\left(x/2\right)\right)^{8}+\left(-16/3\,a^{2}b^{3}-4/3\,b^{5}\right)\left(\tan\left(x\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] -2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(-a^2*b^3*tan(1/2*x)^9-a*b^4*tan(1/2*x)^8+
(-16/3*a^2*b^3-4/3*b^5)*tan(1/2*x)^7+(2*a^5+6*a^3*b^2)*tan(1/2*x)^6+(16/5*a
^4*b-34/15*a^2*b^3+8/15*b^5)*tan(1/2*x)^5+(-2/3*a^5+10/3*a^3*b^2-2*a*b^4)*t
an(1/2*x)^4+(-16/3*a^2*b^3-4/3*b^5)*tan(1/2*x)^3+(2/3*a^5+14/3*a^3*b^2)*tan
(1/2*x)^2-a^2*b^3*tan(1/2*x)+2/15*a^5+14/15*a^3*b^2-1/5*a*b^4)/(tan(1/2*x)^
2+1)^5-16*a^3*b^3/(8*a^6+24*a^4*b^2+24*a^2*b^4+8*b^6)/(a^2+b^2)^(1/2)*arcta
nh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 0.583998, size = 706, normalized size = 3.66

$$\frac{15\sqrt{a^{2}+b^{2}}a^{3}b^{3}\log\left(\frac{2\,ab\cos(x)\sin(x)+\left(a^{2}-b^{2}\right)\cos(x)^{2}-2\,a^{2}-b^{2}-2\,\sqrt{a^{2}+b^{2}}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+\left(a^{2}-b^{2}\right)\cos(x)^{2}+b^{2}}\right)+6\left(a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}\right)\cos\left(x\right)^{5}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(a^2 + b^2)*a^3*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos
(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x
```

```
)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
a*b^6)*cos(x)^5 - 10*(a^7 + 2*a^5*b^2 + a^3*b^4)*cos(x)^3 - 30*(a^5*b^2 + a
^3*b^4)*cos(x) - 2*(3*a^6*b - 11*a^4*b^3 - 16*a^2*b^5 - 2*b^7 + 3*(a^6*b +
3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^4 - (6*a^6*b + 13*a^4*b^3 + 8*a^2*b^5 +
 b^7)*cos(x)^2)*sin(x))/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23626, size = 487, normalized size = 2.52

$$\frac{a^3b^3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^9+15\,ab^4\tan\left(\frac{1}{2}\,x\right)^8+80\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^7+20\,b^5\tan\left(\frac{1}{2}\,x\right)^7 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^3*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x
) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)) + 2/15*(15*a^2*b^3*tan(1/2*x)^9 + 15*a*b^4*tan(1/2*x)^8 + 80*a^2*b
^3*tan(1/2*x)^7 + 20*b^5*tan(1/2*x)^7 - 30*a^5*tan(1/2*x)^6 - 90*a^3*b^2*ta
n(1/2*x)^6 - 48*a^4*b*tan(1/2*x)^5 + 34*a^2*b^3*tan(1/2*x)^5 - 8*b^5*tan(1/
2*x)^5 + 10*a^5*tan(1/2*x)^4 - 50*a^3*b^2*tan(1/2*x)^4 + 30*a*b^4*tan(1/2*x
)^4 + 80*a^2*b^3*tan(1/2*x)^3 + 20*b^5*tan(1/2*x)^3 - 10*a^5*tan(1/2*x)^2 -
 70*a^3*b^2*tan(1/2*x)^2 + 15*a^2*b^3*tan(1/2*x) - 2*a^5 - 14*a^3*b^2 + 3*a
*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^5)
```

3.284 $\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=70

$$\frac{2abx}{\left(a^2+b^2\right)^2}-\frac{b\sin(x)}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}-\frac{\left(a^2-b^2\right)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

```
[Out] (2*a*b*x)/(a^2 + b^2)^2 - ((a^2 - b^2)*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2
)^2 - (b*Sin[x])/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.166461, antiderivative size = 87, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {3111, 3098, 3133, 3097, 3075}

$$\frac{2abx}{\left(a^2+b^2\right)^2}-\frac{b\sin(x)}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}-\frac{a^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}+\frac{b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*b*x)/(a^2 + b^2)^2 - (a^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2 + (b
^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2 - (b*Sin[x])/((a^2 + b^2)*(a*Cos
[x] + b*Sin[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
```

```
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}\\
&= \frac{2abx}{\left(a^2+b^2\right)^2}-\frac{b\sin(x)}{\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}-\frac{a^2\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{b^2\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,d
\end{aligned}
$$

$$
= \frac{2abx}{\left(a^2+b^2\right)^2}-\frac{a^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}+\frac{b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}-\frac{b\sin(x}{\left(a^2+b^2\right)(a\cos(x}
$$

Mathematica [C] time = 0.239182, size = 144, normalized size = 2.06

$$
\frac{a\cos(x)\left(\left(b^2-a^2\right)\log\left((a\cos(x)+b\sin(x))^2\right)-2ix(a+ib)^2\right)+b\sin(x)\left(\left(b^2-a^2\right)\log\left((a\cos(x)+b\sin(x))^2\right)+2(a+ib)\right.}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2]
) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I + x)) + (-a^2 + b^2)*Log[(a*Cos[x]
+ b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin
[x]))/(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.089, size = 120, normalized size = 1.7

$$\frac{a}{\left(a^{2}+b^{2}\right)\left(a+b\tan\left(x\right)\right)}-\frac{\ln\left(a+b\tan\left(x\right)\right)a^{2}}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\ln\left(a+b\tan\left(x\right)\right)b^{2}}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\ln\left(\left(\tan\left(x\right)\right)^{2}+1\right)a^{2}}{2\left(a^{2}+b^{2}\right)^{2}}-\frac{\ln\left(\left(\tan\left(x\right)\right)^{2}+1\right)b^{2}}{2\left(a^{2}+b^{2}\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] a/(a^2+b^2)/(a+b*tan(x))-1/(a^2+b^2)^2*ln(a+b*tan(x))*a^2+1/(a^2+b^2)^2*ln(
a+b*tan(x))*b^2+1/2/(a^2+b^2)^2*ln(tan(x)^2+1)*a^2-1/2/(a^2+b^2)^2*ln(tan(x
)^2+1)*b^2+2/(a^2+b^2)^2*a*b*arctan(tan(x))
```

Maxima [A] time = 1.71112, size = 159, normalized size = 2.27

$$\frac{2\,abx}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{\left(a^{2}-b^{2}\right)\log\left(b\tan\left(x\right)+a\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{\left(a^{2}-b^{2}\right)\log\left(\tan\left(x\right)^{2}+1\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}+\frac{a}{a^{3}+ab^{2}+\left(a^{2}b+b^{3}\right)\tan\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*log(b*tan(x) + a)/(a^4 + 2*a^
2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) +
a/(a^3 + a*b^2 + (a^2*b + b^3)*tan(x))
```

Fricas [A] time = 0.506245, size = 323, normalized size = 4.61

$$\frac{2\left(2\,a^{2}bx+ab^{2}\right)\cos\left(x\right)-\left(\left(a^{3}-ab^{2}\right)\cos\left(x\right)+\left(a^{2}b-b^{3}\right)\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}\right)+2\,(2}{2\left(\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cos\left(x\right)+\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*a^2*b*x + a*b^2)*cos(x) - ((a^3 - a*b^2)*cos(x) + (a^2*b - b^3)*s
in(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*(2*a*b^2*x
 - a^2*b)*sin(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 +
b^5)*sin(x))
```

Sympy [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Exception raised: AttributeError
```

Giac [B] time = 1.09075, size = 194, normalized size = 2.77

$$\frac{2\,abx}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{\left(a^{2}-b^{2}\right)\log\left(\tan\left(x\right)^{2}+1\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}-\frac{\left(a^{2}b-b^{3}\right)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^{4}b+2\,a^{2}b^{3}+b^{5}}+\frac{a^{2}b\tan\left(x\right)-b^{3}\tan\left(x\right)+2\,a^{3}}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(b\tan\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 +
2*a^2*b^2 + b^4) - (a^2*b - b^3)*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3
+ b^5) + (a^2*b*tan(x) - b^3*tan(x) + 2*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*ta
n(x) + a))
```

3.285 $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=110

$$-\frac{(a^2-b^2)\sin(x)}{(a^2+b^2)^2}-\frac{2ab\cos(x)}{(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}-\frac{a(a^2-2b^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

```
[Out] -((a*(a^2 - 2*b^2)*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b
^2)^(5/2)) - (2*a*b*Cos[x])/(a^2 + b^2)^2 - ((a^2 - b^2)*Sin[x])/(a^2 + b^2
)^2 - (a^2*b)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.239339, antiderivative size = 152, normalized size of antiderivative = 1.38, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3109, 2637, 2638, 3074, 206, 3099, 3154}

$$-\frac{a^2\sin(x)}{(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}-\frac{2ab\cos(x)}{(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}-\frac{a^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{2ab^2\tanh^{-1}\left(\right.}{(a^2+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((a^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)) +
 (2*a*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
 - (2*a*b*Cos[x])/(a^2 + b^2)^2 - (a^2*Sin[x])/(a^2 + b^2)^2 + (b^2*Sin[x])
/(a^2 + b^2)^2 - (a^2*b)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
```

```
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= -\frac{a^2\sin(x)}{\left(a^2+b^2\right)^2} - \frac{a^2b}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} + \frac{a^3\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \sin(x)\,d}{\left(a^2+b^2\right)^2}$$

$$= -\frac{2ab\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{b^2\sin(x)}{\left(a^2+b^2\right)^2} - \frac{a^2b}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} - \frac{a^3\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{a^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} + \frac{2ab^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{2ab\cos(x)}{\left(a^2+b^2\right)^2} - \frac{a^2\sin(x)}{\left(a^2+b^2\right)^2} +$$

Mathematica [A] time = 0.589082, size = 111, normalized size = 1.01

$$\frac{2a\left(a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{a\left(a^2+b^2\right)\sin(2x)+b\left(a^2+b^2\right)\cos(2x)+5a^2b-b^3}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*(a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^
(5/2) - (5*a^2*b - b^3 + b*(a^2 + b^2)*Cos[2*x] + a*(a^2 + b^2)*Sin[2*x])/(
2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.114, size = 142, normalized size = 1.3

$$2\,\frac{\left(-a^2+b^2\right)\tan(x/2)-2\,ab}{\left(a^4+2\,a^2b^2+b^4\right)\left((\tan(x/2))^2+1\right)} - 2\,\frac{a}{\left(a^2+b^2\right)^2}\left(\frac{-\tan(x/2)\,b^2-ab}{(\tan(x/2))^2a-2\,b\tan(x/2)-a} - \frac{a^2-2\,b^2}{\sqrt{a^2+b^2}}\mathrm{Artanh}\left(1/2\,\frac{2\,a\tan}{\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*x)-2*a*b)/(tan(1/2*x)^2+1)-2*a/(a
^2+b^2)^2*((-tan(1/2*x)*b^2-a*b)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-(a^2-2*b
^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.544414, size = 597, normalized size = 5.43

$$-\frac{4\,a^4b+2\,a^2b^3-2\,b^5+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(x\right)^2+2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(x\right)\sin\left(x\right)+\sqrt{a^2+b^2}\left(\left(a^4-2\,a^2b^2\right)\cos}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)\cos\left(x\right)+\left(a^6b+3\,a^4b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4*b + 2*a^2*b^3 - 2*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 +
2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)*sin(x) + sqrt(a^2 + b^2)*((a^4 - 2*a^2*b
^2)*cos(x) + (a^3*b - 2*a*b^3)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 - b^
2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.20592, size = 282, normalized size = 2.56

$$-\frac{\left(a^3-2\,ab^2\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,x\right)^3-2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3-a^2b\tan\left(\frac{1}{2}\,x\right)^2+2\,b^3\tan\left(\frac{1}{2}\,x\right)^2-a^3\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3 - 2*a*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
b^2)) - 2*(a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - a^2*b*tan(1/2*x)^2 + 2
*b^3*tan(1/2*x)^2 - a^3*tan(1/2*x) - 4*a*b^2*tan(1/2*x) - 3*a^2*b)/((a*tan(
1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

3.286 $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=129

$$\frac{bx\left(3a^3-ab^2\right)}{\left(a^2+b^2\right)^3}-\frac{\left(a^2-b^2\right)\sin^2(x)}{2\left(a^2+b^2\right)^2}-\frac{a^2b\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}-\frac{ab\sin(x)\cos(x)}{\left(a^2+b^2\right)^2}-\frac{a^2\left(a^2-3b^2\right)\log(a\cos(x)+b\,s}{\left(a^2+b^2\right)^3}$$

```
[Out] (b*(3*a^3 - a*b^2)*x)/(a^2 + b^2)^3 - (a^2*(a^2 - 3*b^2)*Log[a*Cos[x] + b*S
in[x]])/(a^2 + b^2)^3 - (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 - ((a^2 - b^2)*Si
n[x]^2)/(2*(a^2 + b^2)^2) - (a^2*b*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin
[x]))
```

Rubi [A] time = 0.506473, antiderivative size = 198, normalized size of antiderivative = 1.53, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3109, 2564, 30, 2635, 8, 3097, 3133, 3099, 3085, 3483, 3531, 3530}

$$\frac{a^3bx}{\left(a^2+b^2\right)^3}+\frac{abx}{\left(a^2+b^2\right)^2}-\frac{ab^3x}{\left(a^2+b^2\right)^3}+\frac{abx\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3}-\frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2}+\frac{b^2\sin^2(x)}{2\left(a^2+b^2\right)^2}-\frac{a^2b}{\left(a^2+b^2\right)^2(a\cot(x)+b)}-\frac{ab\sin(}{(a^2+}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a^3*b*x)/(a^2 + b^2)^3 - (a*b^3*x)/(a^2 + b^2)^3 + (a*b*(a^2 - b^2)*x)/(a^
2 + b^2)^3 + (a*b*x)/(a^2 + b^2)^2 - (a^2*b)/((a^2 + b^2)^2*(b + a*Cot[x]))
 - (a^4*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (3*a^2*b^2*Log[a*Cos[x] +
 b*Sin[x]])/(a^2 + b^2)^3 - (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 - (a^2*Sin[x]
^2)/(2*(a^2 + b^2)^2) + (b^2*Sin[x]^2)/(2*(a^2 + b^2)^2)
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
```

```
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3085

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= -\frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2} + \frac{a^3\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{(a\ldots}{\ldots}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2} - \frac{a^4\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,d\ldots}{\left(a^2+b^2\right)^3}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^4\log(a\cos(x)+\ldots}{\left(a^2+b^2\right)^3}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^4\log(a\cos(x)+\ldots}{\left(a^2+b^2\right)^3}$$

Mathematica [C] time = 1.43766, size = 226, normalized size = 1.75

$$\frac{a\cos(x)\left(\left(a^4-b^4\right)\cos(2x)+2a\left(-b\left(a^2+b^2\right)\sin(2x)-a\left(a^2-3b^2\right)\log\left((a\cos(x)+b\sin(x))^2\right)+2x(-b+ia)^3\right)\right)-b\sin\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((4*I)*a^2*(a^2 - 3*b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) + a*Cos[x]*((
a^4 - b^4)*Cos[2*x] + 2*a*(2*(I*a - b)^3*x - a*(a^2 - 3*b^2)*Log[(a*Cos[x]
+ b*Sin[x])^2] - b*(a^2 + b^2)*Sin[2*x])) - b*Sin[x]*((-a^4 + b^4)*Cos[2*x]
 + 2*a*(2*(a^3*(1 + I*x) + a*b^2*(1 - (3*I)*x) - 3*a^2*b*x + b^3*x) + a*(a^
2 - 3*b^2)*Log[(a*Cos[x] + b*Sin[x])^2] + b*(a^2 + b^2)*Sin[2*x])))/(4*(a^2
 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.101, size = 243, normalized size = 1.9

$$\frac{a^3}{\left(a^2+b^2\right)^2(a+b\tan(x))} - \frac{a^4\ln(a+b\tan(x))}{\left(a^2+b^2\right)^3} + 3\frac{a^2\ln(a+b\tan(x))b^2}{\left(a^2+b^2\right)^3} - \frac{\tan(x)a^3b}{\left(a^2+b^2\right)^3\left((\tan(x))^2+1\right)} - \frac{a\tan\ldots}{\left(a^2+b^2\right)^3(\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] a^3/(a^2+b^2)^2/(a+b*tan(x))-a^4/(a^2+b^2)^3*ln(a+b*tan(x))+3*a^2/(a^2+b^2)
^3*ln(a+b*tan(x))*b^2-1/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*a^3*b-1/(a^2+b^2)^3
/(tan(x)^2+1)*tan(x)*a*b^3+1/2/(a^2+b^2)^3/(tan(x)^2+1)*a^4-1/2/(a^2+b^2)^3
/(tan(x)^2+1)*b^4+1/2/(a^2+b^2)^3*ln(tan(x)^2+1)*a^4-3/2/(a^2+b^2)^3*ln(tan
(x)^2+1)*a^2*b^2+3/(a^2+b^2)^3*arctan(tan(x))*a^3*b-1/(a^2+b^2)^3*a*arctan(
tan(x))*b^3
```

Maxima [B] time = 1.70432, size = 350, normalized size = 2.71

$$\frac{\left(3\,a^{3}b-ab^{3}\right)x}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}-\frac{\left(a^{4}-3\,a^{2}b^{2}\right)\log\left(b\tan\left(x\right)+a\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{\left(a^{4}-3\,a^{2}b^{2}\right)\log\left(\tan\left(x\right)^{2}+1\right)}{2\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}+\frac{}{2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}+\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] (3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 - 3*a^2*b^2)
*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b
^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^3 - a*
b^2 + 2*(a^3 - a*b^2)*tan(x)^2 - (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2 + a
*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x
)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

Fricas [A] time = 0.559608, size = 531, normalized size = 4.12

$$\frac{2\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cos\left(x\right)^{3}-\left(a^{5}+3\,ab^{4}-4\left(3\,a^{4}b-a^{2}b^{3}\right)x\right)\cos\left(x\right)-2\left(\left(a^{5}-3\,a^{3}b^{2}\right)\cos\left(x\right)+\left(a^{4}b-3\,a^{2}b^{3}\right)\sin\left(x\right)\right)}{4\left(\left(a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}\right)\cos\left(x\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 3*a*b^4 - 4*(3*a^4*b - a
^2*b^3)*x)*cos(x) - 2*((a^5 - 3*a^3*b^2)*cos(x) + (a^4*b - 3*a^2*b^3)*sin(x
```

```
))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (5*a^4*b - b^5 +
 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(3*a^3*b^2 - a*b^4)*x)*sin(x))/((
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^
5 + b^7)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.12032, size = 301, normalized size = 2.33

$$\frac{\left(3\,a^3b - ab^3\right)x}{a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6} + \frac{\left(a^4 - 3\,a^2b^2\right)\log\left(\tan\left(x\right)^2 + 1\right)}{2\left(a^6 + 3\,a^4b^2 + 3\,a^2b^4 + b^6\right)} - \frac{\left(a^4b - 3\,a^2b^3\right)\log\left(|b\tan\left(x\right) + a|\right)}{a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7} + \frac{2\,a^3\tan\left(x\right)^2 - 2\,ab^2 \cdots}{2\left(a^4 + 2\,a^2b^2 + \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*
b^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2
*b^3)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2*(2
*a^3*tan(x)^2 - 2*a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + 3*a^3 - a*b^
2)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

3.287 $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=109

$$\frac{2ab\sin(x)}{(a^2+b^2)^2}-\frac{(a^2-b^2)\cos(x)}{(a^2+b^2)^2}+\frac{ab^2}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}-\frac{b(b^2-2a^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

```
[Out] -((b*(-2*a^2 + b^2)*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 +
b^2)^(5/2)) - ((a^2 - b^2)*Cos[x])/(a^2 + b^2)^2 + (2*a*b*Sin[x])/(a^2 + b^
2)^2 + (a*b^2)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rubi [A] time = 0.259293, antiderivative size = 151, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3111, 3100, 2637, 3074, 206, 3109, 2638, 3155}

$$\frac{2ab\sin(x)}{(a^2+b^2)^2}+\frac{b^2\cos(x)}{(a^2+b^2)^2}-\frac{a^2\cos(x)}{(a^2+b^2)^2}+\frac{ab^2}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}-\frac{b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{2a^2b\tanh^{-1}\left(\frac{b\cos}{}\right.}{(a^2+b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a^2*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
- (b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(a^2*Cos[x])/(a^2 + b^2)^2 + (b^2*Cos[x])/(a^2 + b^2)^2 + (2*a*b*Sin[x])/(a
^2 + b^2)^2 + (a*b^2)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
```

```
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{b^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{ab^2}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} + \frac{a^2\int \sin(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right.}{}$$

$$= -\frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{b^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{2ab\sin(x)}{\left(a^2+b^2\right)^2} + \frac{ab^2}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} + 2\frac{\left(a^2b\right)\text{Subst}}{}$$

$$= \frac{2a^2b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{b^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{2a}{\left(a^2\right.}$$

Mathematica [A] time = 0.682217, size = 110, normalized size = 1.01

$$\frac{2b\left(b^2-2a^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}} - \frac{-b\left(a^2+b^2\right)\sin(2x)+a\left(a^2+b^2\right)\cos(2x)+a^3-5ab^2}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*b*(-2*a^2 + b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)
^(5/2) - (a^3 - 5*a*b^2 + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/
(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.121, size = 144, normalized size = 1.3

$$-4\frac{-ab\tan(x/2)+1/2\,a^2-1/2\,b^2}{\left(a^4+2\,a^2b^2+b^4\right)\left((\tan(x/2))^2+1\right)} + 4\frac{b}{\left(a^2+b^2\right)^2}\left(\frac{-1/2\,\tan(x/2)\,b^2-1/2\,ab}{(\tan(x/2))^2a-2\,b\tan(x/2)-a} - 1/2\,\frac{2\,a^2-b^2}{\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,a}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -4/(a^4+2*a^2*b^2+b^4)*(-a*b*tan(1/2*x)+1/2*a^2-1/2*b^2)/(tan(1/2*x)^2+1)+4
*b/(a^2+b^2)^2*((-1/2*tan(1/2*x)*b^2-1/2*a*b)/(tan(1/2*x)^2*a-2*b*tan(1/2*x
)-a)-1/2*(2*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+
b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.542812, size = 586, normalized size = 5.38

$$\frac{6\,a^3b^2 + 6\,ab^4 - 2\left(a^5 + 2\,a^3b^2 + ab^4\right)\cos\left(x\right)^2 + 2\left(a^4b + 2\,a^2b^3 + b^5\right)\cos\left(x\right)\sin\left(x\right) - \sqrt{a^2+b^2}\left(\left(2\,a^3b - ab^3\right)\cos\left(x\right) + \right.}{2\left(\left(a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6\right)\cos\left(x\right) + \left(a^6b + 3\,a^4b^3 + \right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*b^2 + 6*a*b^4 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + 2*(a^4*b
+ 2*a^2*b^3 + b^5)*cos(x)*sin(x) - sqrt(a^2 + b^2)*((2*a^3*b - a*b^3)*cos(x
) + (2*a^2*b^2 - b^4)*sin(x))*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x
)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24049, size = 275, normalized size = 2.52

$$\frac{\left(2\,a^2b-b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(2\,a^2b\tan\left(\frac{1}{2}\,x\right)^3-b^3\tan\left(\frac{1}{2}\,x\right)^3-a^3\tan\left(\frac{1}{2}\,x\right)^2-4\,ab^2\tan\left(\frac{1}{2}\,x\right)^2-3\,b^3\,\mathrm{ta}\right.}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^4+2\,a^2b^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*b - b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b
^2)) + 2*(2*a^2*b*tan(1/2*x)^3 - b^3*tan(1/2*x)^3 - a^3*tan(1/2*x)^2 - 4*a*
b^2*tan(1/2*x)^2 - 3*b^3*tan(1/2*x) + a^3 - 2*a*b^2)/((a*tan(1/2*x)^4 - 2*b
*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

3.288 $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=131

$$\frac{x\left(-6a^2b^2+a^4+b^4\right)}{2\left(a^2+b^2\right)^3}+\frac{ab\sin^2(x)}{\left(a^2+b^2\right)^2}+\frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{\left(b^2-a^2\right)\sin(x)\cos(x)}{2\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\log(a\cos}{\left(a^2+b^2\right.}$$

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(2*(a^2 + b^2)^3) + (2*a*b*(a^2 - b^2)*Log[a*Co
s[x] + b*Sin[x]])/(a^2 + b^2)^3 + ((-a^2 + b^2)*Cos[x]*Sin[x])/(2*(a^2 + b^
2)^2) + (a*b*Sin[x]^2)/(a^2 + b^2)^2 + (a*b^2*Sin[x])/((a^2 + b^2)^2*(a*Cos
[x] + b*Sin[x]))
```

Rubi [A] time = 0.544877, antiderivative size = 186, normalized size of antiderivative = 1.42, number of steps used = 21, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3111, 3109, 2635, 8, 2564, 30, 3098, 3133, 3097, 3075}

$$\frac{a^2x}{2\left(a^2+b^2\right)^2}-\frac{4a^2b^2x}{\left(a^2+b^2\right)^3}+\frac{b^2x}{2\left(a^2+b^2\right)^2}+\frac{ab\sin^2(x)}{\left(a^2+b^2\right)^2}-\frac{a^2\sin(x)\cos(x)}{2\left(a^2+b^2\right)^2}+\frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{b^2\sin(x)}{2\left(a^2+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-4*a^2*b^2*x)/(a^2 + b^2)^3 + (a^2*x)/(2*(a^2 + b^2)^2) + (b^2*x)/(2*(a^2
+ b^2)^2) + (2*a^3*b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 - (2*a*b^3*Log
[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 - (a^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2
) + (b^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2) + (a*b*Sin[x]^2)/(a^2 + b^2)^2 +
(a*b^2*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
```

```
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \cos^2(x)\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{a^2\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2} + \frac{b^2\cos(x)\sin(x)}{2\left(a^2+b^2\right)^2} + \frac{ab^2\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))} - 2\left(\frac{a^2b^2x}{\left(a^2+b^2\right)^3} - \right.$$

$$= \frac{a^2x}{2\left(a^2+b^2\right)^2} + \frac{b^2x}{2\left(a^2+b^2\right)^2} - 2\left(\frac{a^2b^2x}{\left(a^2+b^2\right)^3} - \frac{a^3b\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}\right) - 2\left(\frac{a^2b^2x}{\left(a^2+b^2\right.}\right.$$

Mathematica [A] time = 1.58527, size = 145, normalized size = 1.11

$$\frac{\sin(x)}{8a(a\cos(x)+b\sin(x))} - \frac{-4x\left(-6a^2b^2+a^4+b^4\right)+2\left(a^4-b^4\right)\sin(2x)+4ab\left(a^2+b^2\right)\cos(2x)+\frac{\left(a^2+b^2\right)\left(-6a^2b^2+a^4+b^4\right)\sin}{a(a\cos(x)+b\sin(x))}}{8\left(a^2+b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] Sin[x]/(8*a*(a*Cos[x] + b*Sin[x])) - (-4*(a^4 - 6*a^2*b^2 + b^4)*x + 4*a*b*
(a^2 + b^2)*Cos[2*x] - 16*a*b*(a^2 - b^2)*Log[a*Cos[x] + b*Sin[x]] + ((a^2
+ b^2)*(a^4 - 6*a^2*b^2 + b^4)*Sin[x])/(a*(a*Cos[x] + b*Sin[x])) + 2*(a^4 -
 b^4)*Sin[2*x])/(8*(a^2 + b^2)^3)
```

Maple [B] time = 0.105, size = 260, normalized size = 2.

$$-\frac{a^2b}{\left(a^2+b^2\right)^2(a+b\tan(x))}+2\frac{a^3b\ln(a+b\tan(x))}{\left(a^2+b^2\right)^3}-2\frac{ab^3\ln(a+b\tan(x))}{\left(a^2+b^2\right)^3}-\frac{\tan(x)a^4}{2\left(a^2+b^2\right)^3\left((\tan(x))^2+1\right)}+\frac{}{2\left(a^2+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -a^2*b/(a^2+b^2)^2/(a+b*tan(x))+2*a^3*b/(a^2+b^2)^3*ln(a+b*tan(x))-2*a*b^3/
(a^2+b^2)^3*ln(a+b*tan(x))-1/2/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*a^4+1/2/(a^2
+b^2)^3/(tan(x)^2+1)*tan(x)*b^4-1/(a^2+b^2)^3/(tan(x)^2+1)*a^3*b-1/(a^2+b^2
)^3/(tan(x)^2+1)*a*b^3-1/(a^2+b^2)^3*ln(tan(x)^2+1)*a^3*b+1/(a^2+b^2)^3*ln(
tan(x)^2+1)*a*b^3-3/(a^2+b^2)^3*arctan(tan(x))*a^2*b^2+1/2/(a^2+b^2)^3*arct
an(tan(x))*b^4+1/2/(a^2+b^2)^3*arctan(tan(x))*a^4
```

Maxima [B] time = 1.69715, size = 347, normalized size = 2.65

$$\frac{\left(a^4-6a^2b^2+b^4\right)x}{2\left(a^6+3a^4b^2+3a^2b^4+b^6\right)}+\frac{2\left(a^3b-ab^3\right)\log(b\tan(x)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{\left(a^3b-ab^3\right)\log\left(\tan(x)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{}{2\left(a^5+2a^3b^2+ab^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3*
b - a*b^3)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b -
 a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(4*a^2*
b + (3*a^2*b - b^3)*tan(x)^2 + (a^3 + a*b^2)*tan(x))/(a^5 + 2*a^3*b^2 + a*b
^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x)^
2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

Fricas [A] time = 0.566495, size = 540, normalized size = 4.12

$$-\frac{\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(x\right)^3+\left(a^2b^3-b^5-\left(a^5-6\,a^3b^2+ab^4\right)x\right)\cos\left(x\right)-2\left(\left(a^4b-a^2b^3\right)\cos\left(x\right)+\left(a^3b^2-ab^4\right)\sin\left(x\right.\right.}{2\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6\right)\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 + (a^2*b^3 - b^5 - (a^5 - 6*a^3*b^
2 + a*b^4)*x)*cos(x) - 2*((a^4*b - a^2*b^3)*cos(x) + (a^3*b^2 - a*b^4)*sin(
x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (3*a^3*b^2 + a*
b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + (a^4*b - 6*a^2*b^3 + b^5)*x)*sin
(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.09116, size = 296, normalized size = 2.26

$$\frac{\left(a^4-6\,a^2b^2+b^4\right)x}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}-\frac{\left(a^3b-ab^3\right)\log\left(\tan\left(x\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\left(a^3b^2-ab^4\right)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{3\,a^2b\tan\left(x\right)^2-b^3}{2\left(a^4+2\,a^2b^2+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b
- a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3*b^2
 - a*b^4)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/
```

```
2*(3*a^2*b*tan(x)^2 - b^3*tan(x)^2 + a^3*tan(x) + a*b^2*tan(x) + 4*a^2*b)/(
(a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

3.289 $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=172

$$\frac{2ab\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{2ab\left(a^2-b^2\right)\sin(x)}{\left(a^2+b^2\right)^3}+\frac{\left(a^2-b^2\right)\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a^2\left(a^2-3b^2\right)\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^3b^2}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}+\frac{a^2b\,(}{}$$

```
[Out] (a^2*b*(2*a^2 - 3*b^2)*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2
 + b^2)^(7/2) - (a^2*(a^2 - 3*b^2)*Cos[x])/(a^2 + b^2)^3 + ((a^2 - b^2)*Cos
[x]^3)/(3*(a^2 + b^2)^2) + (2*a*b*(a^2 - b^2)*Sin[x])/(a^2 + b^2)^3 + (2*a*
b*Sin[x]^3)/(3*(a^2 + b^2)^2) + (a^3*b^2)/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[
x]))
```

Rubi [A] time = 0.677315, antiderivative size = 238, normalized size of antiderivative = 1.38, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3111, 3109, 2565, 30, 2564, 2637, 2638, 3074, 206, 2633, 3099, 3154}

$$\frac{2ab\sin^3(x)}{3\left(a^2+b^2\right)^2}+\frac{2a^3b\sin(x)}{\left(a^2+b^2\right)^3}-\frac{2ab^3\sin(x)}{\left(a^2+b^2\right)^3}+\frac{a^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{b^2\cos^3(x)}{3\left(a^2+b^2\right)^2}-\frac{a^2\cos(x)}{\left(a^2+b^2\right)^2}+\frac{4a^2b^2\cos(x)}{\left(a^2+b^2\right)^3}+\frac{a^3b}{\left(a^2+b^2\right)^3(a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a^4*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
- (3*a^2*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7
/2) + (4*a^2*b^2*Cos[x])/(a^2 + b^2)^3 - (a^2*Cos[x])/(a^2 + b^2)^2 + (a^2*
Cos[x]^3)/(3*(a^2 + b^2)^2) - (b^2*Cos[x]^3)/(3*(a^2 + b^2)^2) + (2*a^3*b*S
in[x])/(a^2 + b^2)^3 - (2*a*b^3*Sin[x])/(a^2 + b^2)^3 + (2*a*b*Sin[x]^3)/(3
*(a^2 + b^2)^2) + (a^3*b^2)/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
```

```
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3099

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Simp[(a*Sin[c + d*x]^(m - 1))/(d*(a^2
+ b^2)*(m - 1)), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*Co
s[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*x]
^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m,
1]
```

Rule 3154

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> -Simp[(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \sin^3(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right)\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \cos^2(x)\sin}{\left(a^2+b^2\right)}$$

$$= \frac{a^3b^2}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))} - 2\left(-\frac{a^3b\sin(x)}{\left(a^2+b^2\right)^3} + \frac{\left(a^4b\right)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(a^2b^2\right)\int \mathrm{si}}{\left(a^2+b\right.}\right.$$

$$= -\frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\cos^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{2ab\sin^3(x)}{3\left(a^2+b^2\right)^2} + \frac{a^3b^2}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x)}$$

$$= -\frac{a^2b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{a^2\cos(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\cos^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{2ab\sin^3(x)}{3\left(a^2+b^2\right)^2} +$$

Mathematica [A] time = 1.21688, size = 200, normalized size = 1.16

$$\frac{16a^2b^3\sin(2x) - 2a^2b^3\sin(4x) + a\left(a^2+b^2\right)^2\cos(4x) + \left(4a^3b^2 - 8a^5 + 12ab^4\right)\cos(2x) + 90a^3b^2 + 18a^4b\sin(2x) - a^4b\sin}{24\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-2*a^2*b*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(7/2) + (-9*a^5 + 90*a^3*b^2 - 21*a*b^4 + (-8*a^5 + 4*a^3*b^2 + 12*a
*b^4)*Cos[2*x] + a*(a^2 + b^2)^2*Cos[4*x] + 18*a^4*b*Sin[2*x] + 16*a^2*b^3*
Sin[2*x] - 2*b^5*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*Sin[4
*x])/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.133, size = 269, normalized size = 1.6

$$-4\frac{\left(-a^3b+ab^3\right)(\tan(x/2))^5 + \left(-3/2\,a^2b^2+1/2\,b^4\right)(\tan(x/2))^4 + \left(-10/3\,a^3b+2/3\,ab^3\right)(\tan(x/2))^3 + \left(a^4-3\,a^2b^2\right)(\tan}{\left(a^2+b^2\right)\left(a^4+2\,a^2b^2+b^4\right)\left((\tan(x/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] -4/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-a^3*b+a*b^3)*tan(1/2*x)^5+(-3/2*a^2*b^2
+1/2*b^4)*tan(1/2*x)^4+(-10/3*a^3*b+2/3*a*b^3)*tan(1/2*x)^3+(a^4-3*a^2*b^2)
*tan(1/2*x)^2+(-a^3*b+a*b^3)*tan(1/2*x)+1/3*a^4-3/2*a^2*b^2+1/6*b^4)/(tan(1
/2*x)^2+1)^3+4*a^2*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-1/2*tan(1/2*x)*b^2-1/
2*a*b)/(tan(1/2*x)^2*a-2*b*tan(1/2*x)-a)-1/2*(2*a^2-3*b^2)/(a^2+b^2)^(1/2)*
arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.600247, size = 818, normalized size = 4.76

$$\frac{22\,a^5b^2 + 14\,a^3b^4 - 8\,ab^6 + 2\left(a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6\right)\cos\left(x\right)^4 - 2\left(3\,a^7 + 4\,a^5b^2 - a^3b^4 - 2\,ab^6\right)\cos\left(x\right)^2 - 3\,\sqrt{a^2 + b}}{6\left(\left(a^9 -\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(22*a^5*b^2 + 14*a^3*b^4 - 8*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*cos(x)^4 - 2*(3*a^7 + 4*a^5*b^2 - a^3*b^4 - 2*a*b^6)*cos(x)^2 - 3*sqr
t(a^2 + b^2)*((2*a^5*b - 3*a^3*b^3)*cos(x) + (2*a^4*b^2 - 3*a^2*b^4)*sin(x)
)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a
^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 + b^2)) - 2*((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^3 - 5*(a^6*b +
2*a^4*b^3 + a^2*b^5)*cos(x))*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sin
```

```
(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2139, size = 462, normalized size = 2.69

$$\frac{\left(2\,a^4b-3\,a^2b^3\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}-\frac{2\left(a^2b^3\tan\left(\frac{1}{2}\,x\right)+a^3b^2\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)}+\frac{2\left(6\,a^3b\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^4*b - 3*a^2*b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*sqrt(a^2 + b^2)) - 2*(a^2*b^3*tan(1/2*x) + a^3*b^2)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) + 2/3*(6*a^3*b*t
an(1/2*x)^5 - 6*a*b^3*tan(1/2*x)^5 + 9*a^2*b^2*tan(1/2*x)^4 - 3*b^4*tan(1/2
*x)^4 + 20*a^3*b*tan(1/2*x)^3 - 4*a*b^3*tan(1/2*x)^3 - 6*a^4*tan(1/2*x)^2 +
 18*a^2*b^2*tan(1/2*x)^2 + 6*a^3*b*tan(1/2*x) - 6*a*b^3*tan(1/2*x) - 2*a^4
+ 9*a^2*b^2 - b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^
3)
```

3.290 $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=128

$$-\frac{abx\left(a^2-3b^2\right)}{\left(a^2+b^2\right)^3}+\frac{\left(a^2-b^2\right)\sin^2(x)}{2\left(a^2+b^2\right)^2}+\frac{ab^2\cos(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}+\frac{ab\sin(x)\cos(x)}{\left(a^2+b^2\right)^2}-\frac{b^2\left(3a^2-b^2\right)\log(a\cos(x)+}{\left(a^2+b^2\right)^3}$$

```
[Out] -((a*b*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3) - (b^2*(3*a^2 - b^2)*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^3 + (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 + ((a^2 - b^2)
*Sin[x]^2)/(2*(a^2 + b^2)^2) + (a*b^2*Cos[x])/((a^2 + b^2)^2*(a*Cos[x] + b*
Sin[x]))
```

Rubi [A] time = 0.407634, antiderivative size = 196, normalized size of antiderivative = 1.53, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3111, 3100, 2635, 8, 3098, 3133, 3109, 2564, 30, 3086, 3483, 3531, 3530}

$$\frac{ab^3x}{\left(a^2+b^2\right)^3}+\frac{abx}{\left(a^2+b^2\right)^2}-\frac{a^3bx}{\left(a^2+b^2\right)^3}-\frac{abx\left(a^2-b^2\right)}{\left(a^2+b^2\right)^3}+\frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2}+\frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2}+\frac{ab^2}{\left(a^2+b^2\right)^2(a+b\tan(x))}+\frac{ab\,\mathrm{si}}{(a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((a^3*b*x)/(a^2 + b^2)^3) + (a*b^3*x)/(a^2 + b^2)^3 - (a*b*(a^2 - b^2)*x)/
(a^2 + b^2)^3 + (a*b*x)/(a^2 + b^2)^2 + (b^2*Cos[x]^2)/(2*(a^2 + b^2)^2) -
(3*a^2*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (b^4*Log[a*Cos[x] + b*
Sin[x]])/(a^2 + b^2)^3 + (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 + (a^2*Sin[x]^2)
/(2*(a^2 + b^2)^2) + (a*b^2)/((a^2 + b^2)^2*(a + b*Tan[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
```

```
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
```

```
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3086

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3483

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(a +
b*Tan[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 + b^2)), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3531

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[((a*c + b*d)*x)/(a^2 + b^2), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rule 3530

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c*Log[RemoveContent[a*Cos[e + f*x] + b*Sin[e + f
*x], x]])/(b*f), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} + \frac{a^2\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos^2(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(a^2b\right)\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{b^3}{}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} + \frac{ab^2}{\left(a^2+b^2\right)^2(a+b\tan(x))} - \frac{\left(a^2b^2\right)\int \frac{b\cos(x)-a\sin}{a\cos(x)+b\sin}}{\left(a^2+b^2\right)^3}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} - \frac{a^2b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3} +$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{ab\left(a^2-b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} - \frac{3a^2b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3}$$

Mathematica [C] time = 1.28507, size = 221, normalized size = 1.73

$$\frac{b\sin(x)\left(\left(b^4-a^4\right)\cos(2x)+2b\left(-2(a+ib)\left(a^2x+a(b+2ibx)-b^2(x+i)\right)+a\left(a^2+b^2\right)\sin(2x)+\left(b^3-3a^2b\right)\log\left((a\cos\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((-4*I)*b^2*(-3*a^2 + b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) - a*Cos[x]*
((a^4 - b^4)*Cos[2*x] + 2*b*(2*(a + I*b)^3*x - b*(-3*a^2 + b^2)*Log[(a*Cos[
x] + b*Sin[x])^2] - a*(a^2 + b^2)*Sin[2*x])) + b*Sin[x]*((-a^4 + b^4)*Cos[2
*x] + 2*b*(-2*(a + I*b)*(a^2*x - b^2*(I + x) + a*(b + (2*I)*b*x)) + (-3*a^2
*b + b^3)*Log[(a*Cos[x] + b*Sin[x])^2] + a*(a^2 + b^2)*Sin[2*x])))/(4*(a^2
+ b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.098, size = 241, normalized size = 1.9

$$\frac{ab^2}{\left(a^2+b^2\right)^2(a+b\tan(x))} - 3\frac{a^2\ln(a+b\tan(x))\,b^2}{\left(a^2+b^2\right)^3} + \frac{b^4\ln(a+b\tan(x))}{\left(a^2+b^2\right)^3} + \frac{\tan(x)\,a^3b}{\left(a^2+b^2\right)^3\left((\tan(x))^2+1\right)} + \frac{a\tan}{\left(a^2+b^2\right)^3\left((\mathrm{t}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] a*b^2/(a^2+b^2)^2/(a+b*tan(x))-3*a^2/(a^2+b^2)^3*ln(a+b*tan(x))*b^2+b^4/(a^
2+b^2)^3*ln(a+b*tan(x))+1/(a^2+b^2)^3/(tan(x)^2+1)*tan(x)*a^3*b+1/(a^2+b^2)
^3/(tan(x)^2+1)*tan(x)*a*b^3-1/2/(a^2+b^2)^3/(tan(x)^2+1)*a^4+1/2/(a^2+b^2)
^3/(tan(x)^2+1)*b^4+3/2/(a^2+b^2)^3*ln(tan(x)^2+1)*a^2*b^2-1/2/(a^2+b^2)^3*
ln(tan(x)^2+1)*b^4-1/(a^2+b^2)^3*arctan(tan(x))*a^3*b+3/(a^2+b^2)^3*a*arcta
n(tan(x))*b^3
```

Maxima [B] time = 1.70046, size = 346, normalized size = 2.7

$$-\frac{\left(a^3b-3\,ab^3\right)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\left(3\,a^2b^2-b^4\right)\log\left(b\tan\left(x\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^2b^2-b^4\right)\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)}+\frac{}{2\left(a^5+2\,a^3b^2+ab^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^2 - b^4
)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2 -
b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(4*a*b^2*t
an(x)^2 - a^3 + 3*a*b^2 + (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2 + a*b^4 +
(a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x)^2 + (
a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

Fricas [A] time = 0.559767, size = 562, normalized size = 4.39

$$-\frac{2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(x\right)^3-\left(a^5+4\,a^3b^2+7\,ab^4-4\left(a^4b-3\,a^2b^3\right)x\right)\cos\left(x\right)+2\left(\left(3\,a^3b^2-ab^4\right)\cos\left(x\right)+\left(3\,a^2b^3-\right.\right.}{4\left(\left(a^7+3\,a^5b^2+3\,a^3b^4+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 4*a^3*b^2 + 7*a*b^4 - 4
*(a^4*b - 3*a^2*b^3)*x)*cos(x) + 2*((3*a^3*b^2 - a*b^4)*cos(x) + (3*a^2*b^3
```

```
 - b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^
4*b - 4*a^2*b^3 - b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(a^3*b^2 -
 3*a*b^4)*x)*sin(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13159, size = 289, normalized size = 2.26

$$-\frac{\left(a^{3} b-3 \, a b^{3}\right) x}{a^{6}+3 \, a^{4} b^{2}+3 \, a^{2} b^{4}+b^{6}}+\frac{\left(3 \, a^{2} b^{2}-b^{4}\right) \log \left(\tan \left(x\right)^{2}+1\right)}{2 \, \left(a^{6}+3 \, a^{4} b^{2}+3 \, a^{2} b^{4}+b^{6}\right)}-\frac{\left(3 \, a^{2} b^{3}-b^{5}\right) \log \left(|b \tan \left(x\right)+a|\right)}{a^{6} b+3 \, a^{4} b^{3}+3 \, a^{2} b^{5}+b^{7}}+\frac{4 \, a b^{2} \tan \left(x\right)^{2}+a^{2}}{2 \, \left(a^{4}+2 \, a^{2} b^{2}+b^{4}\right)\left(b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2 -
 b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^3 -
b^5)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2*(4*
a*b^2*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a^3 + 3*a*b^2)/((a^4 + 2*a^2*b
^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

3.291 $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=176

$$\frac{(a^2-b^2)\sin^3(x)}{3(a^2+b^2)^2}-\frac{b^2(3a^2-b^2)\sin(x)}{(a^2+b^2)^3}-\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}+\frac{2ab(a^2-b^2)\cos(x)}{(a^2+b^2)^3}-\frac{a^2b^3}{(a^2+b^2)^3(a\cos(x)+b\sin(x))}-\frac{ab^2(\cdots}{}$$

```
[Out] -((a*b^2*(3*a^2 - 2*b^2)*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a
^2 + b^2)^(7/2)) + (2*a*b*(a^2 - b^2)*Cos[x])/(a^2 + b^2)^3 - (2*a*b*Cos[x]
^3)/(3*(a^2 + b^2)^2) - (b^2*(3*a^2 - b^2)*Sin[x])/(a^2 + b^2)^3 + ((a^2 -
b^2)*Sin[x]^3)/(3*(a^2 + b^2)^2) - (a^2*b^3)/((a^2 + b^2)^3*(a*Cos[x] + b*S
in[x]))
```

Rubi [A] time = 0.696505, antiderivative size = 238, normalized size of antiderivative = 1.35, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3111, 3109, 2633, 2565, 30, 3100, 2637, 3074, 206, 2564, 2638, 3155}

$$-\frac{b^2\sin^3(x)}{3(a^2+b^2)^2}+\frac{a^2\sin^3(x)}{3(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}-\frac{4a^2b^2\sin(x)}{(a^2+b^2)^3}-\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}-\frac{2ab^3\cos(x)}{(a^2+b^2)^3}+\frac{2a^3b\cos(x)}{(a^2+b^2)^3}-\frac{a\cdots}{(a^2+b^2)^3(a\,c\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-3*a^3*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/
2) + (2*a*b^4*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(
7/2) + (2*a^3*b*Cos[x])/(a^2 + b^2)^3 - (2*a*b^3*Cos[x])/(a^2 + b^2)^3 - (2
*a*b*Cos[x]^3)/(3*(a^2 + b^2)^2) - (4*a^2*b^2*Sin[x])/(a^2 + b^2)^3 + (b^2*
Sin[x])/(a^2 + b^2)^2 + (a^2*Sin[x]^3)/(3*(a^2 + b^2)^2) - (b^2*Sin[x]^3)/(
3*(a^2 + b^2)^2) - (a^2*b^3)/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
```

```
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3100

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*Cos[c + d*x]^(m - 1))/(d*(a^2 +
 b^2)*(m - 1)), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3155

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \cos(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos^2(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \cos}{\left(a^2+\right.}$$

$$= -\frac{a^2b^3}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))} - 2\left(\frac{\left(a^3b\right)\int \sin(x)\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(a^2b^2\right)\int \cos(x)\,dx}{\left(a^2+b^2\right)^3} - \frac{\left(a^3b^2\right)\int}{\left(a^2\right.}\right.$$

$$= -\frac{2ab\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{b^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\sin^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2\sin^3(x)}{3\left(a^2+b^2\right)^2} - \frac{a^2b^3}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}$$

$$= -\frac{a^3b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{2ab\cos^3(x)}{3\left(a^2+b^2\right)^2} + \frac{b^2\sin(x)}{\left(a^2+b^2\right)^2} + \frac{a^2\sin^3(x)}{3\left(a^2+b^2\right)^2} - \frac{b^2\sin^3(x)}{3\left(a^2+b^2\right)^2} -$$

Mathematica [A] time = 1.22336, size = 198, normalized size = 1.12

$$\frac{2ab^2\left(3a^2-2b^2\right)\tanh^{-1}\left(\frac{a\tan\left(\frac{x}{2}\right)-b}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}} - \frac{16a^3b^2\sin(2x)+2a^3b^2\sin(4x)-4b\left(a^2b^2+3a^4-2b^4\right)\cos(2x)+b\left(a^2+b^2\right)^2\cos}{24\left(a^2+b^2\right)^3(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*b^2*(3*a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 +
 b^2)^(7/2) - (-21*a^4*b + 90*a^2*b^3 - 9*b^5 - 4*b*(3*a^4 + a^2*b^2 - 2*b^
4)*Cos[2*x] + b*(a^2 + b^2)^2*Cos[4*x] - 2*a^5*Sin[2*x] + 16*a^3*b^2*Sin[2*
x] + 18*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4*x] + a*b^4*Sin[4*x]
)/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A] time = 0.131, size = 261, normalized size = 1.5

$$2\,\frac{\left(-3\,a^2b^2+b^4\right)(\tan(x/2))^5-4\,ab^3(\tan(x/2))^4+\left(4/3\,a^4-6\,a^2b^2+2/3\,b^4\right)(\tan(x/2))^3+\left(4\,a^3b-4\,ab^3\right)(\tan(x/2))^2+}{\left(a^2+b^2\right)\left(a^4+2\,a^2b^2+b^4\right)\left((\tan(x/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] 2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((-3*a^2*b^2+b^4)*tan(1/2*x)^5-4*a*b^3*tan(
1/2*x)^4+(4/3*a^4-6*a^2*b^2+2/3*b^4)*tan(1/2*x)^3+(4*a^3*b-4*a*b^3)*tan(1/2
*x)^2+(-3*a^2*b^2+b^4)*tan(1/2*x)+4/3*a^3*b-8/3*a*b^3)/(tan(1/2*x)^2+1)^3-2
*a*b^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-tan(1/2*x)*b^2-a*b)/(tan(1/2*x)^2*a
-2*b*tan(1/2*x)-a)-(3*a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x
)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.604008, size = 838, normalized size = 4.76

$$\frac{2\,a^6b - 22\,a^4b^3 - 20\,a^2b^5 + 4\,b^7 - 2\left(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7\right)\cos\left(x\right)^4 + 2\left(4\,a^6b + 7\,a^4b^3 + 2\,a^2b^5 - b^7\right)\cos\left(x\right)^2 - 3}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^6*b - 22*a^4*b^3 - 20*a^2*b^5 + 4*b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*cos(x)^4 + 2*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*cos(x)^2
 - 3*sqrt(a^2 + b^2)*((3*a^4*b^2 - 2*a^2*b^4)*cos(x) + (3*a^3*b^3 - 2*a*b^5
)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2
*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*
cos(x)^2 + b^2)) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^3 - (a^7
 - 2*a^5*b^2 - 7*a^3*b^4 - 4*a*b^6)*cos(x))*sin(x))/((a^9 + 4*a^7*b^2 + 6*a
^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
```

```
*b^7 + b^9)*sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21616, size = 452, normalized size = 2.57

$$-\frac{\left(3\,a^3b^2-2\,ab^4\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(ab^4\tan\left(\frac{1}{2}\,x\right)+a^2b^3\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)}-\frac{2\left(9\,a^2b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(3*a^3*b^2 - 2*a*b^4)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sqrt(a^2 + b^2)) + 2*(a*b^4*tan(1/2*x) + a^2*b^3)/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) - 2/3*(9*a^2*b^2*
tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a*b^3*tan(1/2*x)^4 - 4*a^4*tan(1/2*x
)^3 + 18*a^2*b^2*tan(1/2*x)^3 - 2*b^4*tan(1/2*x)^3 - 12*a^3*b*tan(1/2*x)^2
+ 12*a*b^3*tan(1/2*x)^2 + 9*a^2*b^2*tan(1/2*x) - 3*b^4*tan(1/2*x) - 4*a^3*b
 + 8*a*b^3)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^3)
```

3.292 $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=210

$$-\frac{3abx\left(-6a^2b^2+a^4+b^4\right)}{4\left(a^2+b^2\right)^4}+\frac{a^2\sin^4(x)}{4\left(a^2+b^2\right)^2}-\frac{2a^2b^2\sin^2(x)}{\left(a^2+b^2\right)^3}-\frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2}-\frac{ab\sin(x)\cos^3(x)}{2\left(a^2+b^2\right)^2}+\frac{ab\left(5a^2-3b^2\right)\sin(x)\,\text{co}}{4\left(a^2+b^2\right)^3}$$

```
[Out] (-3*a*b*(a^4 - 6*a^2*b^2 + b^4)*x)/(4*(a^2 + b^2)^4) - (b^2*Cos[x]^4)/(4*(a
^2 + b^2)^2) - (3*a^2*b^2*(a^2 - b^2)*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
^4 + (a*b*(5*a^2 - 3*b^2)*Cos[x]*Sin[x])/(4*(a^2 + b^2)^3) - (a*b*Cos[x]^3*
Sin[x])/(2*(a^2 + b^2)^2) - (2*a^2*b^2*Sin[x]^2)/(a^2 + b^2)^3 + (a^2*Sin[x
]^4)/(4*(a^2 + b^2)^2) - (a^2*b^3*Sin[x])/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[
x]))
```

Rubi [A] time = 1.25073, antiderivative size = 289, normalized size of antiderivative = 1.38, number of steps used = 48, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3111, 3109, 2565, 30, 2568, 2635, 8, 2564, 3098, 3133, 3097, 3075}

$$-\frac{a^3bx}{\left(a^2+b^2\right)^3}+\frac{6a^3b^3x}{\left(a^2+b^2\right)^4}+\frac{abx}{4\left(a^2+b^2\right)^2}-\frac{ab^3x}{\left(a^2+b^2\right)^3}+\frac{a^2\sin^4(x)}{4\left(a^2+b^2\right)^2}-\frac{2a^2b^2\sin^2(x)}{\left(a^2+b^2\right)^3}-\frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2}-\frac{ab\sin(x)\cos^3(x}{2\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (6*a^3*b^3*x)/(a^2 + b^2)^4 - (a^3*b*x)/(a^2 + b^2)^3 - (a*b^3*x)/(a^2 + b^
2)^3 + (a*b*x)/(4*(a^2 + b^2)^2) - (b^2*Cos[x]^4)/(4*(a^2 + b^2)^2) - (3*a^
4*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^4 + (3*a^2*b^4*Log[a*Cos[x] + b
*Sin[x]])/(a^2 + b^2)^4 + (a^3*b*Cos[x]*Sin[x])/(a^2 + b^2)^3 - (a*b^3*Cos[
x]*Sin[x])/(a^2 + b^2)^3 + (a*b*Cos[x]*Sin[x])/(4*(a^2 + b^2)^2) - (a*b*Cos
[x]^3*Sin[x])/(2*(a^2 + b^2)^2) - (2*a^2*b^2*Sin[x]^2)/(a^2 + b^2)^3 + (a^2
*Sin[x]^4)/(4*(a^2 + b^2)^2) - (a^2*b^3*Sin[x])/((a^2 + b^2)^3*(a*Cos[x] +
b*Sin[x]))
```

Rule 3111

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
```

```
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[(a*b)/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3109

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[(a*b)/(a^2
 + b^2), Int[(Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1))/(a*Cos[c + d*x] +
b*Sin[c + d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 3098

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(a*x)/(a^2 + b^2), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3133

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[((b*B + c*C)*x)/(b^2 + c^2), x] + Simp[((c*B - b*C)*Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]])/(e*(b^2 + c^2)), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3097

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[(b*x)/(a^2 + b^2), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3075

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \cos(x)\sin^3(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos^2(x)\sin^2(x)\,dx}{\left(a^2+b^2\right)^2} - 2\frac{\left(a^2b\right)\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \cos\ldots}{(\ldots}$$

$$= -2\left(\frac{\left(a^3b\right)\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^3} + \frac{\left(a^2b^2\right)\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^3} - \frac{\left(a^3b^2\right)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^3}\right) + \frac{\left(a^3b^2\right)\ldots}{\ldots}$$

$$= \frac{2a^3b^3x}{\left(a^2+b^2\right)^4} - \frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2} + \frac{a^2\sin^4(x)}{4\left(a^2+b^2\right)^2} - \frac{a^2b^3\sin(x)}{\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))} - \frac{\left(a^4b^2\right)\int \frac{b\cos\ldots}{a\cos\ldots}}{\left(a^2+\ldots\right.}$$

$$= \frac{2a^3b^3x}{\left(a^2+b^2\right)^4} - \frac{b^2\cos^4(x)}{4\left(a^2+b^2\right)^2} - \frac{a^4b^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^4} + \frac{a^2b^4\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^4} + \ldots$$

Mathematica [C] time = 2.75806, size = 409, normalized size = 1.95

$$\frac{-12abx\left(a^2-3b^2\right)\left(3a^2-b^2\right)+6ix\left(-15a^4b^2+15a^2b^4+a^6-b^6\right)-2ab\left(a^2+b^2\right)^2\sin(4x)+16ab\left(a^4-b^4\right)\sin(2x)+\left(a^2\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-12*a*b*(a^2 - 3*b^2)*(3*a^2 - b^2)*x + (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*b
^4 - b^6)*x - (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*ArcTan[Tan[x]] -
4*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*Cos[2*x] + (a^2 - b^2)*(a^2 + b^2)^2*
Cos[4*x] + 3*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*Log[(a*Cos[x] + b*Sin[x]
)^2] + (2*b*(a^2 + b^2)*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[x])/(a*Cos[x] + b*
Sin[x]) + (3*(a^2 + b^2)^2*(a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 + b^2)*L
og[(a*Cos[x] + b*Sin[x])^2]) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I + x)) +
(-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^2)*ArcTa
n[Tan[x]]*(a*Cos[x] + b*Sin[x])))/(a*Cos[x] + b*Sin[x]) + 16*a*b*(a^4 - b^4
)*Sin[2*x] - 2*a*b*(a^2 + b^2)^2*Sin[4*x])/(32*(a^2 + b^2)^4)
```

Maple [B] time = 0.108, size = 515, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x)
```

```
[Out] a^3*b^2/(a^2+b^2)^3/(a+b*tan(x))-3*a^4*b^2/(a^2+b^2)^4*ln(a+b*tan(x))+3*a^2
*b^4/(a^2+b^2)^4*ln(a+b*tan(x))+1/2/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^3*a^3
*b^3-3/4/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^3*a*b^5+5/4/(a^2+b^2)^4/(tan(x)^
2+1)^2*tan(x)^3*a^5*b-1/2/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^2*a^6+1/(a^2+b^
2)^4/(tan(x)^2+1)^2*tan(x)^2*a^4*b^2+3/2/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)^
2*a^2*b^4+3/4/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)*a^5*b-1/2/(a^2+b^2)^4/(tan(
x)^2+1)^2*tan(x)*a^3*b^3-5/4/(a^2+b^2)^4/(tan(x)^2+1)^2*tan(x)*a*b^5-1/4/(a
^2+b^2)^4/(tan(x)^2+1)^2*a^6+5/4/(a^2+b^2)^4/(tan(x)^2+1)^2*a^4*b^2+5/4/(a^
2+b^2)^4/(tan(x)^2+1)^2*a^2*b^4-1/4/(a^2+b^2)^4/(tan(x)^2+1)^2*b^6+3/2/(a^2
+b^2)^4*ln(tan(x)^2+1)*a^4*b^2-3/2/(a^2+b^2)^4*ln(tan(x)^2+1)*a^2*b^4-3/4/(
a^2+b^2)^4*arctan(tan(x))*a^5*b+9/2/(a^2+b^2)^4*arctan(tan(x))*a^3*b^3-3/4/
(a^2+b^2)^4*arctan(tan(x))*a*b^5
```

Maxima [B] time = 1.65306, size = 616, normalized size = 2.93

$$-\frac{3\left(a^5b-6\,a^3b^3+ab^5\right)x}{4\left(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8\right)}-\frac{3\left(a^4b^2-a^2b^4\right)\log\left(b\tan\left(x\right)+a\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{3\left(a^4b^2-a^2b^4\right)\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) - 3*(a^4*b^2 - a^2*b^4)*log(b*tan(x) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 1/4*(a^5 - 10*a^3*b^2 + a*b^4 - 3
*(3*a^3*b^2 - a*b^4)*tan(x)^4 - 3*(a^4*b + a^2*b^3)*tan(x)^3 + (2*a^5 - 17*
a^3*b^2 + 5*a*b^4)*tan(x)^2 - (2*a^4*b + a^2*b^3 - b^5)*tan(x))/(a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(x)^5
+ (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(x)^4 + 2*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*tan(x)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(x)^2
 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(x))
```

Fricas [A] time = 0.650174, size = 822, normalized size = 3.91

$$\frac{8\left(a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}\right)\cos\left(x\right)^{5}-8\left(2\,a^{7}+3\,a^{5}b^{2}-ab^{6}\right)\cos\left(x\right)^{3}+\left(5\,a^{7}+21\,a^{5}b^{2}+27\,a^{3}b^{4}-21\,ab^{6}-24\left(a^{6}b-6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^5 - 8*(2*a^7 + 3*a^5*b
^2 - a*b^6)*cos(x)^3 + (5*a^7 + 21*a^5*b^2 + 27*a^3*b^4 - 21*a*b^6 - 24*(a^
6*b - 6*a^4*b^3 + a^2*b^5)*x)*cos(x) - 48*((a^5*b^2 - a^3*b^4)*cos(x) + (a^
4*b^3 - a^2*b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b
^2) + (5*a^6*b - 51*a^4*b^3 - 21*a^2*b^5 + 3*b^7 - 8*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*cos(x)^4 + 24*(a^6*b + 2*a^4*b^3 + a^2*b^5)*cos(x)^2 - 24*(
a^5*b^2 - 6*a^3*b^4 + a*b^6)*x)*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a
^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*
sin(x))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.11637, size = 587, normalized size = 2.8

$$-\frac{3\left(a^{5}b-6\,a^{3}b^{3}+ab^{5}\right)x}{4\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)}+\frac{3\left(a^{4}b^{2}-a^{2}b^{4}\right)\log\left(\tan\left(x\right)^{2}+1\right)}{2\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)}-\frac{3\left(a^{4}b^{3}-a^{2}b^{5}\right)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^{8}b+4\,a^{6}b^{3}+6\,a^{4}b^{5}+4\,a^{2}b^{7}+b^{9}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8) - 3*(a^4*b^3 - a^2*b^5)*log(abs(b*tan(x) + a))/(a^
8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (3*a^4*b^3*tan(x) - 3*a^2*
b^5*tan(x) + 4*a^5*b^2 - 2*a^3*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)*(b*tan(x) + a)) - 1/4*(9*a^4*b^2*tan(x)^4 - 9*a^2*b^4*tan(x)^4 -
5*a^5*b*tan(x)^3 - 2*a^3*b^3*tan(x)^3 + 3*a*b^5*tan(x)^3 + 2*a^6*tan(x)^2 +
 14*a^4*b^2*tan(x)^2 - 24*a^2*b^4*tan(x)^2 - 3*a^5*b*tan(x) + 2*a^3*b^3*tan
(x) + 5*a*b^5*tan(x) + a^6 + 4*a^4*b^2 - 14*a^2*b^4 + b^6)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(x)^2 + 1)^2)
```

3.293 $\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=47

$$\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\tanh^{-1}(\sin(x))}{a}$$

```
[Out] ArcTanh[Sin[x]]/a + (b*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(a*S
qrt[a^2 + b^2])
```

Rubi [A] time = 0.0789635, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 206}

$$\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ArcTanh[Sin[x]]/a + (b*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(a*S
qrt[a^2 + b^2])
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx &= \int \left(\frac{\sec(x)}{a}-\frac{b}{a(b\cos(x)+a\sin(x))}\right)dx\\ &= \frac{\int \sec(x)\,dx}{a}-\frac{b\int \frac{1}{b\cos(x)+a\sin(x)}\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}+\frac{b\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, a\cos(x)-b\sin(x)\right)}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.117298, size = 76, normalized size = 1.62

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((-2*b*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[Co
s[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a
```

Maple [A] time = 0.101, size = 63, normalized size = 1.3

$$\frac{1}{a}\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\frac{1}{a}\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-2\,\frac{b}{\sqrt{a^2+b^2}a}\text{Artanh}\left(1/2\,\frac{2\,b\tan(x/2)-2\,a}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] 1/a*ln(tan(1/2*x)+1)-1/a*ln(tan(1/2*x)-1)-2*b/a/(a^2+b^2)^(1/2)*arctanh(1/2
*(2*b*tan(1/2*x)-2*a)/(a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.590684, size = 347, normalized size = 7.38

$$\frac{\sqrt{a^2+b^2}b\log\left(\frac{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2-a^2-2\,b^2-2\,\sqrt{a^2+b^2}(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2}\right)+\left(a^2+b^2\right)\log\left(\sin\left(x\right)+1\right)-\left(a^2+b^2\right)\log\left(-\mathrm{si}\right.}{2\left(a^3+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^
2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) -
 (a^2 - b^2)*cos(x)^2 + a^2)) + (a^2 + b^2)*log(sin(x) + 1) - (a^2 + b^2)*l
og(-sin(x) + 1))/(a^3 + a*b^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tan\left(x\right)}{a\sin\left(x\right)+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Integral(tan(x)/(a*sin(x) + b*cos(x)), x)
```

Giac [B] time = 1.2445, size = 122, normalized size = 2.6

$$\frac{b\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] b*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(tan(1/2*x) + 1))/a -
log(abs(tan(1/2*x) - 1))/a
```

3.294 $\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=48

$$\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{b}$$

```
[Out] -(ArcTanh[Cos[x]]/b) + (a*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(
b*Sqrt[a^2 + b^2])
```

Rubi [A] time = 0.0783539, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3110, 3770, 3074, 206}

$$\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/b) + (a*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(
b*Sqrt[a^2 + b^2])
```

Rule 3110

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[(cos[c + d*x]^m*sin[c + d*x]^n)/(a*cos[c + d*x] + b*sin[c + d*x]),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3074

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> -Dist[d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx &= \int\left(\frac{\csc(x)}{b}-\frac{a}{b(b\cos(x)+a\sin(x))}\right)dx\\
&= \frac{\int \csc(x)\,dx}{b}-\frac{a\int \frac{1}{b\cos(x)+a\sin(x)}\,dx}{b}\\
&= -\frac{\tanh^{-1}(\cos(x))}{b}+\frac{a\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,a\cos(x)-b\sin(x)\right)}{b}\\
&= -\frac{\tanh^{-1}(\cos(x))}{b}+\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A] time = 0.0829771, size = 60, normalized size = 1.25

$$\frac{-\frac{2a\tanh^{-1}\left(\frac{b\tan\left(\frac{x}{2}\right)-a}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ((-2*a*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[Co
s[x/2]] + Log[Sin[x/2]])/b
```

Maple [A] time = 0.106, size = 49, normalized size = 1.

$$\frac{1}{b}\ln\left(\tan\left(\frac{x}{2}\right)\right)-2\,\frac{a}{b\sqrt{a^2+b^2}}\text{Artanh}\left(1/2\,\frac{2\,b\tan(x/2)-2\,a}{\sqrt{a^2+b^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] 1/b*ln(tan(1/2*x))-2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*b*tan(1/2*x)-2*a)/(
a^2+b^2)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 0.573723, size = 363, normalized size = 7.56

$$\frac{\sqrt{a^2+b^2}a\log\left(\frac{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2-a^2-2\,b^2-2\,\sqrt{a^2+b^2}(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-\left(a^2-b^2\right)\cos(x)^2+a^2}\right)-\left(a^2+b^2\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+\left(a^2+b^2\right)\log(}{2\left(a^2b+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^
2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) -
 (a^2 - b^2)*cos(x)^2 + a^2)) - (a^2 + b^2)*log(1/2*cos(x) + 1/2) + (a^2 +
b^2)*log(-1/2*cos(x) + 1/2))/(a^2*b + b^3)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cot(x)}{a\sin(x)+b\cos(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Integral(cot(x)/(a*sin(x) + b*cos(x)), x)
```

Giac [A] time = 1.1932, size = 101, normalized size = 2.1

$$\frac{a\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) + log(abs(tan(1/2*x)))/b
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
```

```
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
```

```

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal, debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=", ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
```

```
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
     as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
```

```
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
```

```
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
```

```
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
```

```
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
    Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
```

```
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len(
    flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                     #since this estimate of leaf count is bit lower than
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```
def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
```

```
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
```

```
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, " leaf_count_optimal=",leaf_count_optimal)
```

```


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```